Symmetries of the nuclear shell model

The nuclear shell model

Racah's pairing model and seniority

Wigner's supermultiplet model

Elliott's SU(3) model and extensions

École Joliot Curie, September 2010

The nuclear shell model

Many-body quantum mechanical problem:

$$\hat{H} = \sum_{k=1}^{A} \frac{p_k^2}{2m_k} + \sum_{k<l}^{A} \hat{V}_2(\boldsymbol{r}_k, \boldsymbol{r}_l)$$

$$= \underbrace{\sum_{k=1}^{A} \left[\frac{p_k^2}{2m_k} + \hat{V}(\boldsymbol{r}_k) \right]}_{\text{mean field}} + \underbrace{\left[\sum_{k<l}^{A} \hat{V}_2(\boldsymbol{r}_k, \boldsymbol{r}_l) - \sum_{k=1}^{A} V(\boldsymbol{r}_k) \right]}_{\text{residual interaction}}$$

Independent-particle assumption. Choose *V* and neglect residual interaction:

$$\hat{H} \approx \hat{H}_{\mathrm{IP}} = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m_k} + \hat{V}(\boldsymbol{r}_k) \right]$$

Independent-particle shell model

Solution for one particle:

$$\left[\frac{p^2}{2m}+\hat{V}(\boldsymbol{r})\right]\phi_i(\boldsymbol{r})=E_i\phi_i(\boldsymbol{r})$$

Solution for many particles:

$$\Phi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A)=\prod_{k=1}^{A}\phi_{i_k}(\boldsymbol{r}_k)$$

$$\hat{H}_{\mathrm{IP}}\Phi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A)=\left(\sum_{k=1}^{A}E_{i_k}\right)\Phi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A)$$

Independent-particle shell model

Anti-symmetric solution for many particles (Slater determinant):

$$\Psi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A)=\frac{1}{\sqrt{A!}}\begin{vmatrix}\phi_{i_1}(\boldsymbol{r}_1) & \phi_{i_1}(\boldsymbol{r}_2) & \cdots & \phi_{i_1}(\boldsymbol{r}_A)\\ \phi_{i_2}(\boldsymbol{r}_1) & \phi_{i_2}(\boldsymbol{r}_2) & \cdots & \phi_{i_2}(\boldsymbol{r}_A)\\ \vdots & \vdots & \ddots & \vdots \\ \phi_{i_A}(\boldsymbol{r}_1) & \phi_{i_A}(\boldsymbol{r}_2) & \cdots & \phi_{i_A}(\boldsymbol{r}_A)\end{vmatrix}$$

Example for *A=2* particles:

$$\Psi_{i_1 i_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\frac{1}{\sqrt{2}}\left[\phi_{i_1}(\boldsymbol{r}_1)\phi_{i_2}(\boldsymbol{r}_2)-\phi_{i_1}(\boldsymbol{r}_2)\phi_{i_2}(\boldsymbol{r}_1)\right]$$

Hartree-Fock approximation

Vary ϕ_i (*i.e. V*) to minimize the expectation value of *H* in a Slater determinant:

$$\delta \frac{\int \Psi^*_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) \hat{H} \Psi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) d\boldsymbol{r}_1 d\boldsymbol{r}_2 \ldots d\boldsymbol{r}_A}{\int \Psi^*_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) \Psi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) d\boldsymbol{r}_1 d\boldsymbol{r}_2 \ldots d\boldsymbol{r}_A} = 0$$

Application requires choice of *H*. Many global parametrizations (Skyrme, Gogny,…) have been developed.

Poor man's Hartree-Fock

Choose a simple, analytically solvable *V* that approximates the microscopic HF potential:

$$\hat{H}_{\text{IP}} = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m} + \frac{m\omega^2}{2} r_k^2 - \zeta \boldsymbol{l}_k \cdot \boldsymbol{s}_k - \kappa l_k^2 \right]$$

Contains

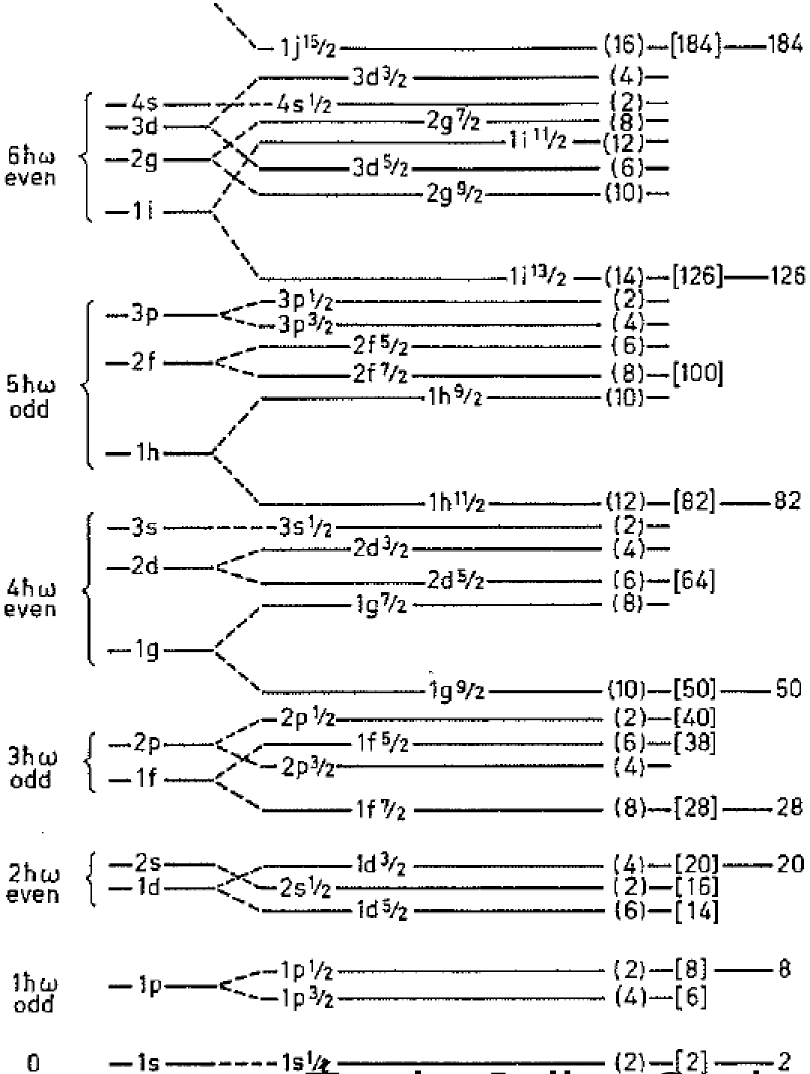
Harmonic oscillator potential with constant ω.

Spin-orbit term with strength ζ.

Orbit-orbit term with strength κ.

Adjust ω, ζ and κ to best reproduce HF.

Single-particle energy levels



Typical parameter values:

$$\hbar\omega \approx 41\, A^{-1/3}\ \text{MeV}$$

$$\zeta\hbar^2 \approx 20\, A^{-2/3}\ \text{MeV}$$

$$\kappa\hbar^2 \approx 0.1\,\text{MeV}$$

$$\therefore b \approx 1.0\, A^{1/6}\ \text{fm}$$

‘Magic’ numbers at 2, 8, 20, 28, 50, 82, 126, 184,…

The nuclear shell model

Hamiltonian with one-body term (mean field) and two-body (residual) interactions:

$$\hat{H}_{\mathrm{SM}} = \sum_{k=1}^{A} \hat{U}(\xi_k) + \sum_{1 \le k < l}^{A} \hat{W}_2(\xi_k, \xi_l)$$

Entirely equivalent form of the same hamiltonian in second quantization:

$$\hat{H}_{\mathrm{SM}} = \sum_{i} \varepsilon_i a_i^{+} a_i + \frac{1}{4} \sum_{ijkl} \upsilon_{ijlk} a_i^{+} a_j^{+} a_k a_l$$

ε, υ: single-particle energies & interactions

ijkl: single-particle quantum numbers

Symmetries of the shell model

Three *bench-mark* solutions:

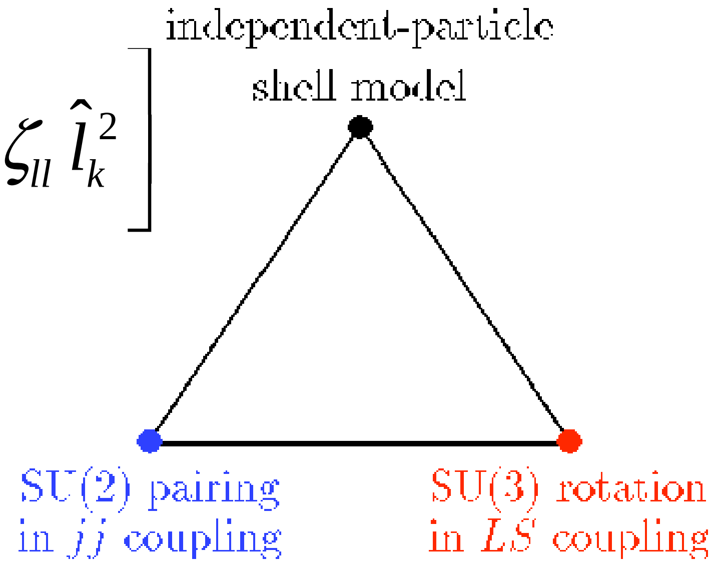
No residual interaction $\Rightarrow$ *IP shell model.*

Pairing (in jj coupling) $\Rightarrow$ *Racah's SU(2).*

Quadrupole (in LS coupling) $\Rightarrow$ *Elliott's SU(3).*

Symmetry triangle:

$$\hat{H} = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m} + \frac{1}{2} m\omega^2 r_k^2 - \zeta_{ls} \hat{\boldsymbol{l}}_k \cdot \hat{\boldsymbol{s}}_k - \zeta_{ll} \hat{l}_k^2 \right] + \sum_{1 \le k < l}^{A} \hat{W}_2(\xi_k, \xi_l)$$



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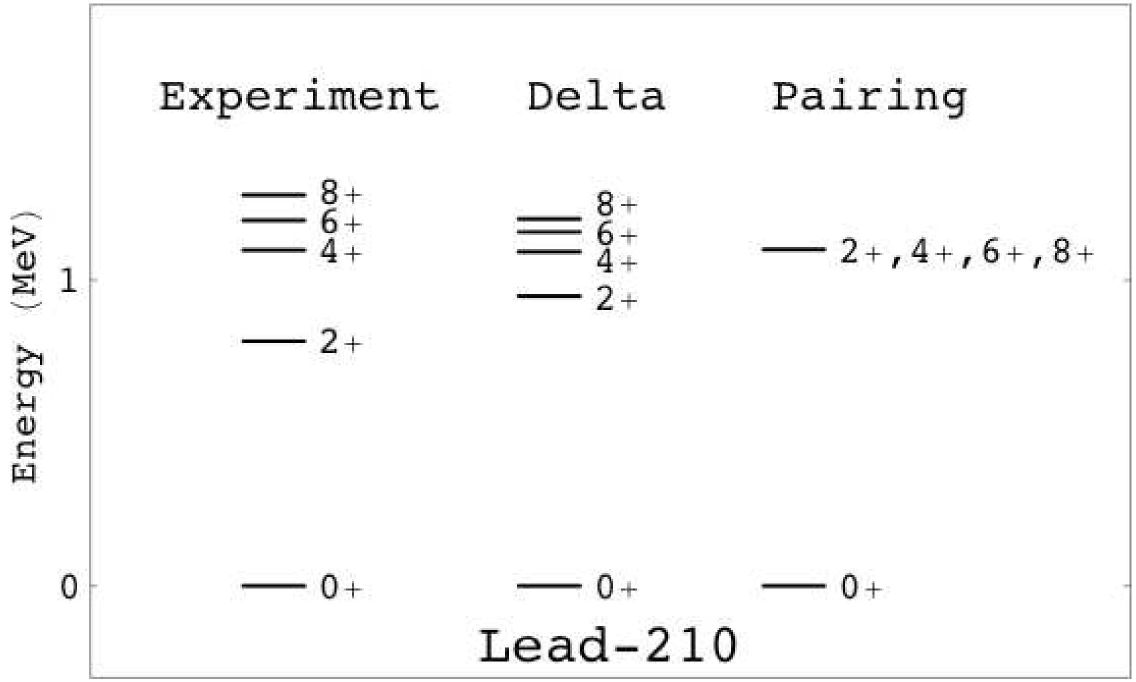
Racah's SU(2) pairing model

Assume pairing interaction in a single-*j* shell:

$$\left\langle j^2 J M_J \left| \hat{V}_{\text{pairing}} \right| j^2 J M_J \right\rangle = \begin{cases} -\frac{1}{2}(2j+1)g_0, & J = 0 \\ 0, & J \neq 0 \end{cases}$$

Spectrum ^{210}Pb:



G. Racah, Phys. Rev. **63** (1943) 367

Pairing SU(2) dynamical symmetry

The pairing hamiltonian,

$$\hat{H} = -g_0 \hat{S}_+ \cdot \hat{S}_-, \quad \hat{S}_+ = \tfrac{1}{2}\sum_m a^+_{jm} a^+_{j\bar{m}}, \quad \hat{S}_- = \left(\hat{S}_+\right)^+$$

…has a *quasi-spin* SU(2) algebraic structure:

$$\left[\hat{S}_+, \hat{S}_-\right] = \tfrac{1}{2}\left(2\hat{n} - 2j - 1\right) \equiv -2\hat{S}_z, \quad \left[\hat{S}_z, \hat{S}_\pm\right] = \pm\hat{S}_\pm$$

H has SU(2) ⊃ SO(2) dynamical symmetry:

$$-g_0 \hat{S}_+ \cdot \hat{S}_- = -g_0\left(\hat{S}^2 - \hat{S}_z^2 + \hat{S}_z\right)$$

Eigensolutions of pairing hamiltonian:

$$-g_0 \hat{S}_+ \cdot \hat{S}_- \left|SM_S\right\rangle = -g_0\left(S(S+1) - M_S\left(M_S - 1\right)\right)\left|SM_S\right\rangle$$

A. Kerman, Ann. Phys. (NY) **12** (1961) 300
K. Helmers, Nucl. Phys. **23** (1961) 594

Interpretation of pairing solution

Quasi-spin labels *S* and M_S are related to nucleon number *n* and seniority υ:

$$S = \tfrac{1}{4}(2j - \upsilon + 1), \quad M_S = \tfrac{1}{4}(2n - 2j - 1)$$

Energy eigenvalues in terms of *n, j* and υ:

$$\left\langle j^n \upsilon J M_J \right| -g_0 \hat{S}_+ \cdot \hat{S}_- \left| j^n \upsilon J M_J \right\rangle = -g_0 \tfrac{1}{4}(n - \upsilon)(2j - n + \upsilon + 3)$$

Eigenstates have an *S*-pair character:

$$\left| j^n \upsilon J M_J \right\rangle \propto \left(\hat{S}_+\right)^{(n-\upsilon)/2} \left| j^\upsilon \upsilon J M_J \right\rangle$$

Seniority υ is the number of nucleons *not* in *S* pairs (pairs coupled to *J*=0).

Pairing between identical nucleons

Analytic solution of the pairing hamiltonian based on SU(2) symmetry. *E.g.* energies:

$$\left\langle j^n \upsilon J \left| \sum_{1\le k<l}^{n} \hat{V}_{\text{pairing}}(k,l) \right| j^n \upsilon J \right\rangle = -g_0 \tfrac{1}{4}(n-\upsilon)(2j-n-\upsilon+3)$$

Seniority υ (number of nucleons not in pairs coupled to J=0) is a good quantum number.

Correlated ground-state solution (*cf.* BCS).

Nuclear superfluidity

Ground states of pairing hamiltonian have the following *correlated* character:

Even-even nucleus (υ=0): $\left(\hat{S}_+\right)^{n/2}|\mathrm{o}\rangle, \quad \hat{S}_+ = \sum_m a^+_{m\downarrow} a^+_{\bar{m}\uparrow}$

Odd-mass nucleus (υ=1): $a^+_{m\updownarrow}\left(\hat{S}_+\right)^{n/2}|\mathrm{o}\rangle$

Nuclear superfluidity leads to

Constant energy of first 2^+ in even-even nuclei.

Odd-even staggering in masses.

Smooth variation of two-nucleon separation energies with nucleon number.

Two-particle (2n or 2p) transfer enhancement.

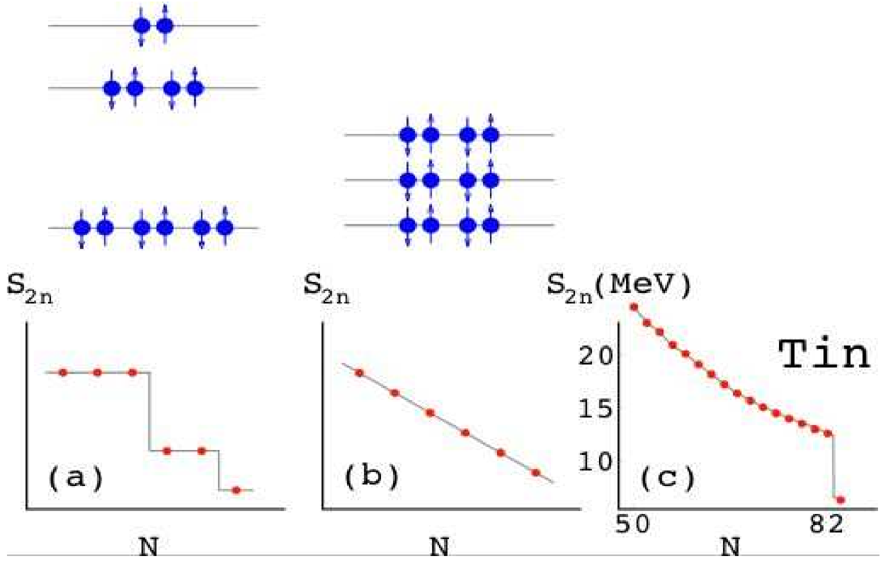
Two-nucleon separation energies

Two-nucleon separation energies S_{2n}:

(a) Shell splitting dominates over interaction.

(b) Interaction dominates over shell splitting.

(c) S_{2n} in tin isotopes.



Pairing gap in semi-magic nuclei

Even-even nuclei:

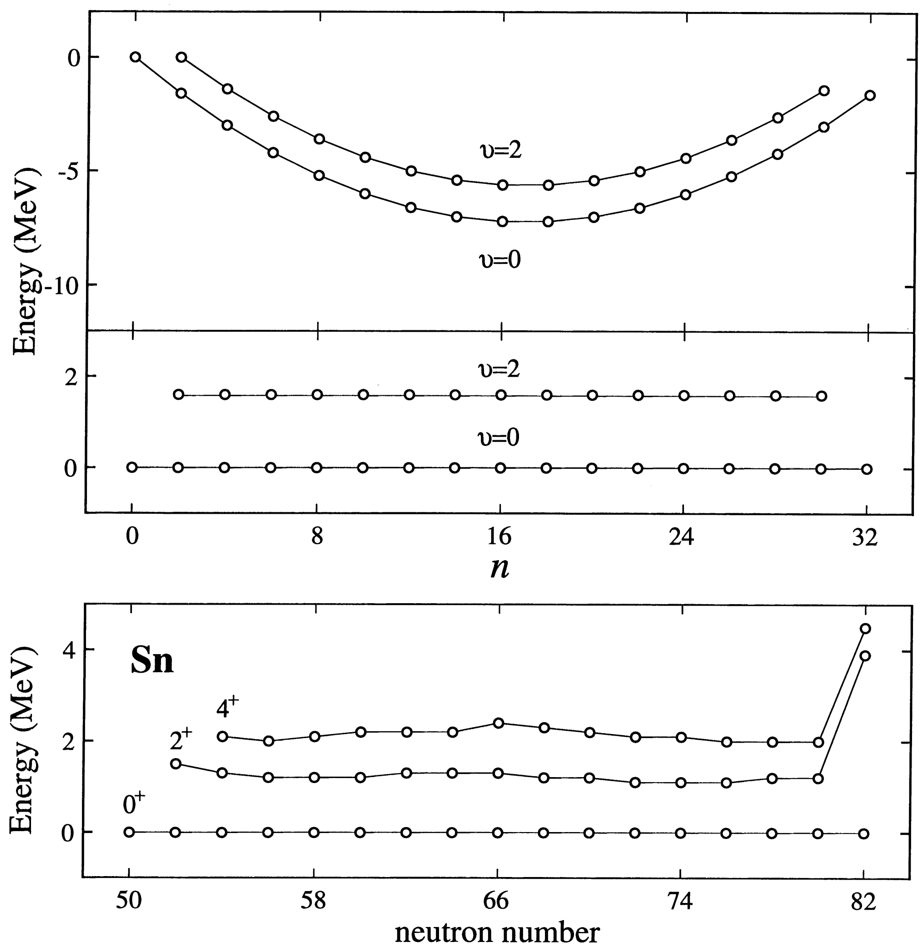
Ground state: υ=0.

First-excited state: υ=2.

Pairing produces constant excitation energy:

$$E_x\left(2_1^+\right)=\frac{1}{2}(2j+1)g_0$$

Example of Sn isotopes:



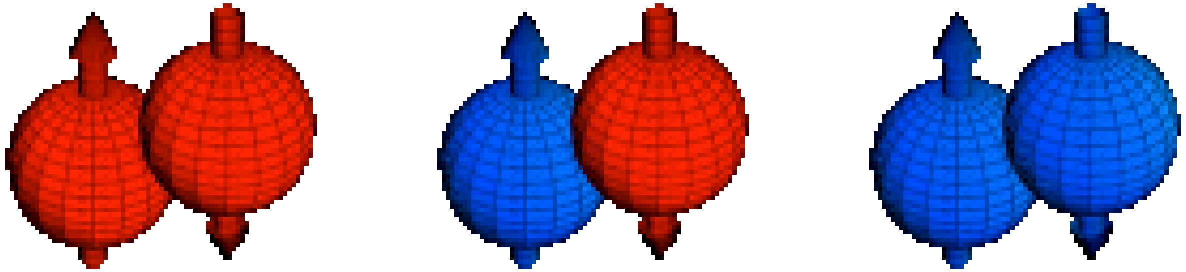
Generalized seniority models

- Trivial generalization from a single-*j* shell to several **degenerate** *j* shells.
- Pairing with neutrons and protons (**isospin**):
  - *SO(5) T=1 pairing (Racah, Flowers, Hecht).*
  - *SO(8) T=0 & T=1 pairing (Flowers and Szpikowski).*
- **Non-degenerate** shells:
  - *Generalized seniority (Talmi).*
  - *Integrable pairing models (Richardson, Gaudin, Dukelsky).*

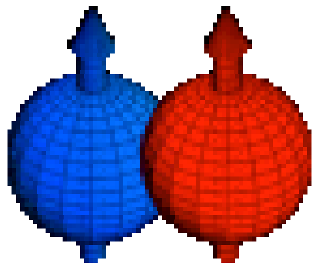
Pairing with neutrons and protons

For neutrons and protons *two* pairs and hence *two* pairing interactions are possible:

$^1S_0$ *isovector or spin singlet (S=0,T=1):* $\hat{S}_+ = \sum_{m>0} a^+_{m\downarrow} a^+_{\bar{m}\uparrow}$



$^3S_1$ *isoscalar or spin triplet (S=1,T=0):* $\hat{P}_+ = \sum_{m>0} a^+_{m\uparrow} a^+_{\bar{m}\uparrow}$



Neutron-proton pairing hamiltonian

The nuclear hamiltonian has two pairing interactions

$$\hat{V}_{\text{pairing}} = -g_0 \hat{S}_+ \cdot \hat{S}_- - g_1 \hat{P}_+ \cdot \hat{P}_-$$

SO(8) algebraic structure.

Integrable and solvable for g_0=0, g_1=0 and g_0=g_1.

B.H. Flowers & S. Szpikowski, Proc. Phys. Soc. **84** (1964) 673

Quartetting in *N*=*Z* nuclei

Pairing ground state of an *N*=*Z* nucleus:

$$\left(\cos\theta\hat{S}_+\cdot\hat{S}_+ - \sin\theta\hat{P}_+\cdot\hat{P}_+\right)^{n/4}|\mathrm{o}\rangle$$

$\Rightarrow$ Condensate of "α-like" objects.

Observations:

Isoscalar component in condensate survives only in N≈Z nuclei, if anywhere at all.

Spin-orbit term reduces *isoscalar component.*

J. Dobes & S. Pittel, Phys. Rev. C **57** (1998) 6883

Wigner's SU(4) symmetry

Assume the nuclear hamiltonian is invariant under spin *and* isospin rotations:

$$\left[\hat{H}_{\mathrm{nucl}},\hat{S}_{\mu}\right]=\left[\hat{H}_{\mathrm{nucl}},\hat{T}_{\nu}\right]=\left[\hat{H}_{\mathrm{nucl}},\hat{Y}_{\mu\nu}\right]=0$$

$$\hat{S}_{\mu}=\sum_{k=1}^{A}\hat{s}_{\mu}(k),\quad \hat{T}_{\nu}=\sum_{k=1}^{A}\hat{t}_{\nu}(k),\quad \hat{Y}_{\mu\nu}=\sum_{k=1}^{A}\hat{s}_{\mu}(k)\hat{t}_{\nu}(k)$$

Since $\{S_{\mu}, T_{\nu}, Y_{\mu\nu}\}$ form an SU(4) algebra:

- H_{nucl} *has SU(4) symmetry.*
- *Total spin S, total orbital angular momentum L, total isospin T and SU(4) labels (λ,μ,ν) are conserved quantum numbers.*

E.P. Wigner, Phys. Rev. **51** (1937) 106
F. Hund, Z. Phys. **105** (1937) 202

Physical origin of SU(4) symmetry

SU(4) labels specify the separate spatial and spin-isospin symmetry of the wave function.

Nuclear interaction is short-range attractive and hence *favours maximal spatial symmetry*.

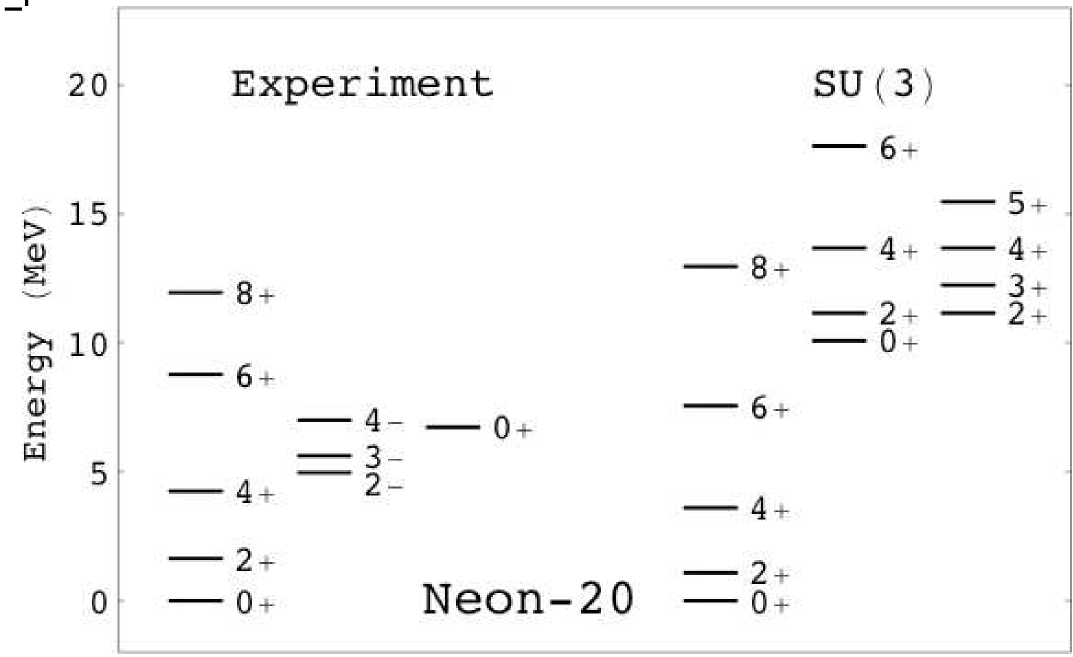
particle number	spatial symmetry	L	spin–isospin symmetry	$(\lambda\mu\nu)$	(S,T)
1		$0, 2$		(100)	$(\frac{1}{2}, \frac{1}{2})$
2	(S)	$0^2, 2^2, 4$	(A)	(010)	(0,1) (1,0)
	(A)	$1, 2, 3$	(S)	(200)	(0,0) (1,1)

Elliott's SU(3) model of rotation

Harmonic oscillator mean field (*no* spin-orbit) with residual interaction of quadrupole type:

$$\hat{H} = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m} + \frac{1}{2} m\omega^2 r_k^2 \right] - g_2 \hat{Q} \cdot \hat{Q},$$

$$\hat{Q}_\mu \propto \sum_{k=1}^{A} r_k^2 Y_{2\mu}(\hat{\boldsymbol{r}}_k) + \sum_{k=1}^{A} p_k^2 Y_{2\mu}(\hat{\boldsymbol{p}}_k)$$



J.P. Elliott, Proc. Roy. Soc. A **245** (1958) 128; 562

Importance & limitations of SU(3)

Historical importance:

Bridge between the spherical shell model and the liquid-drop model through mixing of orbits.

Spectrum generating algebra of Wigner's SU(4) model.

Limitations:

LS (Russell-Saunders) coupling, not jj coupling (no spin-orbit splitting) $\Rightarrow$ *(beginning of) sd shell.*

Q is the algebraic quadrupole operator $\Rightarrow$ *no major-shell mixing.*

Breaking of SU(4) symmetry

SU(4) symmetry breaking as a consequence of

- *Spin-orbit term in nuclear mean field.*
- *Coulomb interaction.*
- *Spin-dependence of the nuclear interaction.*

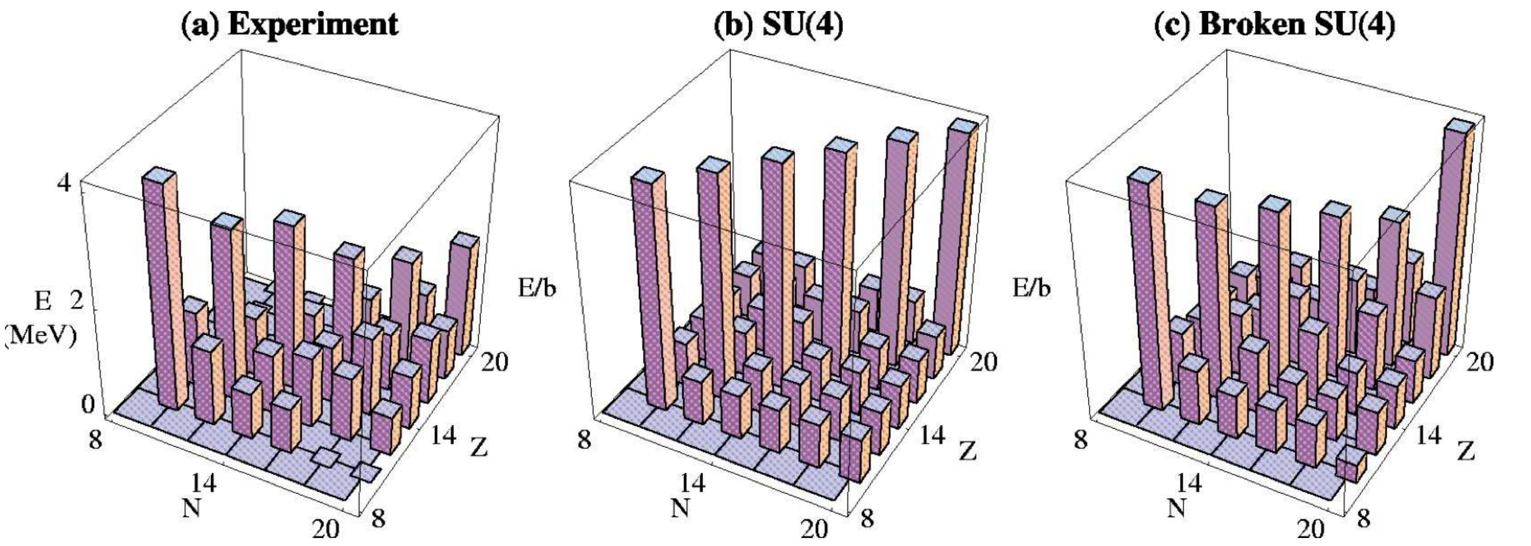
Evidence for SU(4) symmetry breaking from masses and from Gamow-Teller β decay.

SU(4) breaking from masses

Double binding energy difference δV_{np}

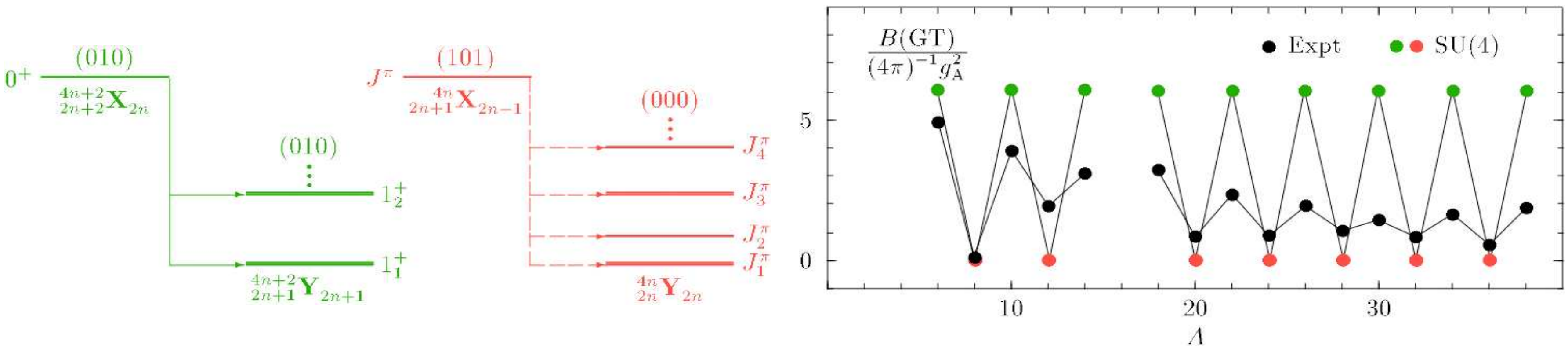
$$\delta V_{\mathrm{np}}(N,Z)=\tfrac{1}{4}\left[B(N,Z)-B(N-2,Z)-B(N,Z-2)+B(N-2,Z-2)\right]$$

δV_{np} in *sd*-shell nuclei:



P. Van Isacker *et al.*, Phys. Rev. Lett. **74** (1995) 4607

SU(4) breaking from β decay



Gamow-Teller decay into odd-odd or even-even N=Z nuclei.

P. Halse & B.R. Barrett, Ann. Phys. (NY) **192** (1989) 204

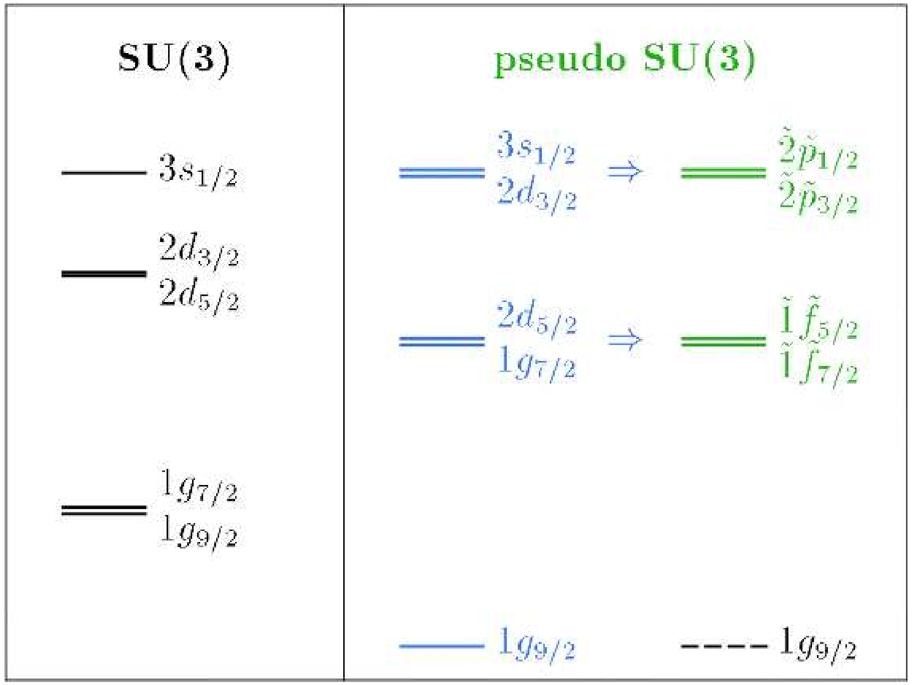
Pseudo-spin symmetry

Apply a *helicity* transformation to the spin-orbit + orbit-orbit nuclear mean field:

$$\hat{u}_k^{-1}\left(\zeta\hat{\boldsymbol{l}}_k\cdot\hat{\boldsymbol{s}}_k+\kappa\hat{\boldsymbol{l}}_k\cdot\hat{\boldsymbol{l}}_k\right)\hat{u}_k=(4\zeta-\kappa)\hat{\boldsymbol{l}}_k\cdot\hat{\boldsymbol{s}}_k+\kappa\hat{\boldsymbol{l}}_k\cdot\hat{\boldsymbol{l}}_k+\mathrm{c}^{\mathrm{te}}$$

$$\hat{u}_k=2i\frac{\hat{\boldsymbol{s}}_k\cdot\boldsymbol{p}_k}{p_k}$$

Degeneracies for $4\zeta=\kappa$.



K.T. Hecht & A. Adler, Nucl. Phys. A **137** (1969) 129

A. Arima *et al.*, Phys. Lett. B **30** (1969) 517

R.D. Ratna *et al.*, Nucl. Phys. A **202** (1973) 433

J.N. Ginocchio, Phys. Rev. Lett. **78** (1998) 436

Pseudo-SU(4) symmetry

Assume the nuclear hamiltonian is invariant under *pseudo*-spin and isospin rotations:

$$\left[\hat{H}_{\text{nucl}},\hat{\tilde{S}}_{\mu}\right]=\left[\hat{H}_{\text{nucl}},\hat{T}_{\nu}\right]=\left[\hat{H}_{\text{nucl}},\hat{\tilde{Y}}_{\mu\nu}\right]=0$$

$$\hat{\tilde{S}}_{\mu}=\sum_{k=1}^{A}\hat{\tilde{s}}_{\mu}(k),\quad \hat{T}_{\nu}=\sum_{k=1}^{A}\hat{t}_{\nu}(k),\quad \hat{\tilde{Y}}_{\mu\nu}=\sum_{k=1}^{A}\hat{\tilde{s}}_{\mu}(k)\hat{t}_{\nu}(k)$$

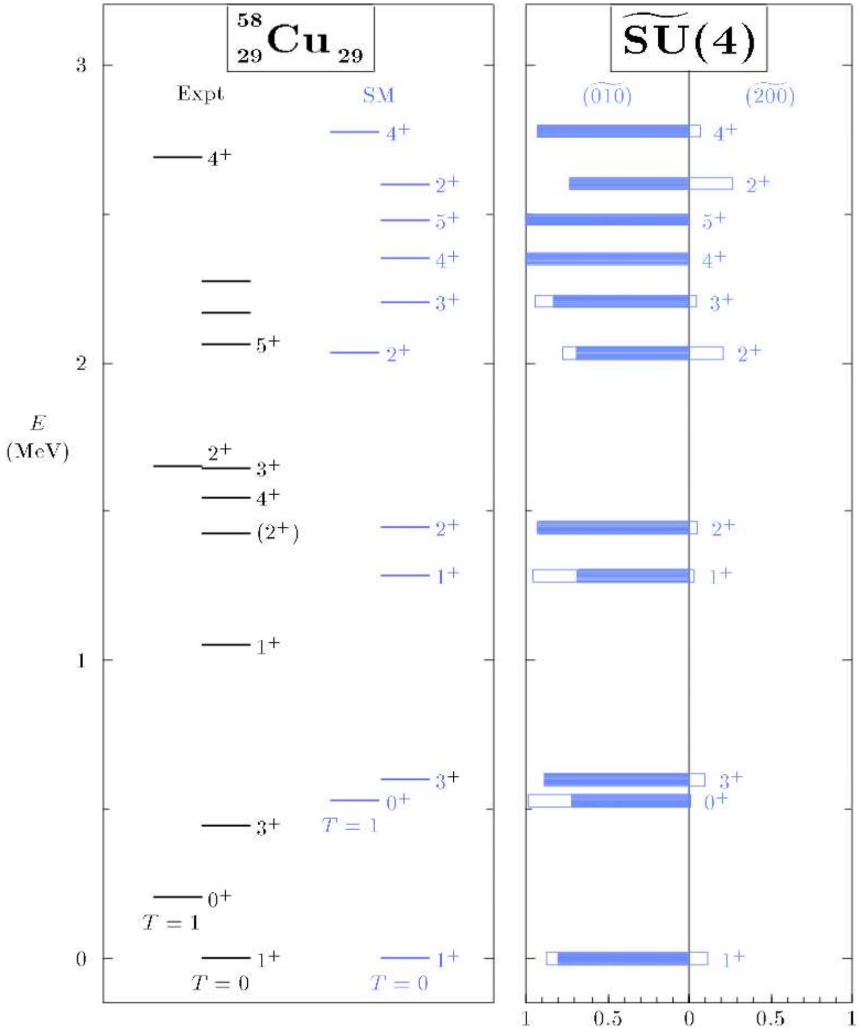
Consequences:

Hamiltonian has pseudo-SU(4) symmetry.

Total pseudo-spin, total pseudo-orbital angular momentum, total isospin and pseudo-SU(4) labels are conserved quantum numbers.

D. Strottman, Nucl. Phys. A **188** (1971) 488

Test of pseudo-SU(4) symmetry



Shell-model test of pseudo-SU(4).

Realistic interaction in $pf_{5/2}g_{9/2}$ space.

Example: ^{58}Cu.

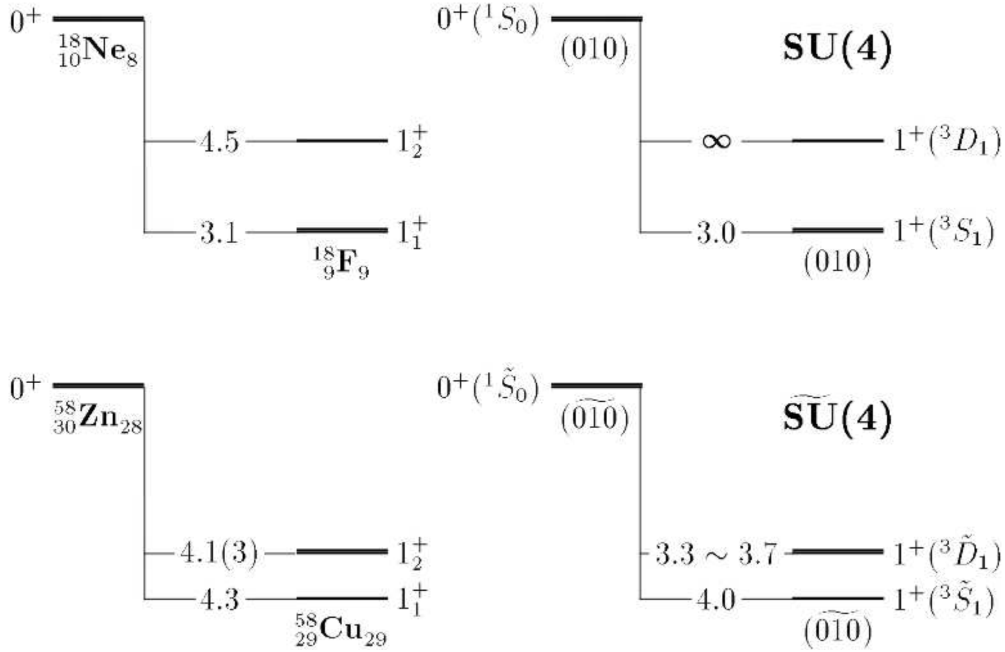
P. Van Isacker *et al.*, Phys. Rev. Lett. **82** (1999) 2060

Pseudo-SU(4) and β decay

Pseudo-spin transformed Gamow-Teller operator is *deformation dependent*:

$$\hat{\tilde{s}}_\mu \hat{t}_\nu \equiv \hat{u}^{-1}\hat{s}_\mu \hat{t}_\nu \hat{u} = -\frac{1}{3}\hat{s}_\mu \hat{t}_\nu + \sqrt{\frac{20}{3}}\,\frac{1}{r^2}\left[(\boldsymbol{r}\times\boldsymbol{r})^{(2)}\times\hat{\boldsymbol{s}}\right]^{(1)}_\mu \hat{t}_\nu$$

Test: β decay of ^{58}Zn.



A. Jokinen *et al.*, Eur. Phys. A. **3** (1998) 271

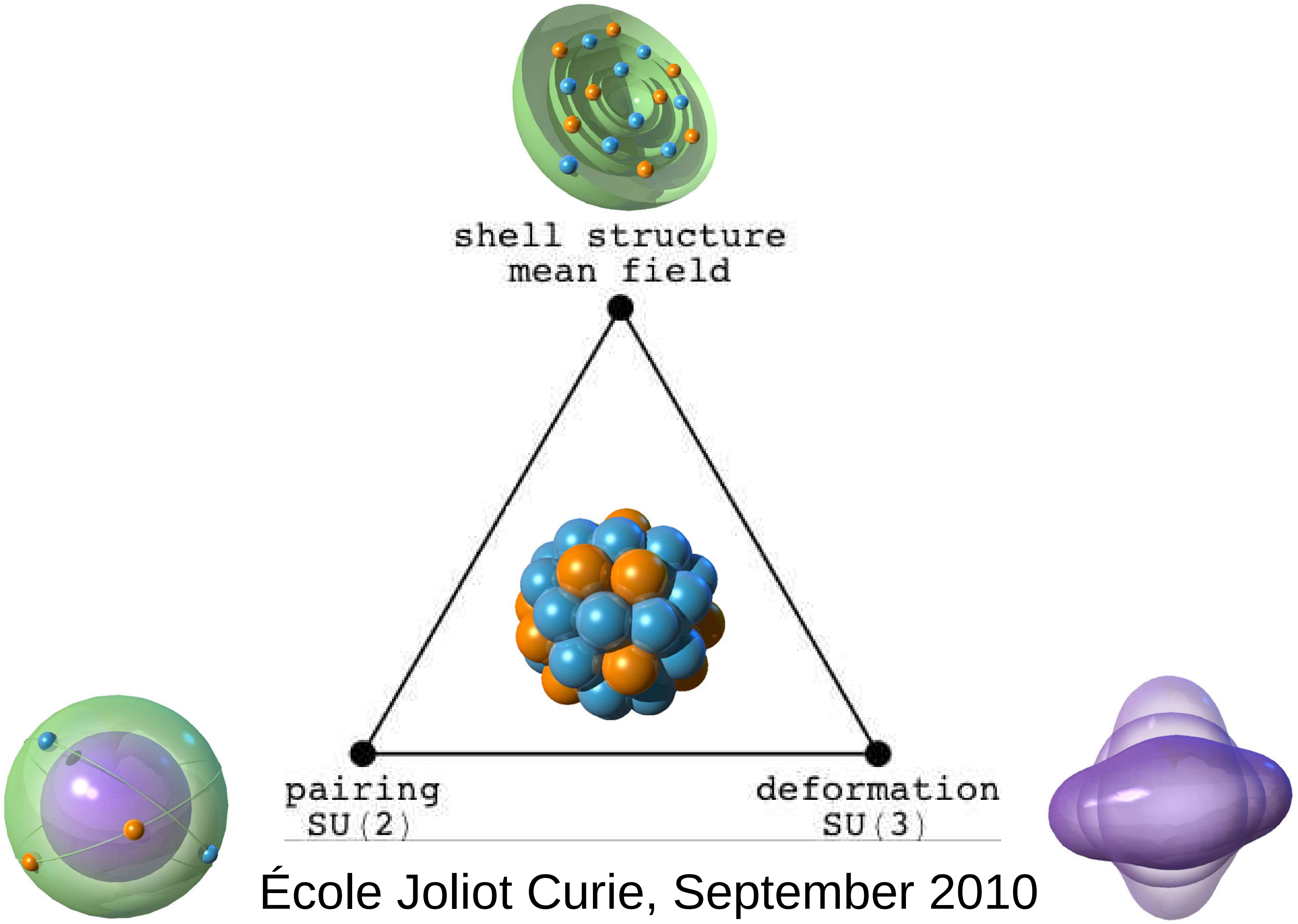
Symmetries in nuclei

Quantum many-body (bosons and/or fermions) systems can be analyzed with algebraic methods.

Two nuclear examples:

- *Pairing vs. quadrupole interaction in the nuclear shell model.*
- *Spherical, deformed and γ-unstable nuclei with s,d-boson IBM.*

Three faces of the shell model



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Boson and fermion statistics

Fermions have half-integer spin and obey Fermi-Dirac statistics:

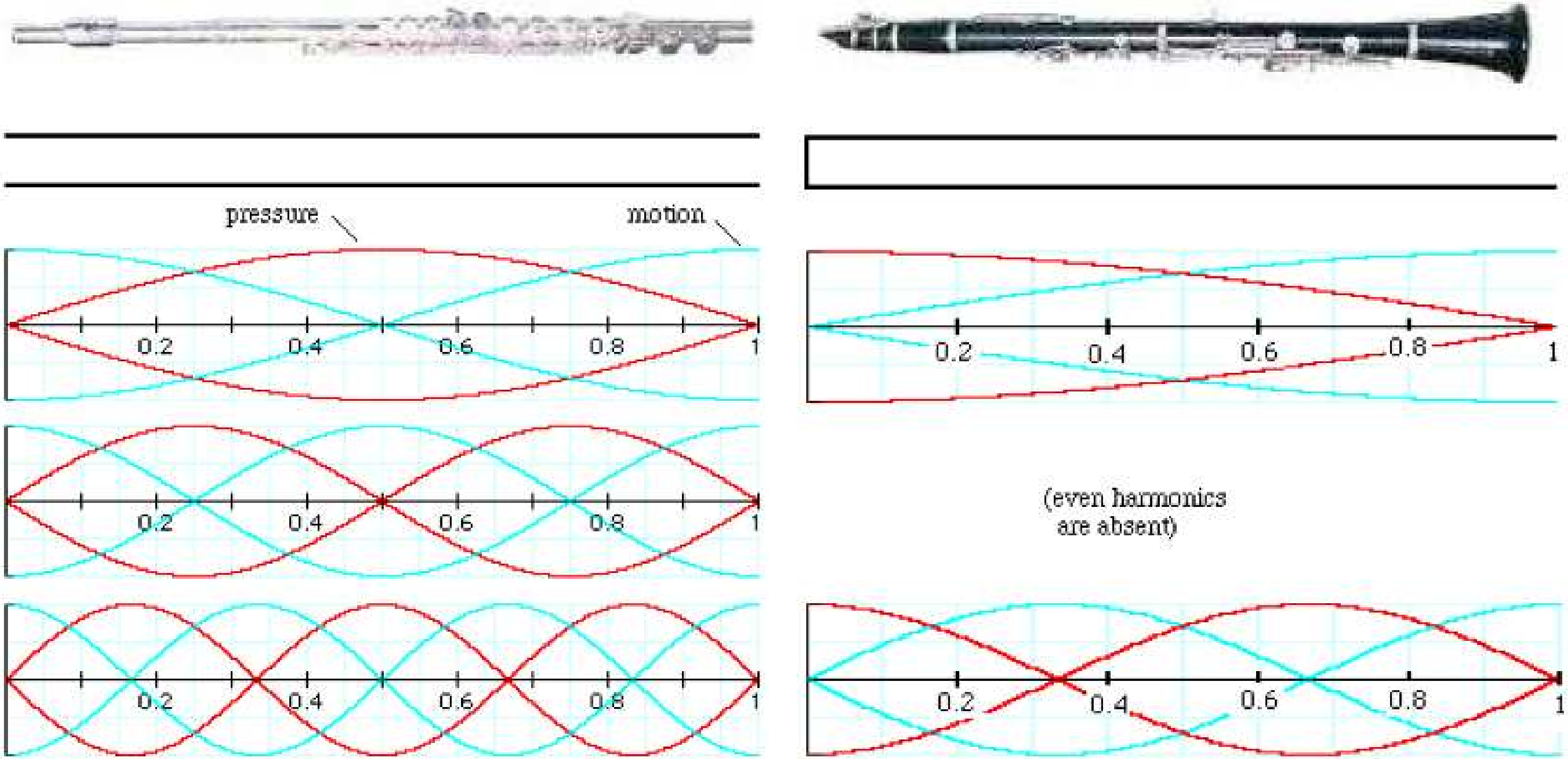
$$\{a_i, a_j^+\} \equiv a_i a_j^+ + a_j^+ a_i = \delta_{ij}, \quad \{a_i, a_j\} = \{a_i^+, a_j^+\} = 0$$

Bosons have integer spin and obey Bose-Einstein statistics:

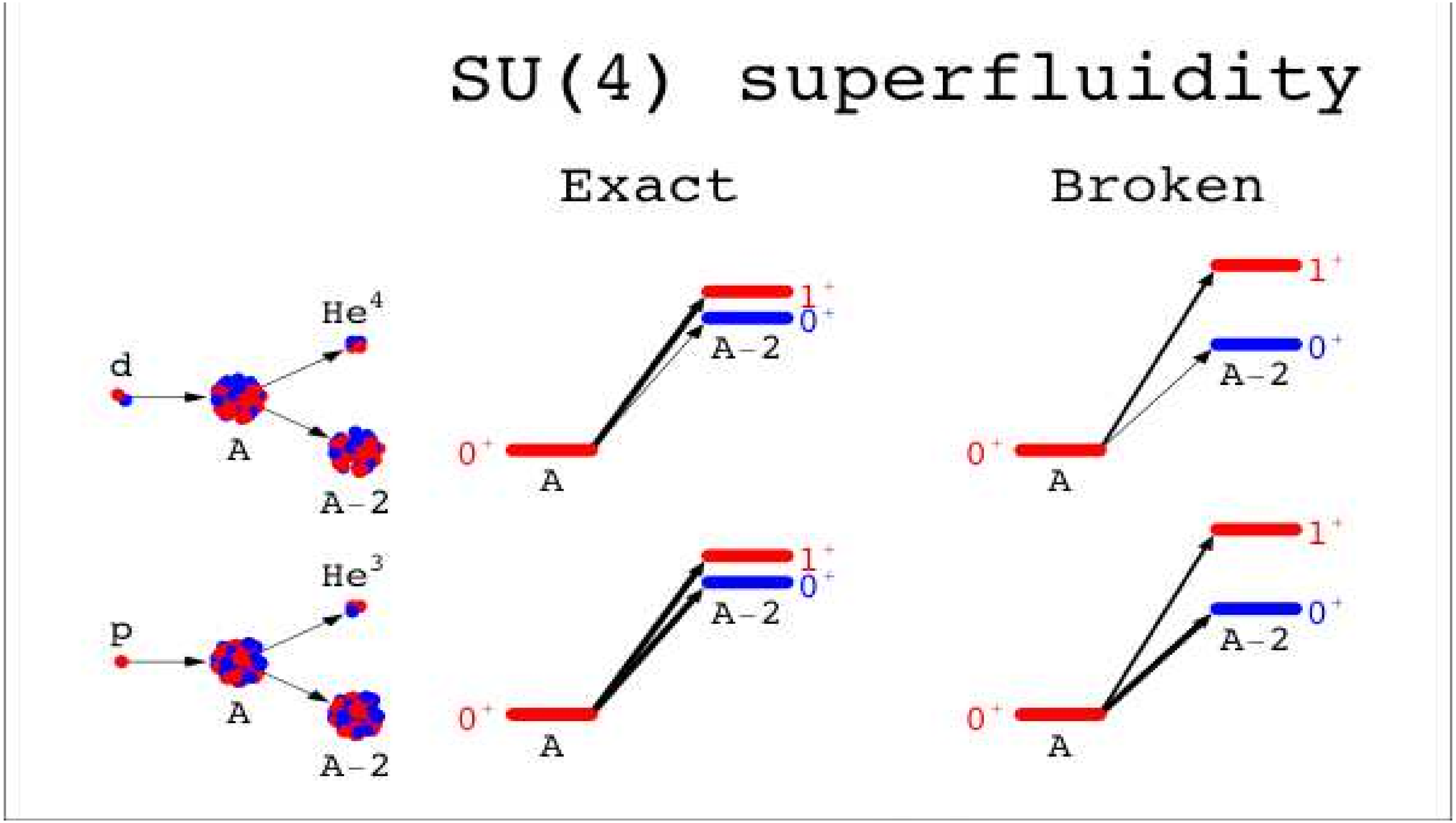
$$[b_i, b_j^+] \equiv b_i b_j^+ - b_j^+ b_i = \delta_{ij}, \quad [b_i, b_j] = [b_i^+, b_j^+] = 0$$

Matter is carried by fermions. Interactions are carried by bosons. Composite matter particles can be fermions or bosons.

Bosons and fermions



(d,α) and (p,^{3}He) transfer



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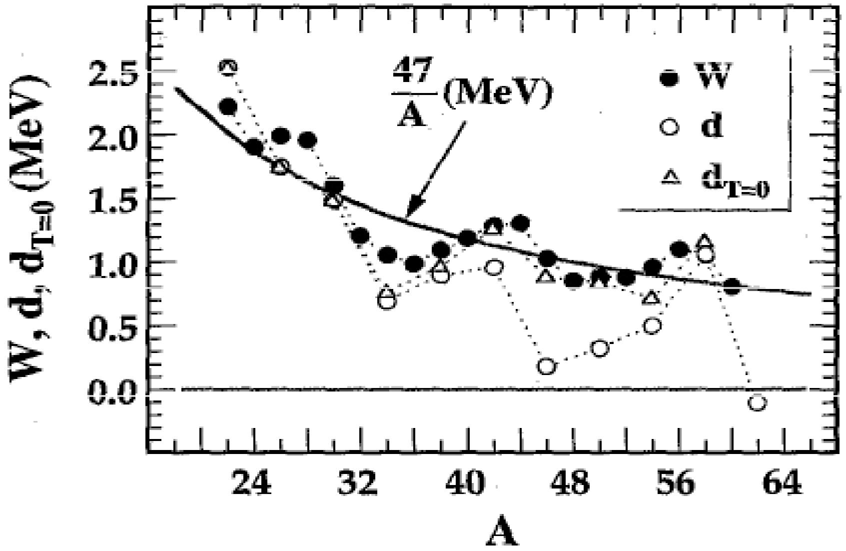
Wigner energy

Extra binding energy of N=Z nuclei (cusp).

Wigner energy B_W is decomposed in two parts:

$$B_W = -W(A)|N-Z| - d(A)\delta_{N,Z}\pi_{np}$$

$W(A)$ and $d(A)$ can be fixed empirically from binding energies.



P. Möller & R. Nix, Nucl. Phys. A **536** (1992) 20
J.-Y. Zhang *et al.*, Phys. Lett. B **227** (1989) 1
W. Satula *et al.*, Phys. Lett. B **407** (1997) 103

Connection with SU(4) model

Wigner's explanation of the 'kinks in the mass defect curve' was based on SU(4) symmetry.

Symmetry contribution to the nuclear binding energy is

$$-K(A)g(\lambda,\mu,\nu)=K(A)\left[(N-Z)^2+8|N-Z|+8\delta_{N,Z}\pi_{\mathrm{np}}+6\delta_{\mathrm{pairing}}\right]$$

SU(4) symmetry is broken by spin-orbit term. Effects of SU(4) mixing must be included.

D.D. Warner *et al.*, Nature Physics **2** (2006) 311

Algebraic definition of seniority

For a system of *n* identical bosons with spin *j*

$$\begin{array}{ccccccc} \mathrm{U}(2j+1) & \supset & \mathrm{SO}(2j+1) & \supset & \cdots & \supset & \mathrm{SO}(3) \\ \downarrow & & \downarrow & & & & \downarrow \\ [n] & & \upsilon & & & & J \end{array}$$

For a system of *n* identical fermions with spin *j*

$$\begin{array}{ccccccc} \mathrm{U}(2j+1) & \supset & \mathrm{Sp}(2j+1) & \supset & \cdots & \supset & \mathrm{SO}(3) \\ \downarrow & & \downarrow & & & & \downarrow \\ [1^n] & & \upsilon & & & & J \end{array}$$

Alternative definition with quasi-spin algebras.

Conservation of seniority

Seniority υ is the number of particles not in pairs coupled to J=0 (Racah).

Conditions for the conservation of seniority by a given (two-body) interaction V can be derived from the analysis of a 3-particle system.

Any interaction between identical fermions with spin j conserves seniority if $j \leq 7/2$.

Any interaction between identical bosons with spin j conserves seniority if $j \leq 2$.

G. Racah, Phys. Rev. **63** (1943) 367
I. Talmi, *Simple Models of Complex Nuclei*

Conservation of seniority

Necessary and sufficient conditions for a two-body interaction v_λ to conserve seniority:

$$\sum_\lambda \sqrt{2\lambda+1}\, a_{jI}^{\lambda} v_\lambda = 0, \quad I = 2,4,\ldots,2\lfloor j \rfloor$$

$$v_\lambda \equiv \left\langle j^2;\lambda \middle| \hat{V} \middle| j^2;\lambda \right\rangle$$

$$a_{jI}^{\lambda} = \delta_{\lambda I} + 2\sqrt{(2\lambda+1)(2I+1)} \begin{Bmatrix} j & j & \lambda \\ j & j & I \end{Bmatrix} - \frac{4\sqrt{(2\lambda+1)(2I+1)}}{(2j+1)(2j+1+2\sigma)}$$

For fermions σ = -1; for bosons σ = +1.

I. Talmi, *Simple Models of Complex Nuclei*

Conservation of seniority

Bosons:

$$j=3: 11v_2 - 18v_4 + 7v_6 = 0,$$

$$j=4: 65v_2 - 30v_4 - 91v_6 + 56v_8 = 0,$$

$$j=5: 3230v_2 - 2717v_6 - 3978v_8 + 3465v_{10} = 0,$$

Fermions:

$$j=9/2: 65v_2 - 315v_4 + 403v_6 - 153v_8 = 0,$$

$$j=11/2: 1020v_2 - 3519v_4 - 637v_6 + 4403v_8 - 2541v_{10} = 0,$$

$$j=13/2: 1615v_2 - 4275v_4 - 1456v_6 + 3196v_8 - 5145v_{10}$$
$$- 4225v_{12} = 0,$$

Is seniority conserved in nuclei?

The interaction between nucleons is “short range”.

A δ interaction is therefore a reasonable approximation to the nucleon two-body force.

A pairing interaction is a further approximation.

Both δ and pairing interaction between **identical** nucleons conserve seniority.

$\therefore$ In **semi-magic** nuclei seniority is conserved to a good approximation.

Partial conservation of seniority

Question: Can we construct interactions for which **some but not all** of the eigenstates have good seniority?

A non-trivial solution occurs for four identical fermions with j=9/2 and J=4 and J=6. These states are solvable for **any** interaction in the j=9/2 shell. They have a wave function which is **independent** of the interactions ν_J.

This finding has relevance for the existence of seniority isomers in nuclei.

P. Van Isacker & S. Heinze, Phys. Rev. Lett. **100** (2008) 052501
L. Zamick & P. Van Isacker, Phys. Rev. C **78** (2008) 044327

Energy matrix for (9/2)^4 J=4

$$\langle a|\hat{V}|a\rangle = \frac{3}{5}v_0 + \frac{67}{99}v_2 + \frac{746}{715}v_4 + \frac{1186}{495}v_6 + \frac{918}{715}v_8,$$

$$\langle a|\hat{V}|b\rangle = \frac{\sqrt{14}\Delta}{495\sqrt{2119}}, \quad \langle a|\hat{V}|c\rangle = \frac{2\sqrt{170}\Delta}{429\sqrt{489}},$$

$$\Delta = -65v_2 + 315v_4 - 403v_6 + 153v_8$$

$$\langle b|\hat{V}|b\rangle = \frac{33161}{16137}v_2 + \frac{1800}{1793}v_4 + \frac{70382}{80685}v_6 + \frac{18547}{8965}v_8,$$

$$\langle b|\hat{V}|c\rangle = \frac{-10\sqrt{595}(13v_2 - 9v_4 - 13v_6 + 9v_8)}{5379\sqrt{39}}$$

$$\langle c|\hat{V}|c\rangle = \frac{2584}{5379}v_2 + \frac{48809}{23309}v_4 + \frac{65809}{26895}v_6 + \frac{114066}{116545}v_8.$$

Energy matrix for $(9/2)^4$ J=6

$$\langle a|\hat{V}|a\rangle = \frac{3}{5}v_0 + \frac{34}{99}v_2 + \frac{1186}{715}v_4 + \frac{658}{495}v_6 + \frac{1479}{715}v_8,$$

$$\langle a|\hat{V}|b\rangle = \frac{-\sqrt{5}\Delta}{1287\sqrt{97}}, \quad \langle a|\hat{V}|c\rangle = \frac{2\sqrt{2261}\Delta}{2145\sqrt{291}},$$

$$\Delta = -65v_2 + 315v_4 - 403v_6 + 153v_8$$

$$\langle b|\hat{V}|b\rangle = \frac{33049}{19206}v_2 + \frac{25733}{27742}v_4 + \frac{19331}{19206}v_6 + \frac{65059}{27742}v_8,$$

$$\langle b|\hat{V}|c\rangle = \frac{5\sqrt{11305}\left(13v_2 - 9v_4 - 13v_6 + 9v_8\right)}{41613\sqrt{3}}$$

$$\langle c|\hat{V}|c\rangle = \frac{1007}{3201}v_2 + \frac{26370}{13871}v_4 + \frac{7723}{3201}v_6 + \frac{19026}{13871}v_8.$$

Energies

Analytic energy expressions:

$$E\left[(9/2)^4, \upsilon=4, J=4\right]=\frac{68}{33}v_2+v_4+\frac{13}{15}v_6+\frac{114}{55}v_8,$$

$$E\left[(9/2)^4, \upsilon=4, J=6\right]=\frac{19}{11}v_2+\frac{12}{13}v_4+v_6+\frac{336}{143}v_8,$$

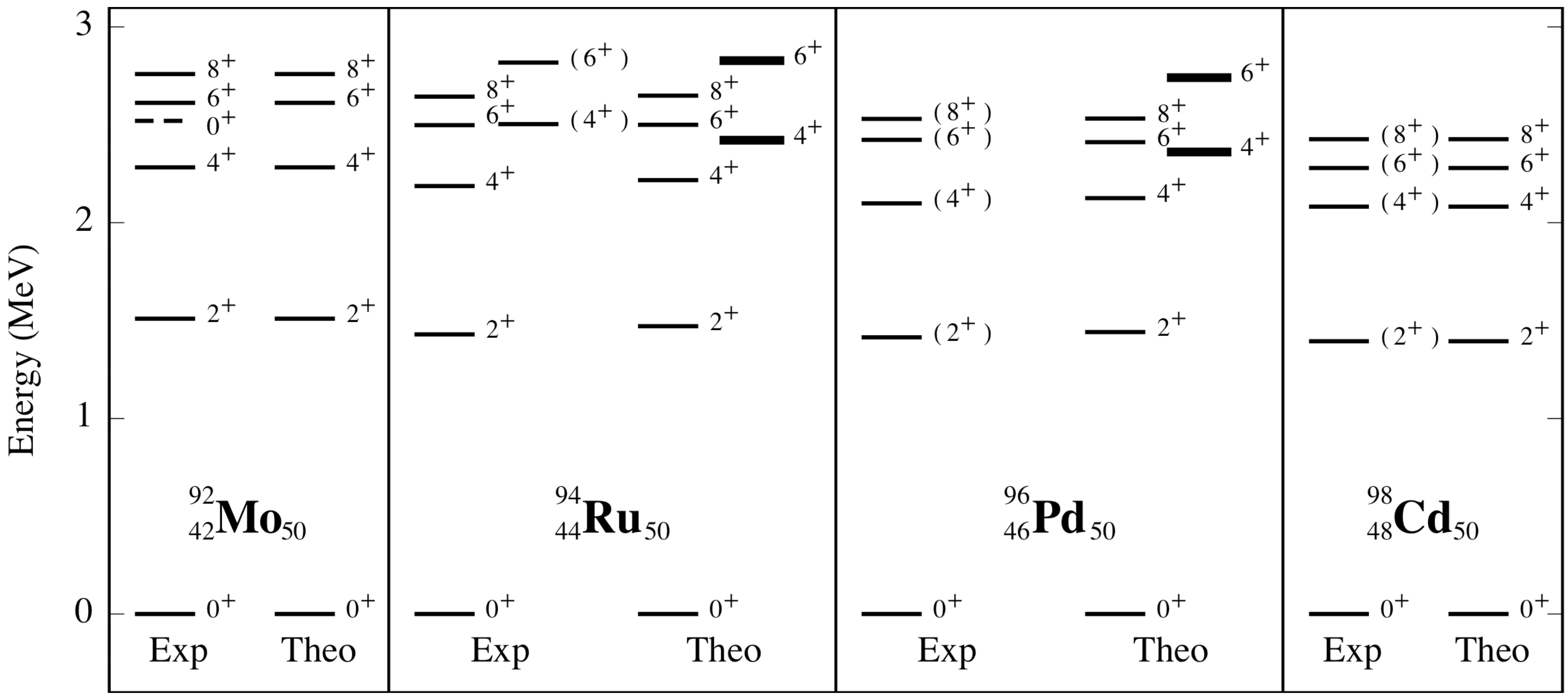
E2 transition rates

Analytic E2 transition rate:

$$B\left(\mathrm{E2};(9/2)^4,\upsilon=4,J=6\rightarrow(9/2)^4,\upsilon=4,J=4\right)$$

$$=\frac{209475}{176468}B\left(\mathrm{E2};(9/2)^2,J=2\rightarrow(9/2)^2,J=0\right)$$

N=50 isotones



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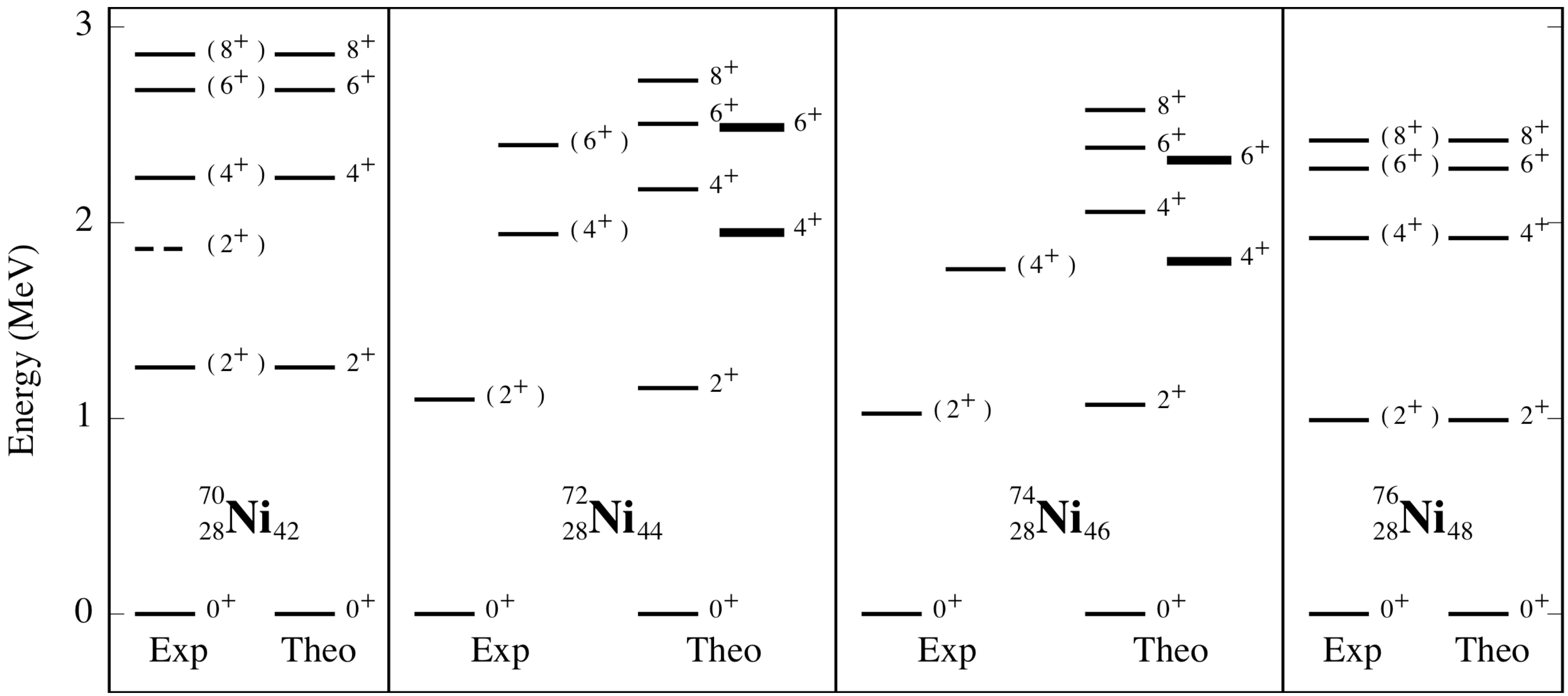
Nickel (*Z*=28) isotopes



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Seniority isomers in the $g_{9/2}$ shell