Joliot Curie School:
Symmetries in subatomic physics:
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G. Chanfray, IPN Lyon, IN2P3/CNRS, Université Lyon 1

QCD and symmetries related to nucleon structure and strongly interacting matter

1- OVERVIEW OF QCD

2- SU(2) x SU(2) CHIRAL SYMMETRY

3- OPERATIONAL APPROACHES AND EFFECTIVE THEORIES FOR LOW ENERGY QCD

4- QCD PHASES AND SYMMETRIES: CHIRAL RESTORATION AND DECONFINEMENT

Units $\hbar = c = 1$ $[Energy] = [Momentum] = [Length]^{-1} = [Time]^{-1}$

System of size R: $E = k\,\frac{1}{R}$ → $E = k\,\frac{\hbar\, c}{R}$

$\hbar\, c = 197.3\, MeV \cdot fm$ → $E\,(Mev) = k\,\frac{200}{R\,(fm)}$

$$1\, fm^{-1} = 200\, MeV$$

I-OVERVIEW OF QCD

1-QCD as a SU(3) gauge theory

Historical introduction: the rationale for QCD

- In the standard model, there are 6 species (flavors) of quarks.

  In the 60th:quark model: Baryon=QQQ, Mesons= $Q\bar{Q}$, but conceptual problems

  $\Delta^{++} = u\uparrow u\uparrow u\uparrow$ violates Pauli principle

- Introduce 3 colors: each quark exists with three color states (1,2,3)=(r,y,b)

  Hadrons are « white »= « color singlet »

  Mathematically: H invariant under permutation of quark color

  $q_i \to V_{ik}\, q_k$ | $V^\dagger V = 1$ | Det V=1 ➡ $V = e^{i\theta_a T_a}$

  V : SU(3) matrix generated by eight generators: $T_a = \lambda_a / 2 \quad a=1,..,8$

  ➡ $M = \sum_i q_i \bar{q}_i = J\bar{J} + B\bar{B} + R\bar{R}$

  $B = \sum_{i,j,k=1} \epsilon_{ijk}\, q_i\, q_j\, q_k$ → $\Delta^{++} = \sum_{i,j,k} \epsilon_{ijk}\, u_i\uparrow\, u_j\uparrow\, u_k\uparrow$

- Dynamical theory for color: SU(3) gauge field theory

  Fondamental object: Dirac field for each color

  $$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

  $$\bar{\psi} = \psi^\dagger \gamma^0 = (\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3)$$

  $(1,2,3) = (r,y,b)$

  Gauge principle: invariance of the theory (lagrangian) under a local transform

  $$\psi(x) \to V(x)\, \psi(x) \qquad V(x) = e^{i\theta_a(x)\, T_a}$$

SU(3) group

SU(3) transformation: $V = e^{i\theta_a T_a}$

Generator of the SU(3) group

$t_a = \lambda_a/2$

$$\lambda_{a=1,2,3} = \begin{pmatrix} \sigma_a & 0 \\ 0 & 0 \end{pmatrix} \qquad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Generator of the SU(2) group

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

They satisfy the Lie algebra

$$[t_a, t_b] = i\, f_{abc}\, t_c$$

f_{abc} are the structure constants of SU(3), fully antisymmetric under permutation

$$\begin{aligned} 1 &= f_{123} = 2\,f_{147} = 2\,f_{246} = 2\,f_{257} = 2\,f_{345} \\ &= -2\,f_{156} = -2\,f_{367} = \frac{2}{\sqrt{3}}\,f_{458} = \frac{2}{\sqrt{3}}\,f_{678} \end{aligned}$$

Building of the gauge theory

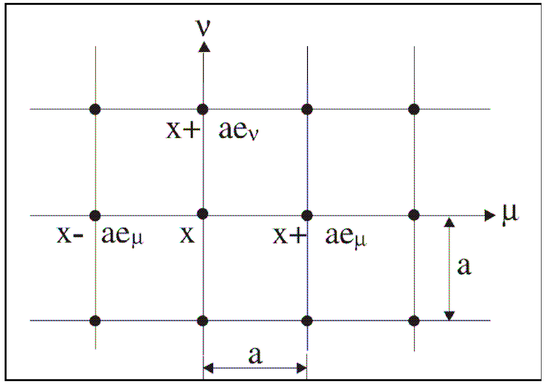
- Start with a free fermion theory $\mathcal{L} = i\,\overline{\psi}\gamma^\mu\partial_\mu\psi - m\,\overline{\psi}\psi$

  Discretize the theory on a lattice (spacing a)

$$\mathcal{L}_c = i\,\overline{\psi}\gamma^\mu\partial_\mu\psi = \frac{i}{a}\sum_\mu \left(\overline{\psi}(x)\gamma^\mu\psi(x+a\,e_\mu) - \overline{\psi}(x)\gamma^\mu\psi(x)\right)$$

  But not « gauge invariant » due to « non locality » !

$$\overline{\psi}(x)\gamma^\mu\psi(x+a\,e_\mu) \to \overline{\psi}(x)\gamma^\mu V^\dagger(x)V(x+a\,e_\mu)\psi(x+a\,e_\mu)$$



- To cure the problem introduce link variables (SU(3) matrices)

$U(x;y)$ with $U(x;x) = 1$ $U^{-1}(x;y) = U(y;x)$

$$\mathcal{L}_c = \frac{i}{a}\sum_\mu \left(\overline{\psi}(x)\gamma^\mu U(x;x+a\,e_\mu)\,\psi(x+a\,e_\mu) - \overline{\psi}(x)\gamma^\mu\psi(x)\right)$$

Invariant if

$$U(x;y) \to V(x)\,U(x;y)\,V^\dagger(y)$$

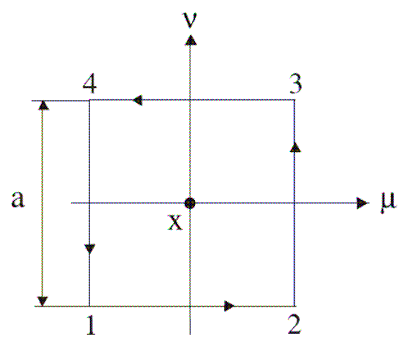
$$U(x;x+dx) = e^{iB^\mu(x)\,dx_\mu}$$

8 Gauge fields

$$B^\mu(x) = B^\mu_a(x)\,T^a$$

- Dynamic of the link variables: gauge invariant « Plaquette term »

$$\mathcal{L}_J = \frac{\beta}{N}\frac{1}{a^4}\sum_{(\mu,\nu)} tr(U_{12}\,U_{23}\,U_{34}\,U_{41})$$



The QCD Lagrangian

Back to continuum limit: a → 0

From link to gluon field gauge transformation

$$U(x;y) \to V(x)\, U(x;y)\, V^\dagger(y) \qquad B_\mu(x) \to V(x)\, B_\mu(x) V^\dagger(x) - iV(x)\partial_\mu V^\dagger(x)$$

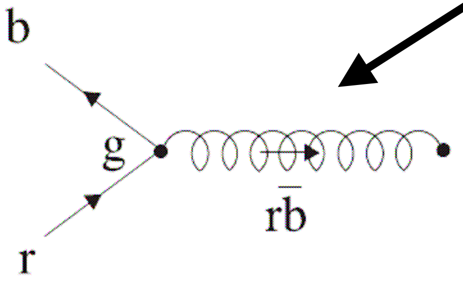
Lagrangian

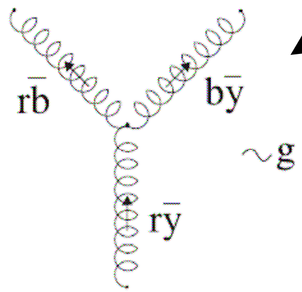
$$g = \sqrt{N_c/\beta} \qquad B^\mu = gA^\mu = g \sum_{a=1}^{N_c^2-1} t_a\, A_a^\mu$$

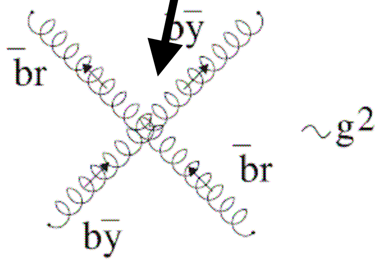
$$G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu] \equiv t_a\, G_a^{\mu\nu} \qquad G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g\, f_{abc}\, A_b^\mu A_c^\mu$$

f=u,d,s,…

$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f \gamma_\mu (i\partial^\mu - g\, t_a\, A_a^\mu)\psi_f - \sum_f m_f\, \bar\psi_f \psi_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$







8 gluons: $(r, y, b) \otimes (\bar{r}, \bar{y}, \bar{b}) - \frac{1}{\sqrt{3}}(r\bar{r} + y\bar{y} + b\bar{b})$

m_u= 4 MeV, m_d=8 MeV, m_s=150 MeV, $m_{c,b,t}$ >> 1 GeV

2- Symmetries of QCD

$$\mathcal{L}_{\text{QCD}} = \sum_f \overline{\psi}_f \gamma_\mu (i\partial^\mu - g\, t_a\, A^\mu_a)\psi_f - \sum_f m_f\, \overline{\psi}_f \psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Observationnal facts

-All the observed hadrons are compatible with QQQ and $Q\overline{Q}$ color singlet states (color confinement)

- M_H~1 GeV, R_H~ 1 fm
- Exception m_π

-Light hadrons classified in isospin multiplets with no degenerate parity partners

-Nuclei are made of « packages of nucleons »

-At high T: Deconfinement and chiral restoration (Lattice + RHIC)

Symmetries

-Gauge (color) symmetry

-Light quark sector (m_u,m_d small)

- No scale in the QCD Lagrangian (scale invariance)
- Invariance under global « chiral » transformations

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

$$\psi \rightarrow e^{i\alpha_k \frac{\tau_k}{2}}\, \psi$$

$$\psi \rightarrow e^{i\alpha_k \frac{\tau_k}{2}\gamma_5}\, \psi$$

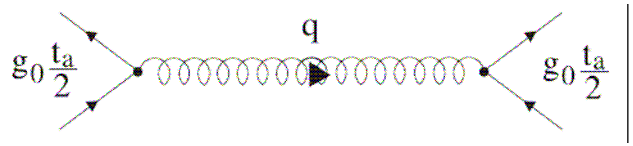
-Center symmetry (pure gauge)

Breaking of scale invariance: running coupling constant

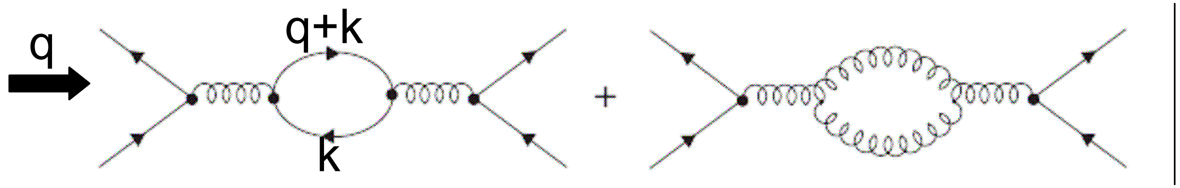
•In the light quark sector the QCD lagrangian is « classically » scale invariant
•However in a QFT the existence of quantum fluctuations of arbitrary size breaks scale invariance
•The QFT must be formulated at each scale μ or at each resolution $a=1/\mu$

•Quark-quark scattering

first order



The exchanged gluon may fluctuate



•Formulate the theory at scale μ: all the high momentum fluctuations $k>\mu$ are included in the definition of the degrees of freedom (field variables) and parameters (coupling constant)

$g(\mu + \delta\mu) \longrightarrow g(\mu)$: the fluctuations between μ and $\mu+\delta\mu$ are now included in the definition of $g(\mu)$

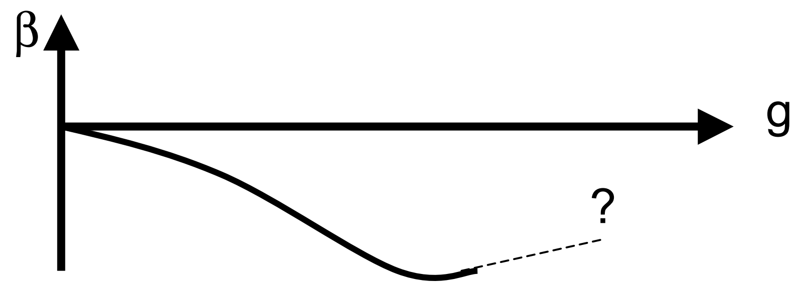
•The evolution of $g(\mu)$ can be calculated if we know

$$\beta(g) = \mu \frac{dg}{d\mu} \equiv \frac{dg}{d\,ln\mu}$$

- β(g) is an intrinsic property of the theory: it can be calculated in the small g domain perturbative QCD

$$\beta(g) = -\beta_0 g^3 + \beta_1 g^5 + \ldots$$

$$(4\pi)^2\beta_0 = \frac{11}{3}N_C - \frac{2}{3}N_f > 0$$



- Increasing μ, g decreases to a minimum value g=0 obtained for large momentum scale. g=0 is an attractive ultraviolet fixed point
QCD is an asymptotically free theory: ASYMPTOPTIC FREEDOM

$$\beta(g) = \mu\frac{dg}{d\mu} \equiv \frac{dg}{d\,ln\mu} \quad \longrightarrow \quad \mu = \Lambda\, exp\left(\frac{1}{2\beta_0\, g^2}\right)\left(\beta_0\, g^2\right)^{-\beta_1/2\beta_0}$$

- Lets us calculate an observable O with dimension D (hadron mass D=1)

$$O = \mu^D f(g(\mu)) \;\longrightarrow\; \frac{df}{f} = -D\frac{dg}{\beta(g)} \;\longrightarrow\; O = C\,\mu^D exp\left(\frac{-D}{2\beta_0\, g^2}\right)\left(\beta_0\, g^2\right)^{D\beta_1/2\beta_0}$$

$$\downarrow$$

$$O = C\,\Lambda_{QCD}^D$$

Λ_{QCD} is the fundamental scale of QCD

- The QCD pb is to find the numerical constants C
- Lattice calculation: μ=1/a: check the scaling law and extract C
- From data Λ_{QCD}=200 MeV

- $Q \gg \Lambda_{QCD}$ Pertubative QCD

$$g^2(Q^2) = \frac{1}{\beta_0\, ln\left(Q^2/\Lambda^2_{QCD}\right)}$$

$$\Lambda_{QCD} \simeq 200\; MeV$$

$$R_H = 1/\Lambda_{QCD} \simeq 1\; fm$$

$Q < \Lambda_{QCD}$ NON Perturbative QCD

Trace anomaly and gluon condensate

- QCD action $S = \int d^4x\, \mathcal{L} = \int d^4x \left[\bar\psi i \gamma^\mu \left(i\partial_\mu - t_a \bar{A}_{a\mu} \right) \psi - m\bar\psi\psi - \frac{1}{4g^2} \bar{G}^{\mu\nu}_a \bar{G}_{a\mu\nu} \right] \quad \bar{A}^\mu_a = g\, A^\mu_a$
- Scale transformation

$$x \to (1-\delta\lambda)x \qquad \psi \to (1+D_\psi \delta\lambda)\psi \qquad \bar{A}^\mu_a \to (1+D_A\delta\lambda)\bar{A}^\mu_a \qquad D_\psi = 3/2,\ D_A = 1$$

- The variation of the action and trace of the stress tensor

$$\delta S = \int d^4x\, \delta\lambda \partial_\mu D^\mu_{dil} \quad \text{with} \quad \partial_\mu D^\mu_{dil} \equiv \partial_\mu \left(x_\nu T^{\nu\mu} \right) = T^\mu_\mu$$

- Explicit variation of the (effective) action taken at scale μ $\quad \mu \to (1+\delta\lambda)\mu.$

$$\delta S = \int d^4x\, \delta\lambda \sum_i (D_i - 4)\mathcal{L}_i + \delta\lambda \frac{\delta\mu}{\delta\lambda} \frac{1}{2g^3} \frac{dg}{d\mu} \left(g^2 G^{\mu\nu}_a G_{a\mu\nu} \right)$$

$$\longrightarrow \quad T^\mu_\mu = m\bar\psi\psi + \frac{\beta(g)}{2g^3} \left(g^2 G^{\mu\nu}_a G_{a\mu\nu} \right)$$

- QCD vacuum $\quad \langle T^{\mu\nu} \rangle = \epsilon g^{\mu\nu} \quad \longrightarrow \quad 4\epsilon = 4\pi^2 \frac{\beta(g)}{2g^3} \left\langle \frac{g^2}{4\pi^2} G^{\mu\nu}_a G_{a\mu\nu} \right\rangle$

Gluon condensate:
QCD sum rule
(hadron spectral Function)

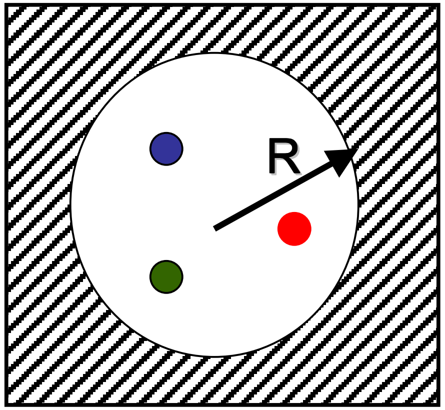
$$\langle (\alpha_S/\pi)\, GG \rangle \simeq 0.012\, GeV^4$$

Energy density of QCD vacuum

$$\epsilon = -0.5\, GeV \cdot fm^{-3}$$

Some consequences for the nucleon structure

MIT bag picture: nucleon = bubble of perturbative vacuum with freely moving current quarks (m=0)



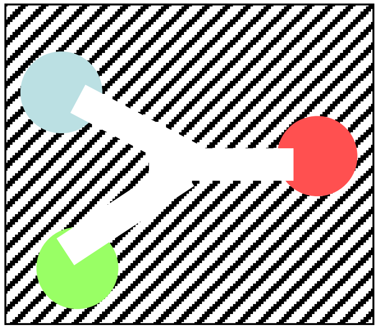
$$M = \frac{3\,\Omega_0}{R} + \frac{4}{3}\pi R^3 B \qquad \Omega_0 = 2.04$$

$$\rightarrow \quad R_h = \left(\frac{3\,\Omega_0}{4\pi}\right)^{1/4} \frac{1}{B^{1/4}} \simeq 1/\Lambda_{QCD} \simeq 1\,fm$$

$$\rightarrow \quad B \simeq 0.1\,Gev\,fm^{-3} << \epsilon$$

$\downarrow$

Alternative picture: non perturbative vacuum fluctuations expelled from a much smaller domain:

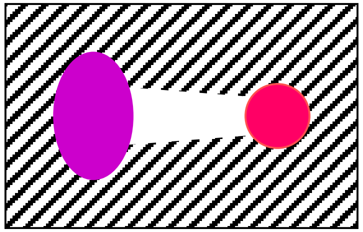


Constituant quark with size

$$R_q = 1/\Lambda_{\chi SB}$$

At the endpoint of strings

$$R_h \simeq 1/\Lambda_{QCD}$$



Evidence for diquark formation

II- $SU(N_f)_L$ X $SU(N_f)_R$

CHIRAL SYMMETRY

N_f=2 : u, d
N_f=3 : u,d,s

$m_{u,d} \sim$ 5 MeV
$m_s \sim$ 150 MeV

to be compared with $M_H \sim$ 1 GeV

1- Vector, axial and chiral symmetry

m_u= 4 MeV
m_d= 8 MeV

QCD Lagrangian without glue

$$\mathcal{L}_{QCD} = i\bar{\psi}_u\gamma^\mu\partial_\mu\psi_u + i\bar{\psi}_d\gamma^\mu\partial_\mu\psi_d - m_u\bar{\psi}_u\psi_u - m_d\bar{\psi}_d\psi_d$$
$$= i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{m_u+m_d}{2}\bar{\psi}\psi - \frac{m_u-m_d}{2}\bar{\psi}\tau_3\psi$$

$$\psi = \begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix}$$

Vector symmetry

L almost exactly invariant under the SU(2) transformation

$$\psi \rightarrow e^{i\alpha_k\frac{\tau_k}{2}}\psi$$

Very small breaking

$$(m_d - m_u)/2 \simeq 2\,MeV \ll M_H \sim 1\,GeV$$

•Conserved charge (isospin)

$$Q_k = \int d\mathbf{r}\,\psi^\dagger\frac{\tau_k}{2}\psi \equiv I_k$$

•Quantum states transform as

$$U = e^{-i\alpha_j Q_j}$$

$$|\Phi\rangle \rightarrow |\Phi'\rangle = U\,|\Phi\rangle$$

$$[Q_i, Q_j] = i\,\epsilon_{ijk}\,Q_k$$
$$[Q_i, H_{QCD}] = 0$$

→ Multiplet structure (with given, P,J)

$$|\alpha IM> = \phi_{\alpha IM}|0>$$

$$U|\alpha IM> = U\,\phi_{\alpha IM}|0> = U\,\phi_{\alpha IM}U^\dagger\,U|0>$$
$$= \Sigma_{M'}\,\phi_{\alpha IM'}\,U|0> \;<\alpha IM'|U|\alpha IM>$$
$$= \Sigma_{M'}|\alpha IM'> \;<\alpha IM'|U|\alpha IM>$$

True because

$$U\,|\,0\rangle = |\,0\rangle \quad\Longleftrightarrow\quad Q_j\,|\,0\rangle = 0$$

Axial symmetry

L almost exactly invariant under

$$\psi \rightarrow e^{i\alpha_k\frac{\tau_k}{2}\gamma_5}\psi$$

$$m = m_u + m_d/2 \simeq 7\,MeV << M_H$$

•Conserved axial charge

$$Q_k^5 = \int d\mathbf{r}\,\psi^\dagger\gamma_5\frac{\tau_k}{2}\psi$$

$$U_5 = e^{-i\alpha_j Q_5^j}$$

Chiral symmetry

$$\psi_R = \frac{1+\gamma_5}{2}\psi \qquad \psi_L = \frac{1-\gamma_5}{2}\psi$$

$$\mathcal{L}_{QCD} = i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R - m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

QCD Lagrangian almost exactly invariant (m~7 MeV) under transformations in the light quark sector (u-d) acting separately on left and right quarks

$$SU(2)_L \;:\quad \psi_L \to e^{i\alpha_k\frac{\tau_k}{2}}\psi_L, \qquad \psi_R \to \psi_R$$
$$SU(2)_R \;:\quad \psi_R \to e^{i\beta_k\frac{\tau_k}{2}}\psi_R, \qquad \psi_L \to \psi_L$$

$$Q_L^k = \int dr\, \psi_L^\dagger \frac{\tau_k}{2}\psi_L = \frac{1}{2}\left(Q_k - Q_k^5\right)$$
$$Q_R^k = \int dr\, \psi_R^\dagger \frac{\tau_k}{2}\psi_R = \frac{1}{2}\left(Q_k + Q_k^5\right)$$

$$[Q_L^i, Q_L^j] = i\,\epsilon_{ijk}\,Q_L^k \qquad [Q_R^i, Q_R^j] = i\,\epsilon_{ijk}\,Q_R^k \qquad [Q_R^i, Q_L^j] = 0$$

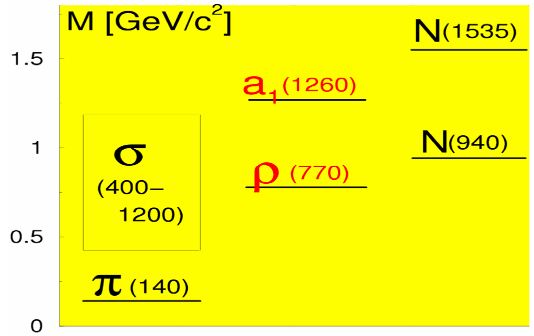
2 degenerate L and R worlds: Two sets of independant identical multiplets

or

Vector symmetry (isospin) multiplets +degenerate partners with opposite parity

kind of « doubling » of isospin symmetry

OBVIOUSLY NO !!!



2- Spontaneous breaking of chiral symmetry

Chiral limit : m=0

[H, Q_k]=0 and Q_k |0>=0 → Isospin multiplets

[H, Q^5_k]=0 but Q^5_k |0> ≠ 0 and $U_5|0> \neq |0>$

The vacuum does not have the symmetry

This is the spontaneous breaking of chiral (axial) symmetry

No Chiral partner multiplet

$$
\begin{aligned}
U_5|\alpha IM> &= U_5\, \phi_{\alpha IM}|0> = U_5\, \phi_{\alpha IM}\, U_5^\dagger\, U_5|0> \\
&= \Sigma_{M'}\, \phi_{\alpha IM'}\, U_5|0> \;\; <\alpha IM'|U|\alpha IM> \\
&\neq \Sigma_{M'}|\alpha IM'> \;\; <\alpha IM'|U|\alpha IM>
\end{aligned}
$$

Goldstone theorem

Since the axial charge commutes with H, its action on the vacuum should gives states having the same energy.

$$|\pi_j\rangle = Q^5_j\,|0\rangle \qquad H\,|\pi_j\rangle = H\,Q^5_j\,|0\rangle = Q^5_j\,H\,|0\rangle = 0$$

This implies the existence of soft (i.e. massless) modes: THE PION

Basis for chiral perturbation theory

Low energy pion do not (or weakly) interact

$$H\,|(\pi)^n\rangle = H\,(Q^5)^n\,|0\rangle = (Q^5)^n\,H\,|0\rangle = 0$$

Explicit chiral symmetry breaking

m_π=140 MeV

Order parameters : Charged pion decay $\pi^+ \rightarrow \mu^+ \nu_\mu$

$$|\pi^\pm> = \mp\frac{1}{\sqrt{2}}(|\pi_1> \pm i\,|\pi_2>)$$

$$< 0|\mathcal{A}_i^\mu(x)\;|\pi_j(q)> = -i\;\delta_{ij}\;f_\pi\;q^\mu\;e^{-iqx}$$

f_π=94 MeV, pion decay constant

$$< 0|Q_5^i(t)|\pi_j(\mathbf{q})> \;=\; \int d\mathbf{r} < 0|\mathcal{A}_i^0(\mathbf{r},t)|\pi_j(\mathbf{q})> = -\frac{i}{2}\,f_\pi\,e^{-im_\pi t} < \pi^i(0)|\pi^j(\mathbf{q})>$$

$$f_\pi \neq 0 \Leftrightarrow Q_i^5\,|0\rangle \neq 0$$

The GOR Relation

- Operatorial identity $$[Q_i^5,[Q_j^5,H] = \delta_{ij}\int d\mathbf{r}\,m\,\bar{\psi}\psi(\mathbf{r})$$
- Insert a complete set of states
  Energy weighted sum rule $$\sum_n 2E_n\,|<n|Q_i^5|0>|^2 = -\int d\mathbf{r}\,2m\;<\bar{q}q>$$
- Single pion dominance $$m_\pi^2\,f_\pi^2 = -2m\;<\bar{q}q>$$

	Microscopic (Quark)	Macroscopic (Hadron)
Explicit breaking	m	m_π
Order parameter	$<\bar{q}q>$	f_π

Quark condensate

m= 6 MeV $\quad < \bar{q}q > \simeq -(240\,MeV)^3$

Scalar density of quarks

$-1.76\,fm^{-3} \approx 10\rho_0$

Order parameter: not invariant under the symmetry group: it should vanish in the symmetric phase

Heisenberg ferromagnet

$$H = -J\sum_{i,j}\vec{S}_i\cdot\vec{S}_j - \mu\sum_i \vec{S}_i\cdot\vec{B}$$

B=0: H rotationnal invariant but non vanishing order parameter

$$\vec{M} = \frac{1}{N}\sum_i \vec{S}_i \neq 0$$

Spontaneous breaking
Ground state infinitely degenerate

In presence of B: magnetization along the magnetic field

Chiral symmetry of QCD

$$H = H_0 + \int d^3r\, m\, \left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)(\mathbf{r})$$

m=0: H chirally invariant but non vanishing order parameter

$$M^{ij} = \langle 0|\,\bar{\psi}_L^j\psi_R^i\,|0\rangle = \frac{1}{2}\,\Sigma\,\delta^{ij} \neq 0$$

$$\Sigma = \frac{1}{2}\langle 0|\,\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\,|0\rangle \equiv \frac{1}{2}\left\langle \bar{u}u + \bar{d}d\right\rangle \equiv \langle \bar{q}q\rangle$$

Chiral transformation $|0\rangle \rightarrow |\alpha,\beta\rangle = U_L(\alpha)U_R(\beta)\,|0\rangle$

$$M^{ij} \rightarrow M'^{ij} = \langle \alpha,\beta,|\,\bar{\psi}_L^j\psi_R^i\,|\alpha,\beta\rangle = \left(V_R(\beta)\,M\,V_L^\dagger(\alpha)\right)_{ij} = \left(V_R(\beta)V_L^\dagger(\alpha)\right)_{ij}\frac{\Sigma}{2}$$

Vector transf. $\alpha=\beta$: M invariant

Axial transf. $\alpha=-\beta$: $M = \frac{\Sigma}{2} \rightarrow \frac{\Sigma}{2}(cos\alpha + i\vec{\tau}\cdot\hat{\alpha}sin\alpha)$ not invariant

Correlators and hadron spectral functions

Correlation function: correlators between two currents or two fields with given quantum numbers characteristic of a hadron

$$\Pi_R(q) \equiv \Pi_R(q_0, \vec{q}) = \int d^4x \, e^{iq\cdot x} \, \Theta(x_0) \, \langle 0 \, | [J(x), \, J(0)] | \, 0 \rangle$$

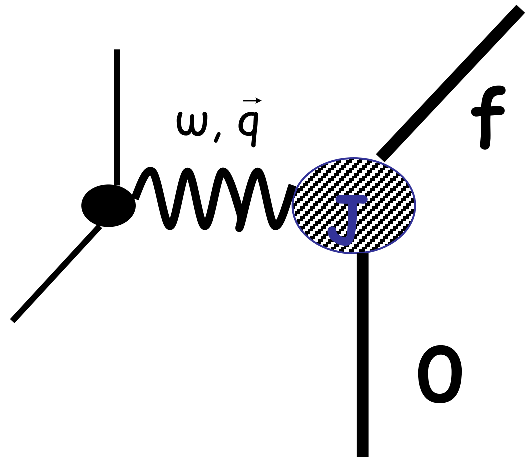
Dispersive analysis
and spectral functions

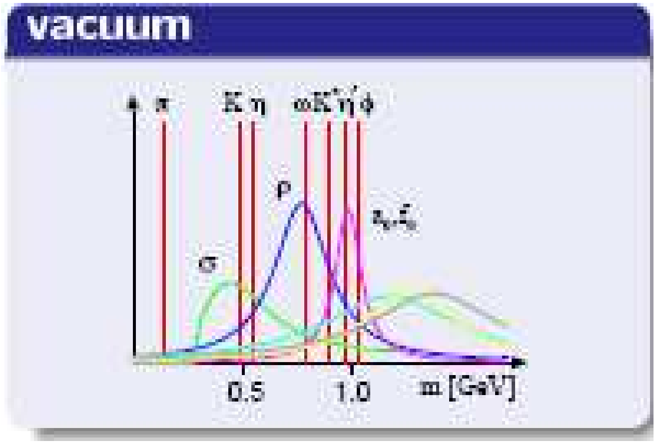
$$\Pi_R(q_0, \vec{q}) = \int_{-\infty}^{+\infty} d\omega \, \frac{S(\omega, \vec{q})}{q_0 - \omega + i\eta}$$

$$S(\omega, \vec{q}) = -\frac{1}{\pi} Im\Pi_R(\omega, \vec{q}) = \sum_f |\langle f \, | J(0) | \, 0 \rangle|^2 \, (2\pi)^3 \delta^{(3)} \, (\vec{q} + \vec{p}_i - \vec{p}_f) \; \delta(\omega + E_0 - E_f)$$

-Response to a probe which couples to a current J(x) carrying the quantum numbers of a hadron.

-Accessible experimentally at energy momentum transfer (ω,q)





Fluctuation currents

Hadrons

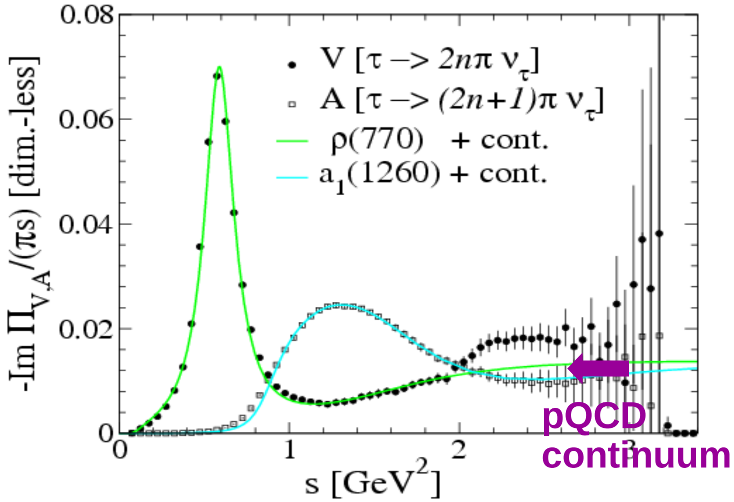
$\langle\langle J_s(x)J_s(0)\rangle\rangle - \langle\langle J_{ps}(x)J_{ps}(0)\rangle\rangle$	$M_{f_0}(600) \to M_\pi(138)$
$\langle\langle V^a_\mu(x)V^a_\mu(0)\rangle\rangle - \langle\langle A^a_\mu(x)A^a_\mu(0)\rangle\rangle$	$M_{a_1}(1250) \to M_\rho(770)$
$\langle\langle \Psi_+(x)\Psi_+(0)\rangle\rangle - \langle\langle \Psi_-(x)\Psi_-(0)\rangle\rangle$	$M_{S_{11}}(1535) \to M_p(938)$

Correlation functions associated with chiral partners

$$\mathcal{V}^\mu_k = \bar\psi\gamma^\mu\frac{\tau_k}{2}\psi$$ $J^\pi=1^-$, I=1 (rho)

$$\mathcal{A}^\mu_k = \bar\psi\gamma^\mu\gamma_5\frac{\tau_k}{2}\psi$$ $J^\pi=1^+$, I=1 (a_1)

Axialvector / Vector in Vacuum



Vector and axial-vector spectral functions accessible from τ decay with even and odd number of pions

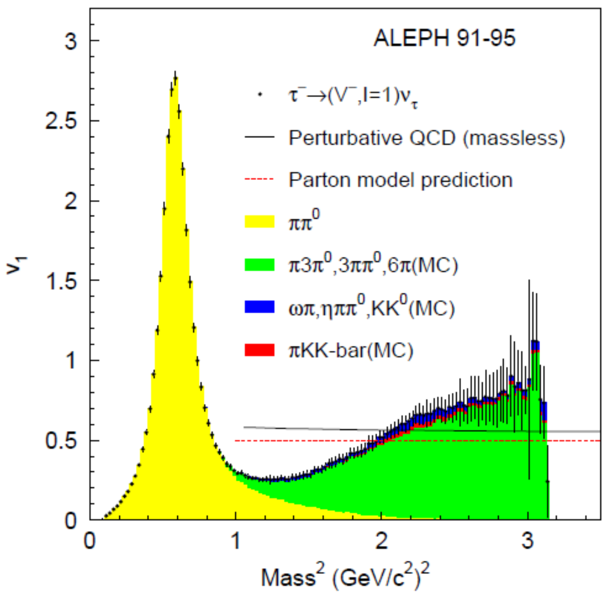
Very different at low energy rho peak and a_1 bump: Spontaneous breaking of chiral symmetry

Become identical at high energy: quark hadron duality: High momentum quarks decouple from the condensate

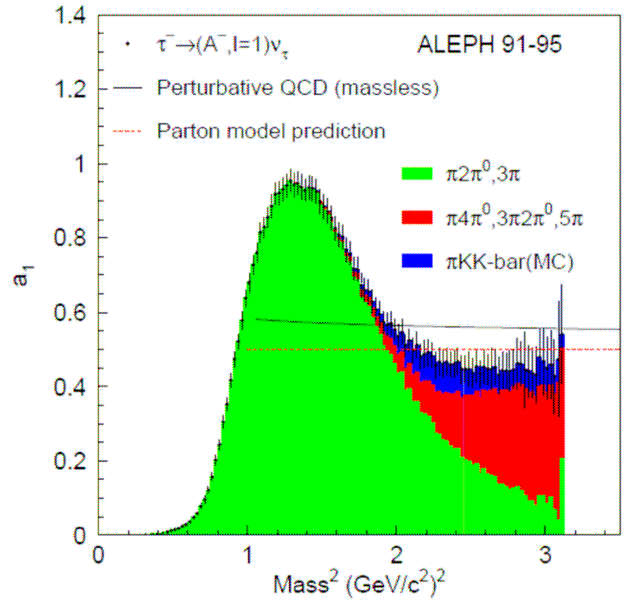
Chiral symmetry breaking is a non perturbative low energy phenomena

ALEPH data

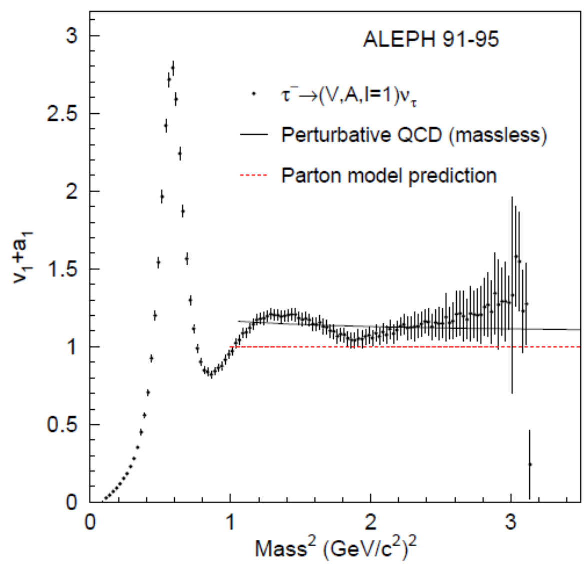
Vector



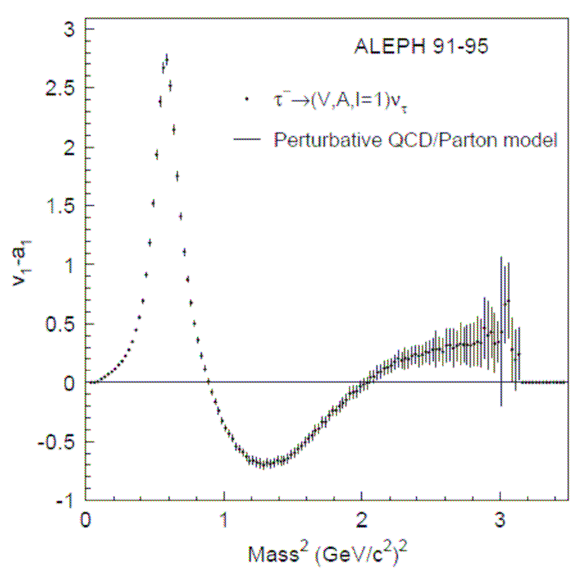
Axial Vector



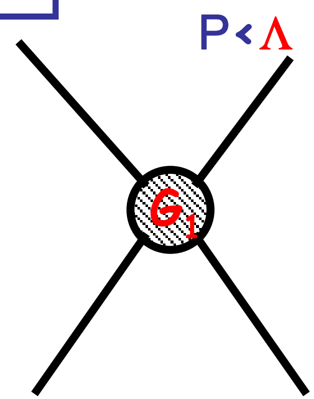
V +AV



V- AV



3- Explicit realization : the Nambu-Jona-Lasinio model



•Basis of the model

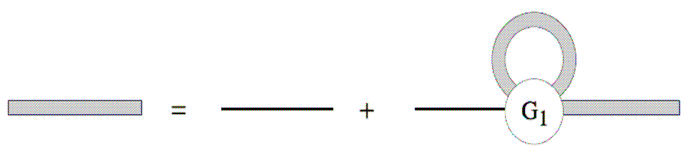
Quartic chiral invariant interaction (P<Λ~1 GeV) simulating non pertubative QCD: (3 parameters: G_1, Λ, m)

$$\mathcal{L}_{NJL} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\,\bar{\psi}\psi + \frac{G_1}{2}\left[(\bar{\psi}\psi)^2 + \left(i\bar{\psi}\gamma^5\vec{\tau}\psi\right)^2\right]$$

•Spontaneous breaking of chiral symmetry: constituants quarks

Mean-field $\quad (\bar{\psi}\psi)^2 \simeq 2\,(\bar{\psi}\psi)\;<<\bar{\psi}\psi>>= 4\,(\bar{\psi}\psi)\;<<\bar{q}q>>$

$$\begin{aligned} M &= m - 2G_1 <<\bar{q}q>> \\ &= m + 4\,N_c\,G_1 \int_{p<\Lambda} \frac{d\mathbf{p}}{(2\pi)^3}\,\frac{M}{E_p} \end{aligned} \qquad E_p = \sqrt{p^2 + M^2}$$



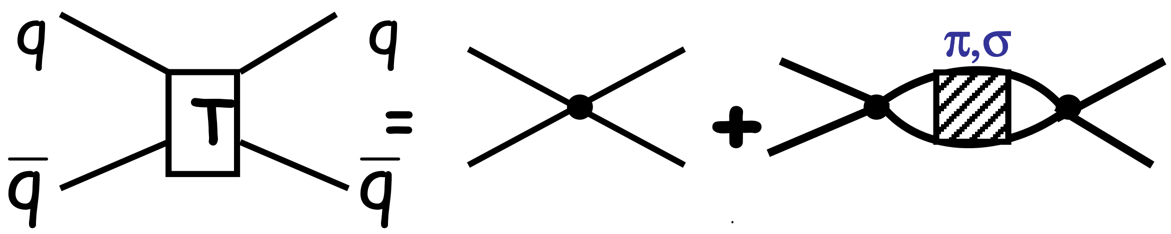
Spontaneous symmetry breaking; the quarks acquire a mass M~350 MeV

The broken vacuum is made of interacting quark-antiquark pairs; the (BCS type) ground state wave function is:

$$|\phi(M)> \;=\; C\,exp\left(-\sum_{s,p<\Lambda}\gamma_{ps}\,b^{\dagger}_{\mathbf{p}s}\,d^{\dagger}_{-\mathbf{p}-s}\right)|\phi_0> = \prod_{s,p<\Lambda}\left(\alpha_p + s\beta_p\,b^{\dagger}_{\mathbf{p}s}\,d^{\dagger}_{-\mathbf{p}-s}\right)|\phi_0>$$

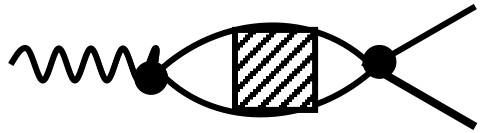
•Mesons; Goldstone theorem

Mesons generated as collective $\bar{q}q$ excitations, i.e., the unitarized interaction is mediated by the corresponding meson exchange



$$\approx \frac{g_B^2}{q^2 - m_B^2}$$

Pion decay constant

$A^\mu =$  $= f_\pi \; q^\mu \frac{1}{q^2 - m_\pi^2} g$

Adjust f_π m_π, $<\bar{q}q>$:

$$M = g f_\pi, \quad m_\pi^2 = m g^2 / M G_1, \quad m_\sigma^2 = 4M^2 + m_\pi^2$$

•Effective potential:

integrate out quarks in the Dirac sea

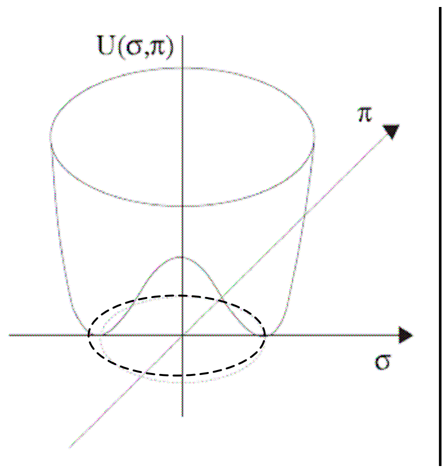
$$Z \equiv \int D\psi D\bar{\psi} \quad \exp\left(i \int d^4x L_{NJL}\right) \left[D\Sigma D\vec{\Pi} \, \delta(\Sigma - \bar{\psi}\psi) \, \delta(\vec{\Pi} - i\bar{\psi}\gamma_5\vec{\tau}\psi) \right]$$

$$Z = \int D\Sigma D\vec{\Pi} \quad \exp\left(i \int d^4x \; L_{eff}(\vec{\Pi}, \Sigma)\right)$$

$$L_{eff} = \frac{Z(\phi)}{2} (\partial^\mu \Sigma \partial_\mu \Sigma + \partial^\mu \vec{\Pi} \partial_\mu \vec{\Pi}) - U(\phi)$$

$$\phi = m + \Sigma + i\vec{\tau}.\vec{\Pi}\gamma_5$$

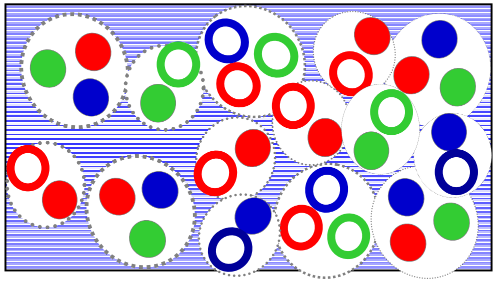
$$U(\phi) = \frac{\Sigma^2 + \vec{\Pi}^2}{4G_1} - 2N_c N_f \int_0^\Lambda \frac{d^3k}{(2\pi)^4} \sqrt{\vec{k}^2 + \phi^2}$$



4- Chiral restoration

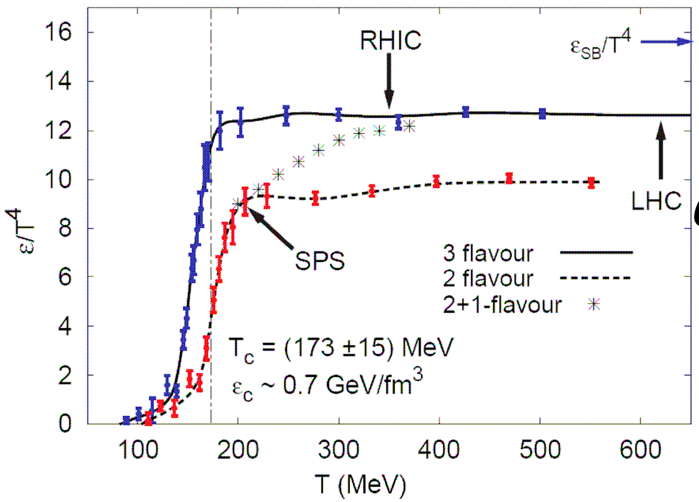
Lattice results

Hadronic matter heated or compressed
quarks «percolate» / liberated

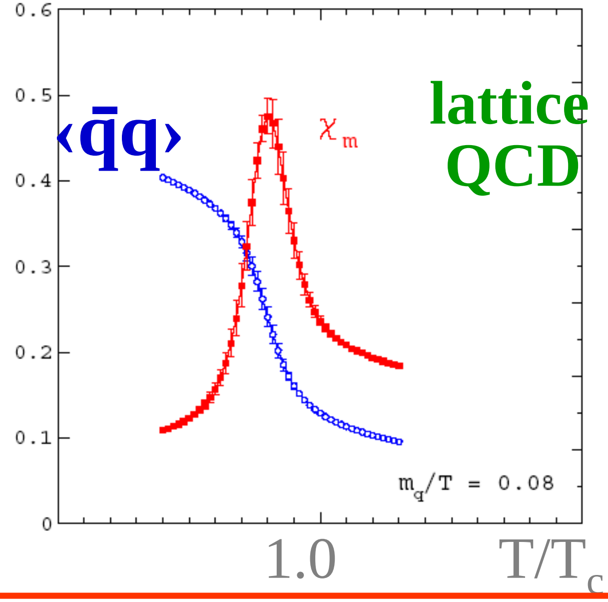


Sudden change of energy density
Sharp decrease of the quark condensate $\langle\bar{q}q\rangle$

DECONFINEMENT
AND
CHIRAL RESTORATION



(F. Karsch et al)



What are the mechanism of chiral restoration, interplay with confinement ?
What about finite density (finite chemical potential, low T) ?

Partial restoration of chiral symmetry

•Dropping of the quark condensate

The quark condensate, i.e., the scalar density of the QCD vacuum is negative. The hadrons have a positive scalar density originating from valence constituant quarks (scalar field) and pion cloud

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{vac} + \sum_h \rho_h Q_S^h \quad \text{with} \quad Q_S^h = \int d^3r \, \langle \bar{q}q(\vec{r}) \rangle_h$$

Scalar charge of the hadron h

Introduce the sigma commutator of the hadron

$$\Sigma_h =< h[Q_i^5, [Q_i^5, H]]|h >_{conn} =< h|H_{\chi SB}|h > \equiv = \int \mathrm{d}\mathbf{r} \, m \, < h|\bar{\psi}\psi(\mathbf{r})|h > \equiv m \, Q_S^h = m \frac{\partial M_h}{\partial m}$$

Assuming the GOR valid ⇒

$$R = \frac{<< \bar{q}q >> (\rho, T)}{< \bar{q}q >} \simeq 1 - \sum_h \frac{\rho_{sh} \Sigma_h}{f_\pi^2 m_\pi^2}$$

•The quark condensate from the equation of state

$$\Omega(V, T, \mu_B) = -T \, lnZ = -T \, ln \left(Tr \left[e^{-\beta(H_{QCD} - \mu_B N_B)} \right] \right) = \Omega_{vac} - V \, P(T, \mu_B) \equiv V \, \omega(T, \mu_B)$$

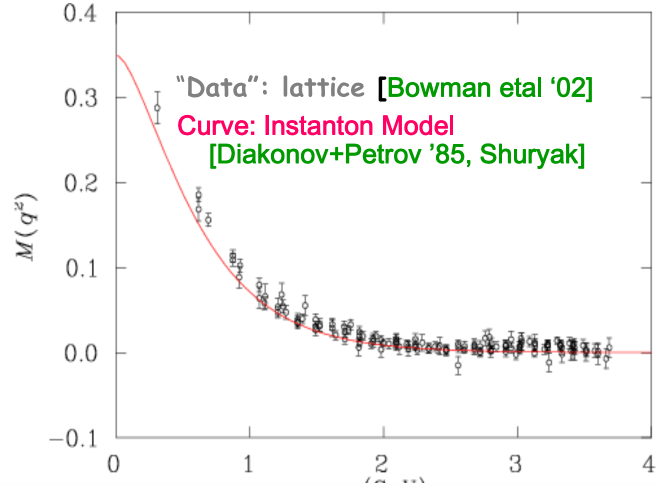
Feynman-Hellmann theorem ⇒

$$\langle\langle \bar{q}q \rangle\rangle (T, \mu_B) = \frac{1}{2} \left(\frac{\partial \omega}{\partial m} \right)_{\mu_B} = \langle \bar{q}q \rangle_{vac} - \frac{1}{2} \left(\frac{\partial P}{\partial m} \right)_{\mu_B}$$

Chiral restoration and hadron structure

$$\frac{\langle\langle \bar{q}q\rangle\rangle(\mu_B,T)}{\langle \bar{q}q\rangle_{vac}} = 1 - \sum_h \frac{\rho_s(\mu_B,T)\sigma_h}{f_\pi^2 m_\pi^2}$$

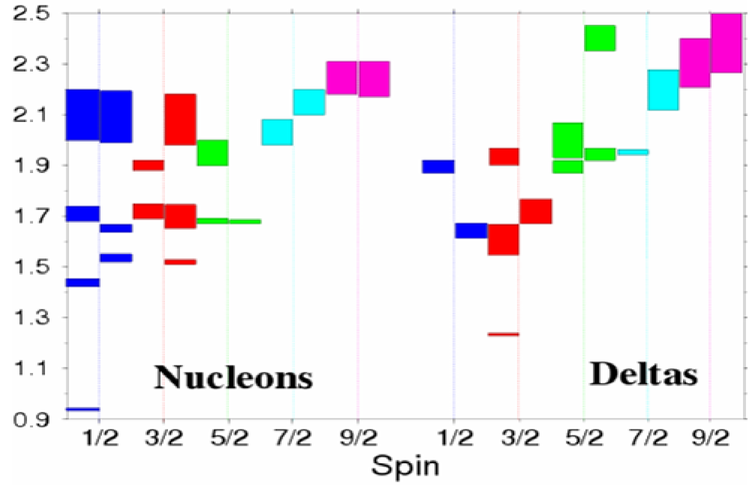
$$\sigma_h = m\frac{\partial M_h}{\partial m} = m_\pi^2 \frac{\partial M_h}{\partial m_\pi^2}$$



Only the lightest hadrons contribute, heavy hadrons (with large momenta valence quarks) decouple from the condensate

- Leading order in T (pion gas)
- Dilute nuclear matter (nucleons)

$$\frac{\langle\langle \bar{q}q\rangle\rangle(\rho)}{\langle \bar{q}q\rangle_{vac}} \simeq 1 - 0.35\frac{\rho}{\rho_0} - \frac{T^2}{8 f_\pi^2}$$



Possibility of quarkyonic matter at high density and low T where chiral symmetry is restored but confinement still there;

•Annex: the pion-nucleon sigma term from lattice

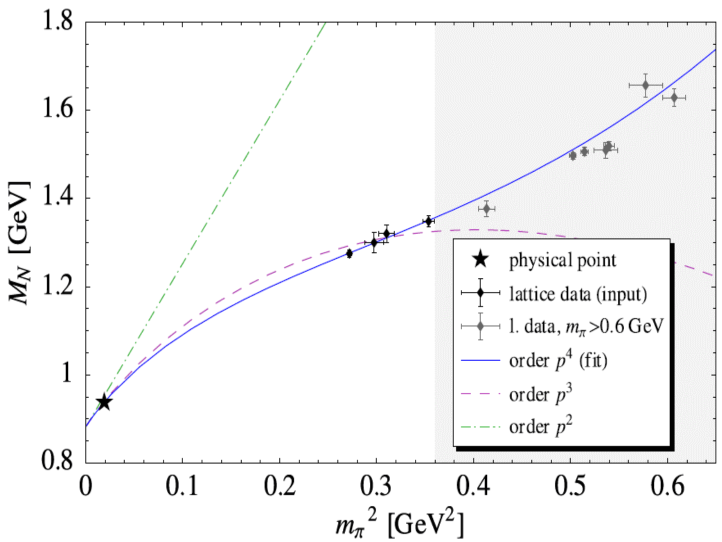
M_N, σ_N obtainable in principle from lattice, but lattice data avalaible only for m_π> 400 MeV ⟶ Use ChiPT to extrapolate

But extrapolation of ChiPT at order m^3_π fails:

$$M_N^{(3)} = M_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3$$

1 - Use Higher order ChiPT

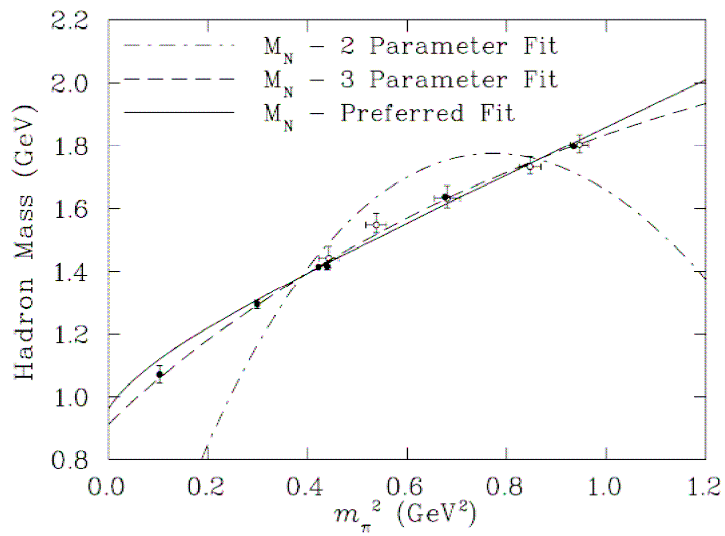
(Procura et al)



2 - Use chiral model to extrapolate lattice data

(Thomas et al)

$$M_{N,\Delta} = \alpha + \beta m_\pi^2 + \quad Pion\, loop(\Lambda)$$



The specific contribution of the pion cloud is very important and depends on a scale : the nucleon size (hidden in χPT)

$\sigma_N \simeq 50 MeV$, half of it from the pion cloud

Fluctuations of the condensate and chiral susceptibilities

•Scalar susceptibility : from the scalar correlator *i.e.* the correlator of the scalar quark density fluctuations

$$\chi_S = \frac{\partial\langle\bar{q}q\rangle}{\partial m_q} = 2\int dt'\, d\mathbf{r}'\, \Theta(t - t')\langle -i\,[\delta\bar{q}\,q(0)\,,\,\delta\bar{q}\,q(\mathbf{r}'\,t')]\rangle$$

(Obtainable from the EOS)

•Compare susceptibilities associated with chiral partners

Scalar (sigma) : $\bar{q}\,q$ → Pseudoscalar (pion) : $\bar{q}\,i\gamma_5\,\frac{\tau_\alpha}{2}\,q$

SCALAR SUSCEPTIBILITY

$$\chi_S = \frac{\partial\langle\bar{q}q\rangle}{\partial m_q} = 2\int dt'\, d\mathbf{r}'\, \Theta(t - t')\langle -i\left[\bar{q}\,q(0)\,,\,\bar{q}\,q(\mathbf{r}'\,t')\right]\rangle$$

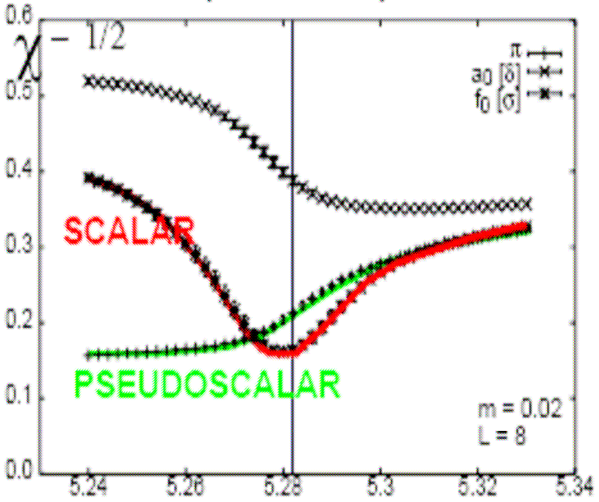
$$\chi_S = \left(\frac{\partial^2\omega}{\partial m_q^2}\right)_\mu = ReG_S(\omega = 0, \vec{q} \to 0) = \int_0^\infty d\omega\,\left(-\frac{2}{\pi\omega}\right)\, ImG_S(\omega, \vec{q} = 0)$$

At finite density a strong contribution of low energy nuclear excitations is expected

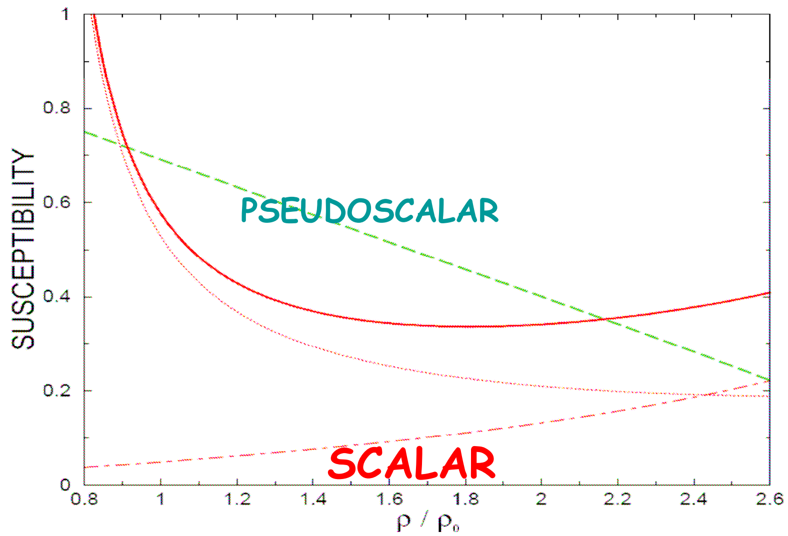
PSEUDOSCALAR SUSCEPTIBILITY

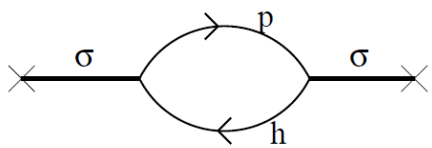
$$\chi_{PS} = 2\int dt'\, d\mathbf{r}'\, \Theta(t - t')\langle -i\left[\bar{q}\,i\gamma_5\frac{\tau_\alpha}{2}q(0)\,,\,\bar{q}\,i\gamma_5\frac{\tau_\alpha}{2}q(\mathbf{r}'\,t')\right]\rangle = \frac{\langle\bar{q}q\rangle(\rho)}{m_q}$$

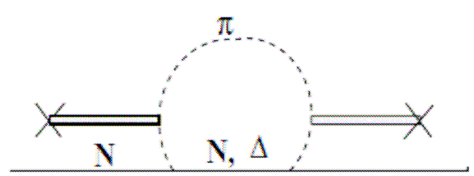
Thermal susceptibilty on Lattice (Karsch)



Finite density : effective chiral theory (M. Ericson, G. C)







Chiral Restoration : $\chi_S \rightarrow \chi_{PS}$

Correlator mixing

Vector and axial-vector correlators

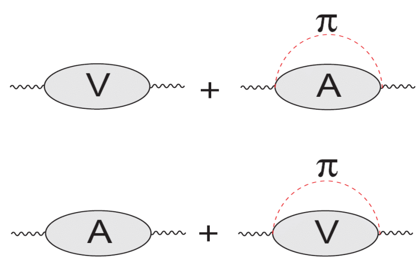
$$\Pi_V^{\mu\nu}(q) = -i \int d^4x\, e^{iq.x} << \mathcal{T}\left(\mathcal{V}_k^{\mu}(x),\, \mathcal{V}_k^{\nu}(0)\right) >>$$
$$\Pi_A^{\mu\nu}(q) = -i \int d^4x\, e^{iq.x} << \mathcal{T}\left(\mathcal{A}_k^{\mu}(x),\, \mathcal{A}_k^{\nu}(0)\right) >>$$

and the corresponding spectral functions (known from data in the vacuum)

$$-\frac{1}{\pi} Im\, \Pi_V^{\mu\nu}(q; T=0) = -(q^2 g^{\mu\nu} - q^\mu q^\nu)\, \rho_V(q^2)$$
$$-\frac{1}{\pi} Im\, \Pi_A^{\mu\nu}(q; T=0) = q^\mu q^\nu\, f_\pi^2\, \delta(q^2 - m_\pi^2) - \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right) \rho_A(q^2)$$

From chiral symmetry alone to order T^2 (chiral limit), the only medium effect is the « mixing » of the correlators (no mass shift)

$$\Pi_V^{\mu\nu}(q;T) = (1-\epsilon)\, \Pi_V^{\mu\nu}(q; T=0) + \epsilon\, \Pi_A^{\mu\nu}(q; T=0)$$
$$\Pi_A^{\mu\nu}(q;T) = (1-\epsilon)\, \Pi_A^{\mu\nu}(q; T=0) + \epsilon\, \Pi_V^{\mu\nu}(q; T=0)$$



The mixing is driven by the pion scalar density

$$\epsilon = \frac{T^2}{6\, f_\pi^2} = \frac{2}{f_\pi^2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{n(\omega_k)}{\omega_k} = \frac{2}{3} \frac{<< \Phi^2 >>}{f_\pi^2}$$

$$\frac{f_\pi^*(T)}{f_\pi} = \sqrt{1-\epsilon} \simeq 1 - \frac{T^2}{12\, f_\pi^2} = 1 - \frac{1}{3} \frac{<< \Phi^2 >>}{f_\pi^2}$$

This axial-vector mixing driven by in-medium pion loop effects
can be generalized for finite density and
is at the heart of the interpretation of the dilepton data (NA60)

Weinberg sum rules

Chiral symmetry breaking: a low energy long range phenomena

Vacuum

$$\int_0^\infty ds \left(\rho_V(s) - \rho_A(s)\right) = f_\pi^2, \qquad \int_0^\infty ds\, s \left(\rho_V(s) - \rho_A(s)\right) = 0$$

In-medium

$$\int_0^\infty d\omega^2 \left[\left(-\frac{Im\,\Pi_V(\omega, \mathbf{q}=0)}{\pi\,\omega^2}\right) - \left(-\frac{Im\,\Pi_A(\omega, \mathbf{q}=0)}{\pi\,\omega^2}\right)\right] = 0$$

$$\int_0^\infty d\omega^2\,\omega^2 \left[\left(-\frac{Im\,\Pi_V(\omega, \mathbf{q}=0)}{\pi\,\omega^2}\right) - \left(-\frac{Im\,\Pi_A(\omega, \mathbf{q}=0)}{\pi\,\omega^2}\right)\right] = 0$$

Pole ansatz

$$-\frac{Im\,\Pi_V(\omega, \mathbf{q}=0)}{\pi\,\omega^2} = \frac{m_\rho^4}{g_\rho^2}\frac{Z_\rho(T)}{\omega^2}\,\delta(\omega^2 - m_\rho^{*2}(T))$$

$$-\frac{Im\,\Pi_A(\omega, \mathbf{q}=0)}{\pi\,\omega^2} = \frac{m_A^4}{g_A^2}\frac{Z_A(T)}{\omega^2}\,\delta(\omega^2 - m_A^{*2}(T)) + f_\pi^{*2}(T)\,\delta(\omega^2)$$

Vacuum:

$$\frac{m_\rho^4}{g_\rho^2} = \frac{m_A^4}{g_A^2}, \qquad m_\rho^2 = a\, g_\rho^2\, f_\pi^2 \quad \text{avec} \quad a = \left(1 - \frac{m_\rho^2}{m_A^2}\right)^{-1}$$

(KSFR relation)

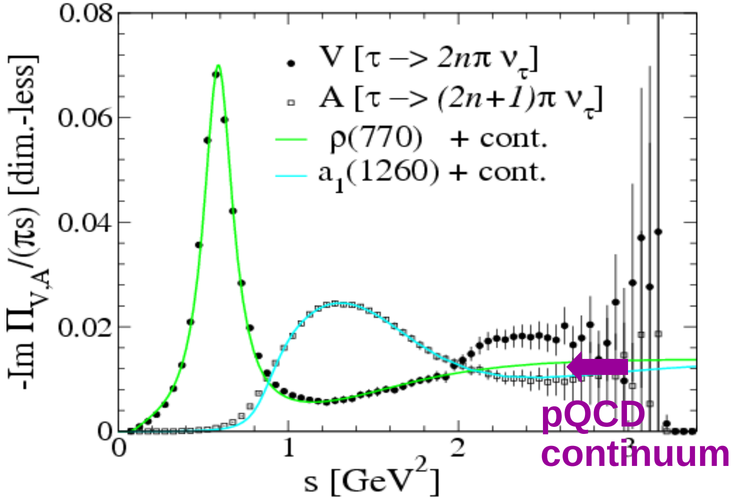
In-medium

$$\frac{f_\pi^{*2}(T)}{f_\pi^2} = a\, Z_\rho(T)\left(\frac{m_\rho^2}{m_\rho^{*2}(T)} - \frac{m_\rho^2}{m_A^{*2}(T)}\right)$$

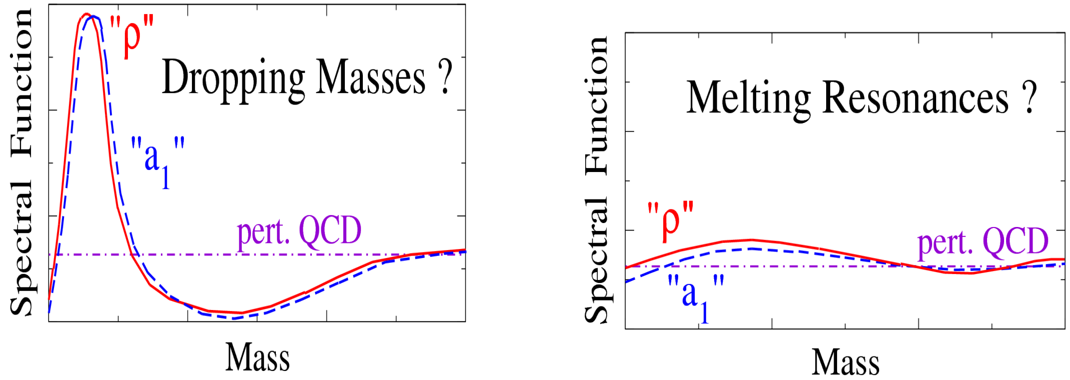
The centroids $m_\rho^{*2}(T)$ and $m_A^{*2}(t)$ becomes identical at full restoration

But we do not know the scenario !

Axial-vector / Vector in Vacuum

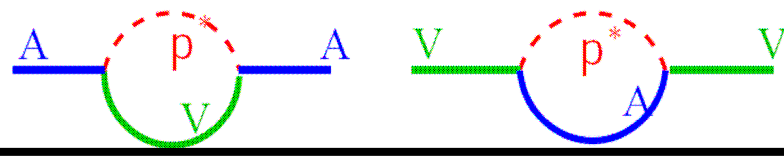


Axial-vector / Vector near Tc



Axial-vector / Vector at finite density

Axial =Vector + 1 pion from the medium



- Low-Mass Dilepton Rate:

$$\frac{dN_{ee}}{d^4x\,d^4q} = \frac{-\alpha^2}{\pi^3 M^2} f^B(T)\ Im\Pi_{em} \sim [ImD_\rho + ImD_\omega/10 + ImD_\phi/5]$$

ρ -meson dominated!

- Axialvector Channel: $\pi^\pm\gamma$ invariant mass-spectra ~ $Im\ D_{a1}(M)$?!

OPERATIONAL APPROACHES

AND EFFECTIVE THEORIES

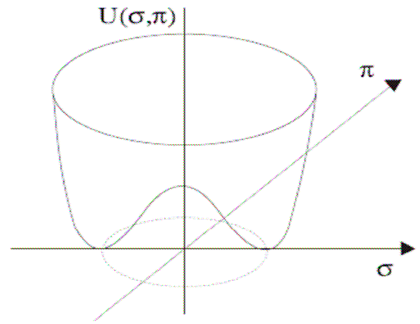
FOR LOW ENERGY QCD

1 - Chiral perturbation theory

Construct an « exact » copy of QCD in the low energy sector for light particles whereas heavy particles are frozen or taken as static sources.
Possible since there is a clear separation (mass gap) $\Lambda=4\pi f_\pi$ between light particles (Goldstone bosons) and heavy particle (ρ, σ, ω,…).

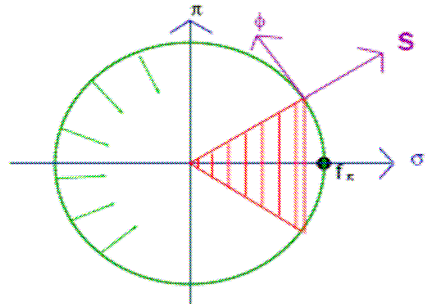
- Only colorless state (hadrons) + Chiral symmetry breaking
- Dynamical modes: fluctuations of the chiral condensate in the bottom of the chiral effective potential

$$M = \sigma + i\vec{\tau}\cdot\vec{\pi} \equiv S\,U$$



- In ChiPT, S frozen to is vacuum value on the « chiral circle »
- U has well defined chiral properties

$$U \rightarrow V_L\,U\,V_R^\dagger$$



THE CHIRAL CIRCLE

⇨ The QCD lagrangian is replaced by an effective one involving the U matrix representing the pions

$$\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{eff}(U, \partial U, \partial^2 U, \ldots)$$

$$U(x) = e^{i\vec{\tau}\cdot\vec{\phi}(x)/f}$$

Expansion of the lagrangian in powers of derivatives **(p_π/Λ)** and in power of the quark mass or the pion mass **(m_π/Λ)**

•Leading term

$$\mathcal{L}^{(2)} = \frac{f^2}{4} Tr[\partial_\mu U^\dagger \partial^\mu U] + \frac{f^2}{2} B_0 Tr[m(U + U^\dagger)]$$

The first term is highly constrained by symmetry (QCD LσM, NJL): f=fπ,
The « mass » term is not universal and depends on the (chiral symmetry breaking) QCD dynamics.

$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -f^2 B_0$ in the chiral limit $\quad m_\pi^2 = (m_u + m_d) B_0 \quad$ **(GOR relation)**

•Fourth order term

$$\mathcal{L}^{(4)} = \frac{l_1}{4}(Tr[\partial_\mu U^\dagger \partial^\mu U])^2 + \frac{l_2}{4} Tr[\partial_\mu U^\dagger \partial_\nu U] Tr[\partial^\mu U^\dagger \partial^\nu U]$$
$$+ \frac{l_3}{4} B_0^2 \left(Tr[m(U + U^\dagger)]\right)^2 + \frac{l_4}{4} B_0 Tr[\partial_\mu U^\dagger \partial^\mu U] Tr[m(U + U^\dagger)] + ...$$

Collect all Feynman diagrams generated by $\mathcal{L}_{eff}$. Classify all terms according to powers of a variable Q which stands generically for three-momentum or energy of the Goldstone bosons, or for the pion mass m_π. The small expansion parameter is $Q/4\pi f_\pi$. Loops are subject to dimensional regularisation and renormalisation.

The unknown coefficients are fixed phenomenologically (no real matching of the EFT to QCD)

- ππ, KK (SU(3) extension) scatterings → many successes
- Unitarized χPT
- Confirmation of the strong condensate scenario (validity of GOR)

•Inclusion of baryons

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \; u^2 = U$$
$$\chi = 2B\mathcal{M}$$

$$\mathcal{L}_N^{(1)} = \bar{\Psi}\left(i\gamma_\mu D^\mu - M_0\right)\Psi + \frac{1}{2} g_A \bar{\Psi}\gamma_\mu\gamma_5 u^\mu \Psi \,,$$
$$\mathcal{L}_N^{(2)} = c_1 \,\mathrm{Tr}(\chi_+)\bar{\Psi}\Psi - \frac{c_2}{4M_0^2}\mathrm{Tr}(u_\mu u_\nu)\left(\bar{\Psi}D^\mu D^\nu\Psi + \mathrm{h.c.}\right) + \frac{c_3}{2}\mathrm{Tr}(u_\mu u^\mu)\,\bar{\Psi}\Psi + \ldots$$

g_A=1.27, c_1 related to σ_N= 50 MeV which is the pion-nucleon sigma term

$$\sigma_N = m_q \frac{\partial M_N}{\partial m_q} = \langle N | m_q(\bar{u}u + \bar{d}d) | N \rangle$$
$$M_N = M_0 + \sigma_N$$

Many successfull applications

- Threshold pion photo et electroproduction, Compton scattering on nucleon
- Pion-nucleon scattering
- $K\bar{N}$ scattering,coupling to resonances via unitarized coupled channels
- NN interaction

•Limitations of ChiPT

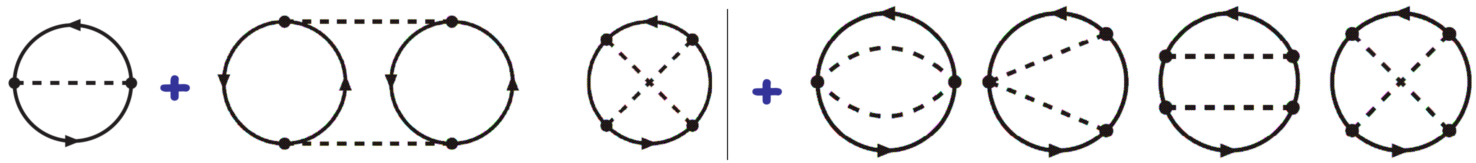
- The structure and the size of the nucleon is hidden: relative role of the pion cloud vs scalar field not known
- The scalar radial field is frozen: ChiPT has little to say for mass evolution, in-medium scalar polarization of the nucleon, mechanisms for chiral restoration
- Unitarization (account of resonances) by hand destroying power counting

2- In-medium Chiral perturbation theory

Three loop approximation *(Kaiser et al)*

πN interaction from ChiPT + expansion in m_π and $k_F \sim 2\, m_\pi$ ⟺ diagr. loop expansion

E/A= T +



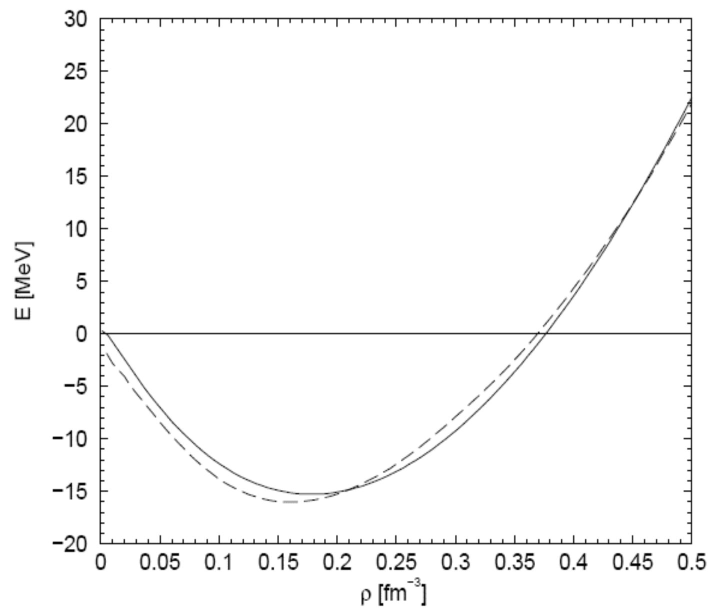
+ 23.4 +18.2 -68.3 +11.5

No short-range correlations but depend on one cutoff parameter Λ

The bulk of the attraction comes from two-pion exchange through a contact cut-off dependent term

Gives a correct asymetry a_4=34 MeV but inclusion of Δ(1232) improves isospin properties

But the spin-orbit not reproduced



Density functional theory

Relativistic mean field ($\sigma+\omega$) gives the correct (enhanced) spin-orbit

$$U(\vec{r}) = U_V + U_S \approx (+200 - 250)\frac{\rho(\vec{r})}{\rho_0}\, MeV \qquad U_{so}(\vec{r}) = \frac{\vec{l}\cdot\vec{s}}{2\,M_N\,r}\,\frac{d(U_V - U_S)}{dr} \approx \left(\frac{+200+250}{+200-250}\right)\frac{\vec{l}\cdot\vec{s}}{2\,M_N\,r}\,\frac{dU}{dr}$$

Hohenberg-Kohn energy density functionnal *(Finelli et al)*

$$F_{\rm HK}[\rho] = E_{\rm kin}[\rho] + E_{\rm H}[\rho] + E_{\rm xc}[\rho]$$

Pion loops (ChiPT)

From scalar and vector mean fields

Constrained by low energy QCD !

(QCD sum rules and condensate evolution)

$$\Sigma_S^{(0)} = -\frac{\sigma_N M_N}{m_\pi^2 f_\pi^2}\rho_S\,, \qquad \Sigma_V^{(0)} = \frac{4(m_u+m_d)M_N}{m_\pi^2 f_\pi^2}\rho$$

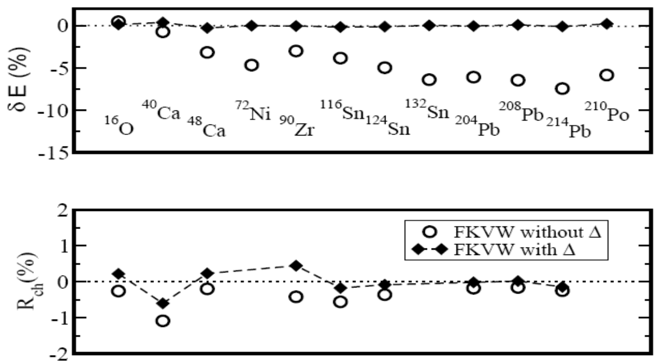
$$\frac{\Sigma_S^{(0)}}{\Sigma_V^{(0)}} = -\frac{\sigma_N}{4(m_u+m_d)}\frac{\rho_S}{\rho} \approx -1$$

But the pion cloud contribution to σ_N should be removed since it cannot contribute to the scalar self-energy, *i.e.*, to the mass !

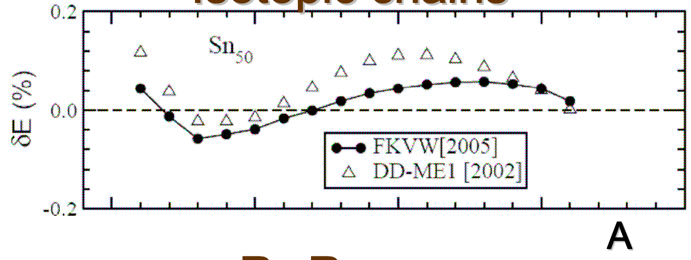
Practical solution / Kohn Sham DFT

-The Ground state density is build with auxiliary single particle orbitals in a self-consistent local potential built from the functionnal
-The Kohn-Sham equations are solved in an equivalent point coupling model which reproduces the self-energies resulting from $E_H(r)$ et Exc(r)

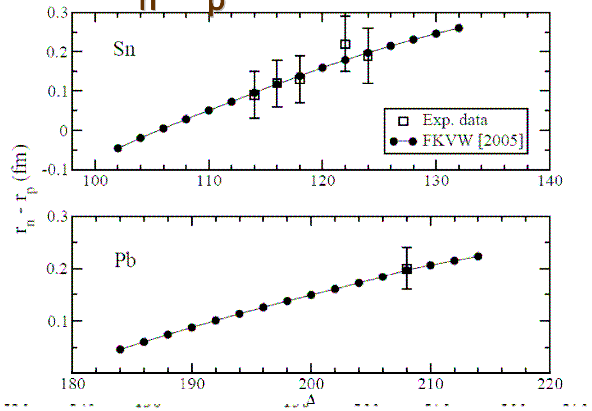
Influence of the Δ



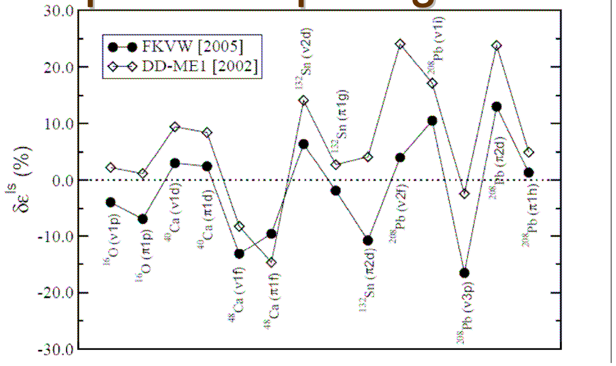
Isotopic chains



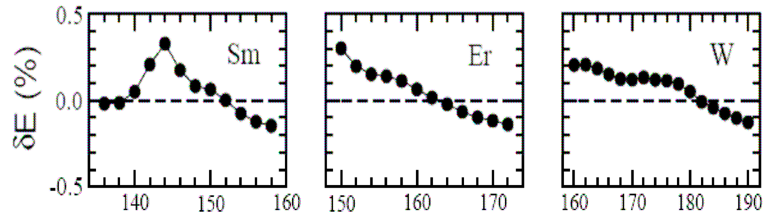
R_n-R_p



Spin-orbit spacings



Deformed nuclei



3- The QCD sum rules (QCDSR)

Basics of QCDSR

Relate the hadron spectral function (ρ,ω,ϕ) to quarks (and gluons) condensates

Current-current correlation function from its spectral representation

$$\Pi(q^2) = \frac{i}{3}\int d^4x\, e^{iq.x} < o|\mathcal{T}\left(J^\mu(x), J_\mu(0)\right)|0 >= \Pi(0) + q^2 \int_0^\infty \frac{ds}{s}\frac{\left(-\frac{1}{\pi}\right) Im\,\Pi(s)}{q^2 - s + i\eta}$$

Currents with quantum numbers of the hadron

$$J^\mu_\rho = \frac{1}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right), \quad J^\mu_\omega = \frac{1}{6}\left(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d\right), \quad J^\mu_\Phi = -\frac{1}{3}\bar{s}\gamma^\mu s$$

For large space-like momenta (Q^2=-q^2>0), use « OPE » (FT of Taylor expansion around x=0)

$$\frac{\Pi(q^2 = -Q^2)}{Q^2} = \int_0^\infty \frac{ds}{s}\frac{\left(-\frac{1}{\pi}\right) Im\,\Pi(s)}{s + Q^2}$$
$$= \frac{d_V}{12\,\pi^2}\left[-c_0\, ln\left(\frac{Q^2}{\mu^2}\right) + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \frac{c_3}{Q^6} + ...\right]$$

Spectral function

C_i~condensates

Problem: OPE valid at large Q^2 where many resonnances contribute to the dispersive integrals

The trick: Borel transform: $Q^2 \rightarrow M_B$

$$\frac{1}{M_B^2}\int \frac{ds}{s} e^{-s/M_B^2}\left(-\frac{1}{\pi}\right) Im\,\Pi(s) = \frac{d_V}{12\pi^2}\left[c_0 + \frac{c_1}{M_B^2} + \frac{c_2}{M_B^4} + \frac{c_3}{2\,M_B^6} + \ldots\right]$$

High part of the spectrum exponentially supressed: favorize low energy resonances

Choice for M_B: convergence of the integral and OPE: 1 GeV< M_B<1.5 GeV

Application: vector mesons in vacuum

$$R(s) = -\frac{12\pi}{s} Im\,\Pi(s) = \rho_{VDM}(s) + d_V\left(1+\frac{\alpha_S}{\pi}\right)\Theta(s-s_V)$$

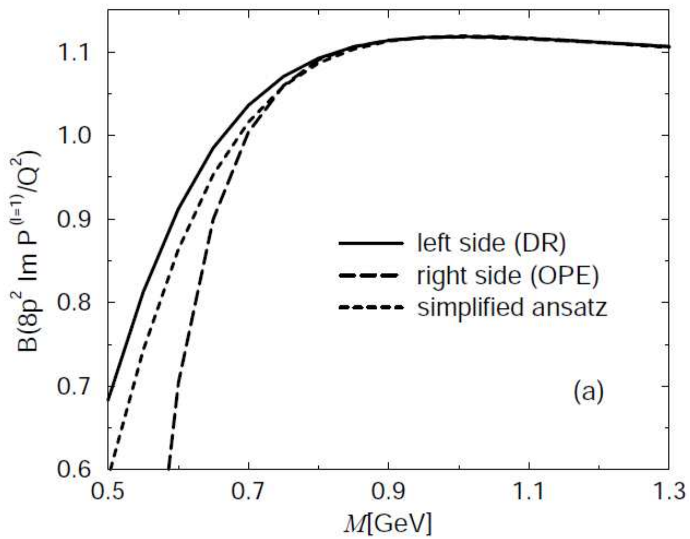
$$c_0^\rho = 1 + \frac{\alpha_S(Q^2)}{\pi}, \qquad c_1^\rho = -3(m_u^2 + m_d^2),$$

$$c_2^\rho = \frac{\pi^2}{3} < G\cdot G > + 4\pi^2 (m_u < \bar{u}u > + m_d < \bar{d}d >)$$

$$c_3^\rho \sim \text{“Condensats à quatre quarks”} \sim < (\bar{q}q)^2 >$$

Pole ansatz: $\rho_{VDM}(s) = \mathcal{F}_V\,\delta(s - m_V^2)$

$$m_{\rho,\omega} = 0.77\,GeV\,,\; m_\Phi = 1.02\,GeV$$



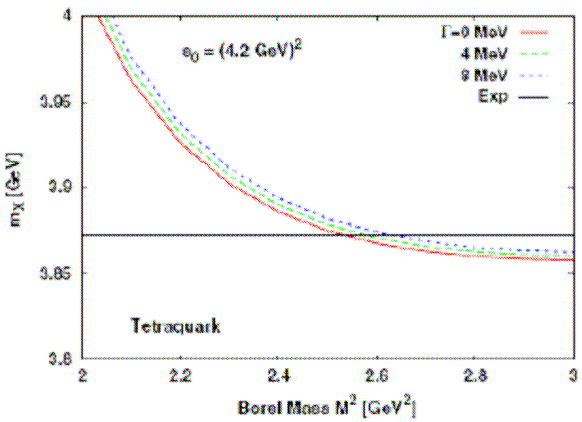
Analysis of the X(3872): Many new « exotic » charmonium states recently discovered: not (easily) explainable by quark models (BaBar, Belle)

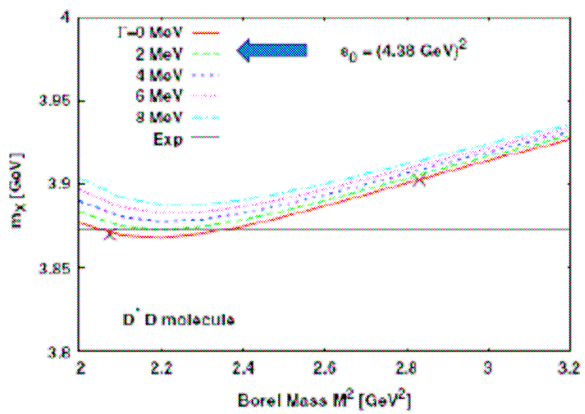
	J^{PC}	Special feature	QSR tetraquark	QSR molecule
X(3872) Γ<2.3	1^{++}	**B(X→Jπππ)/B(X→ππ)=1** **B(X→ψ'γ)/B(X→ψγ)=3**	[AV][S] m=3.92 (Nielsen ..)	DD* m=3.87 (Nielsen, ..)

$$[S]=q_a^T C\gamma_5 c_b,\quad [PS]=q_a^T C c_b,\quad [V]=q_a^T C\gamma_5\gamma_\mu c_b,\quad [AV]=q_a^T C\gamma_\mu c_b,$$

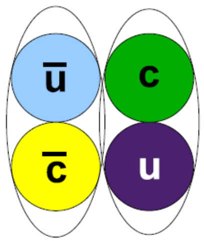
$$D0=\bar{q}c,\qquad D=i\bar{q}\gamma_5 c,\qquad D^*=\bar{q}\gamma_\mu c,\qquad D1=\bar{q}\gamma_\mu\gamma_5 c,$$

- J=[s][V] Tetraquark current vs. J=DD* Molecular current



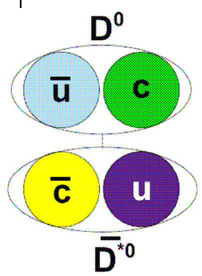


Nielsen
Navarra
SH Lee



antidiquark - diquark

X(3872) most probably a molecular state



D^0-$\bar{D}^{*0}$ "molecule"

In-medium hadrons

In the strongly interacting medium (finite ρ or T) the fundamental QCD condensates are modified:
The QCD Ground state is modified (change in the symmetry pattern of QCD):
we thus expect that the elementary excitations (hadrons) of the changed vacuum are also modified: change of mass, width, coupling constants,…

On the other hand in-medium changes of hadron (associated correlation functions) are usually calculated within hadronic many-body approach, i.e; chiral dynamics

The central question is thus to make a connection between the (observed or calculated) in-medium changes of hadrons and (precursor) effects of chiral symmetry restoration

Use QCD sum rules generalized at finite density
Bottom to top attitude: follow the evolution of the hadron spectral functions and relate it to the evolution of the QCD properties (condensates)

Pion propagation in nuclear matter

(Torino, Lyon, …)

- Pion-nucleon p-wave coupling

$$H_{\pi NN} = -\int d\mathbf{r}\, \frac{g_A}{f_\pi} \bar{N}\, \gamma^\mu\, \gamma_5\, \frac{\vec{\tau}}{2} . \partial_\mu \vec{\Phi}\, N \simeq -\int d\mathbf{r}\, \frac{g_A}{2\, f_\pi}\, N^\dagger\, \boldsymbol{\sigma} . \boldsymbol{\nabla} \vec{\Phi} . \vec{\tau}\, N$$

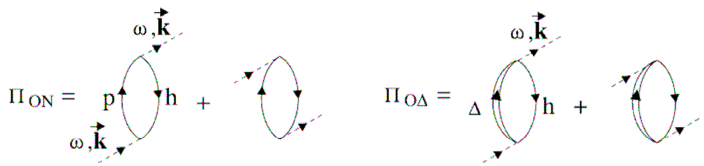
Coupling to Δ states

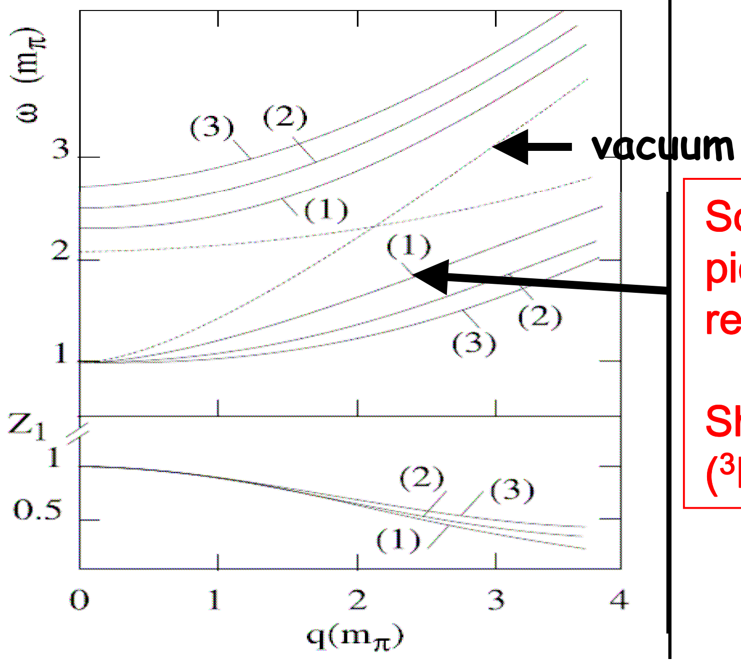
$$\boldsymbol{\sigma} . \mathbf{q}\, \tau_j \longrightarrow (g_{\pi N \Delta} / g_{\pi NN})\, \mathbf{S} . \mathbf{q}\ \mathbf{T_j}$$

- In-medium pion propagator

$$D(\omega, \mathbf{k}) = \left(\omega^2 - \omega_k^2 - S(\omega, \mathbf{k}) \right)^{-1}$$

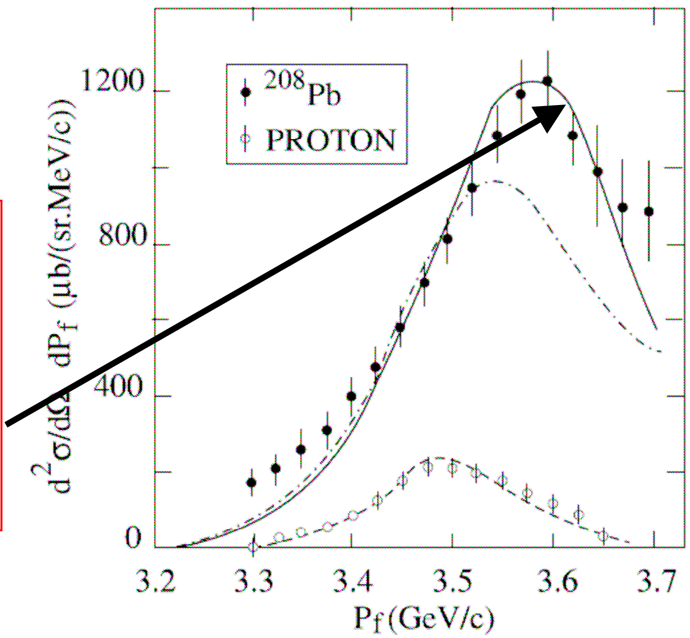
At high energy the strength function is dominated by two (collective) excitations
- The pionic branch Ω_1
- The Delta branch Ω_2





Softening of the pion dispersion relation

Shift of the strength (^{3}He,T), SATURNE



Scalar-isoscalar modes (« sigma meson ») at finite density

•Original idea: Softening of the pion dispersion relation
→ Modification of the two-pion propagator and the unitarized pion-pion interaction → modification of two pion states : correlator in the sigma meson and rho meson channels

•Scalar-isoscalar modes

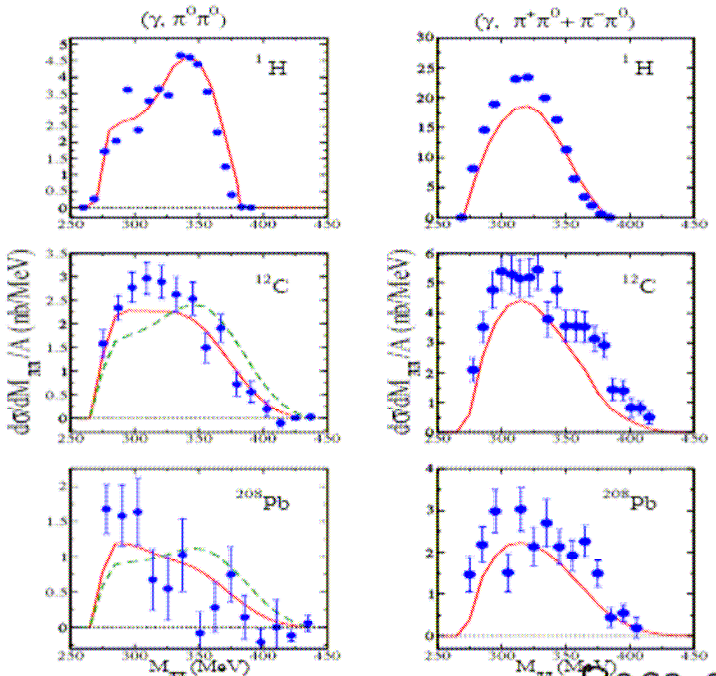
- $A(\pi,\pi\pi)$ (CHAOS, CB), $A(\gamma,\pi\pi)$(TAPS)

Downwards shift of the $\pi\pi$ invariant mass distribution in the scalar-isoscalar channel $I = J = 0$

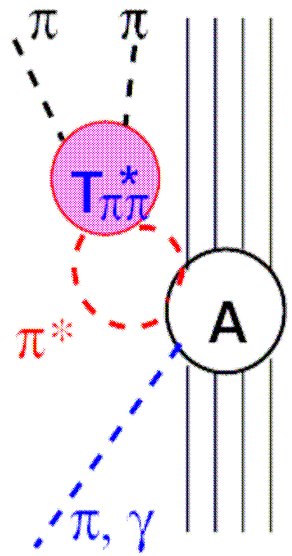
- What is the role of

- Chiral Dynamics

- Chiral restoration ?



Roca et al



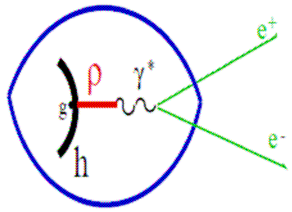
Connection with the pionic enhancement of the scalar (chiral) susceptibility

The rho meson at finite density

Rho meson propagator from the Vector dominance phenomenology

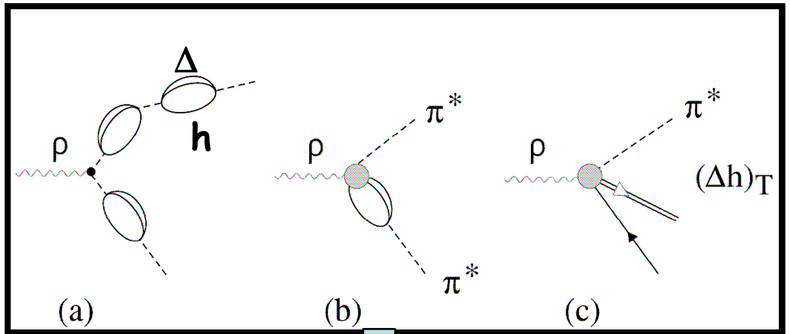
$$V = \rho, \omega, \Phi$$

$$J_V^\mu = \frac{m_V^2}{g_V} \mathcal{V}^\mu$$

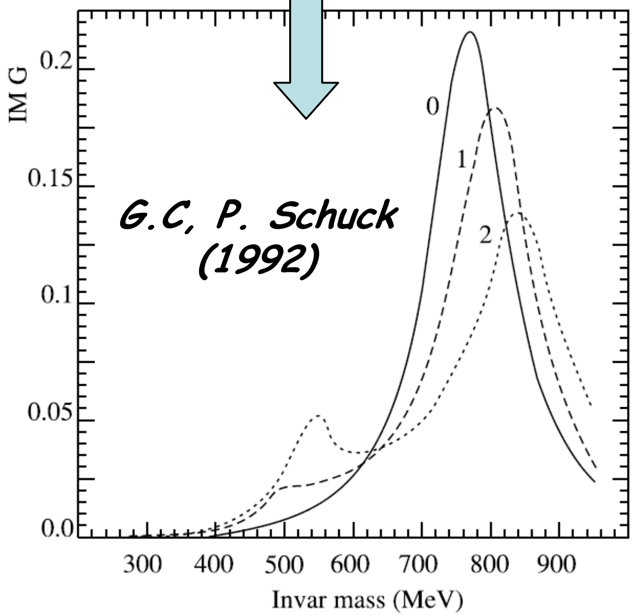


Gauged non linear sigma model Lagrangian

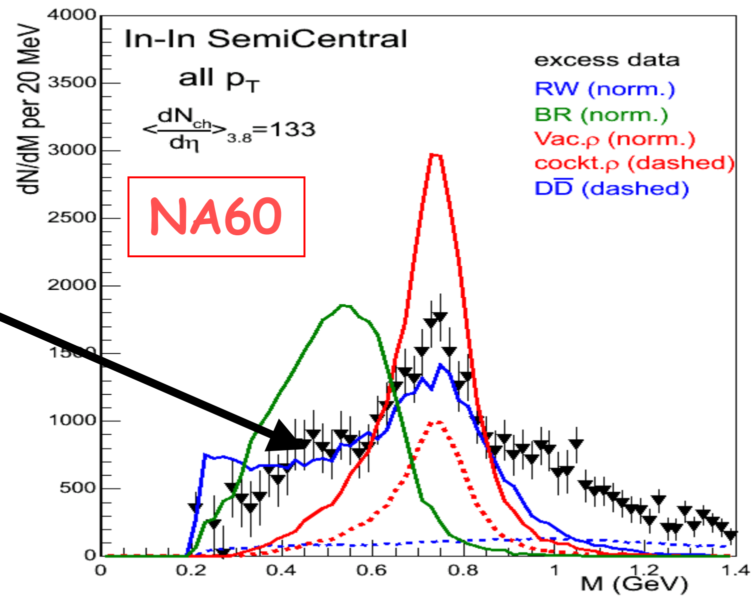
$$\mathcal{L}_{\rho h} = -g\,\vec{\rho}^{\mu}\left(\vec{\Phi}\times\partial_\mu\vec{\Phi}\right) - g\,\bar{N}\gamma_\mu\vec{\rho}^{\mu}.\frac{\vec{\tau}}{2}N - g\,\frac{g_{\pi NN}}{M_N}\bar{N}\gamma_\mu\gamma_5\left(\vec{\rho}^{\mu}\times\vec{\Phi}\right).\frac{\vec{\tau}}{2}N + \frac{g^2}{2}\left(\vec{\rho}^{\mu}\times\vec{\Phi}\right).\left(\vec{\rho}_{\mu}\times\vec{\Phi}\right)$$



(a) Decay of the ρ into two softened collective quasi-pions
(b) From gauge invariance: kill the accumulation of strength near $2m_\pi$
(c) Structure at $\Omega_\Delta + m_\pi \sim 500$ MeV



+ direct coupling of the rho to resonances



In-medium QCD sum rules

$$\Pi^{\mu\nu}(q) = -i \int d^4x \, e^{iq.x} \, < A(\rho) | \mathcal{T}(J^\mu(x), J^\nu(0)) | A(\rho) > \Big|$$

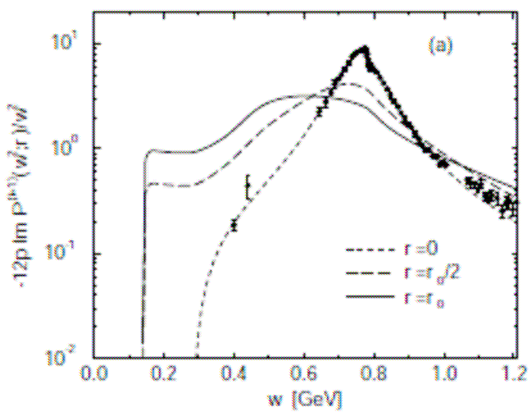
$$\frac{1}{M_B^2}\left[\Pi(0) + \int \frac{d\omega^2}{\omega^2} e^{-\omega^2/M_B^2} \left(-\frac{1}{\pi}\right) Im\,\Pi(\omega,0)\right] = \frac{d_V}{12\,\pi^2}\left[c_0 + \frac{c_1(\rho)}{M_B^2} + \frac{c_2(\rho)}{M_B^4} + \frac{c_3(\rho)}{2\,M_B^6} + ...\right]$$

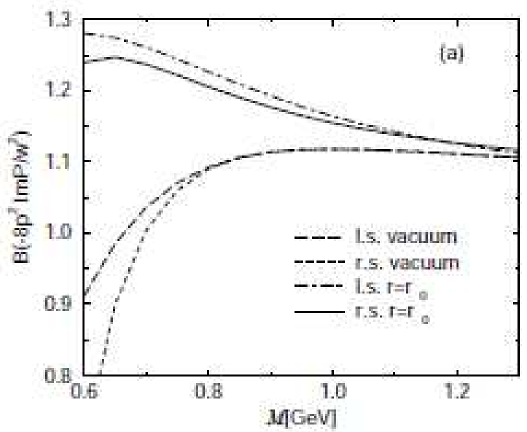
Early analysis

ρ meson: loses its quasiparticle status : a simple pole ansatz leads to an erroneous dropping mass scenario !

$$\frac{m_\rho^*}{m_\rho} = \frac{m_\omega^*}{m_\omega} = 1 - 0.18\,\frac{\rho}{\rho_0}$$

Hadronic calculation: QCDSR analysis comptible with a broadenning





(Klingl et al)

The rho meson in a scenario of pure chiral restoration *(Leupold et al)*

Aim: isolate the role of chiral restoration on rho meson spectral function)

$$\frac{1}{\pi}\int_0^\infty ds\, s^{-1}\,\mathrm{Im}\,\Pi(s)\, e^{-s/M^2} = c_0 M^2 + \sum_{i=1}^{\infty} \frac{c_i}{(i-1)!\, M^{2(i-1)}}$$

$$\begin{aligned}
c_0 &= \frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right), \\
c_1 &= -\frac{3}{8\pi^2}(m_u^2+m_d^2), \\
c_2 &= \frac{1}{2}\left(1+\frac{\alpha_s}{4\pi}C_F\right)\left(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle\right) + \frac{1}{24}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + N_2 \\
c_3 &= -\frac{112}{81}\pi\alpha_s\langle\mathcal{O}_4^V\rangle - 4N_4
\end{aligned}$$

Lhs: $\mathrm{Im}\Pi(s\le s_+) = \frac{F_0}{\pi}\frac{\sqrt{s}\Gamma(s)}{(s-m_0^2)^2+s\Gamma^2(s)}$ $\Gamma(s) = \Theta(s-4m_\pi^2)\Gamma_0\left(1-\frac{4m_\pi^2}{s}\right)^{\frac{3}{2}}\left(1-\frac{4m_\pi^2}{m_0^2}\right)^{-\frac{3}{2}}$

Rhs Drop only « chirally odd » (not chiral invariant) condensates

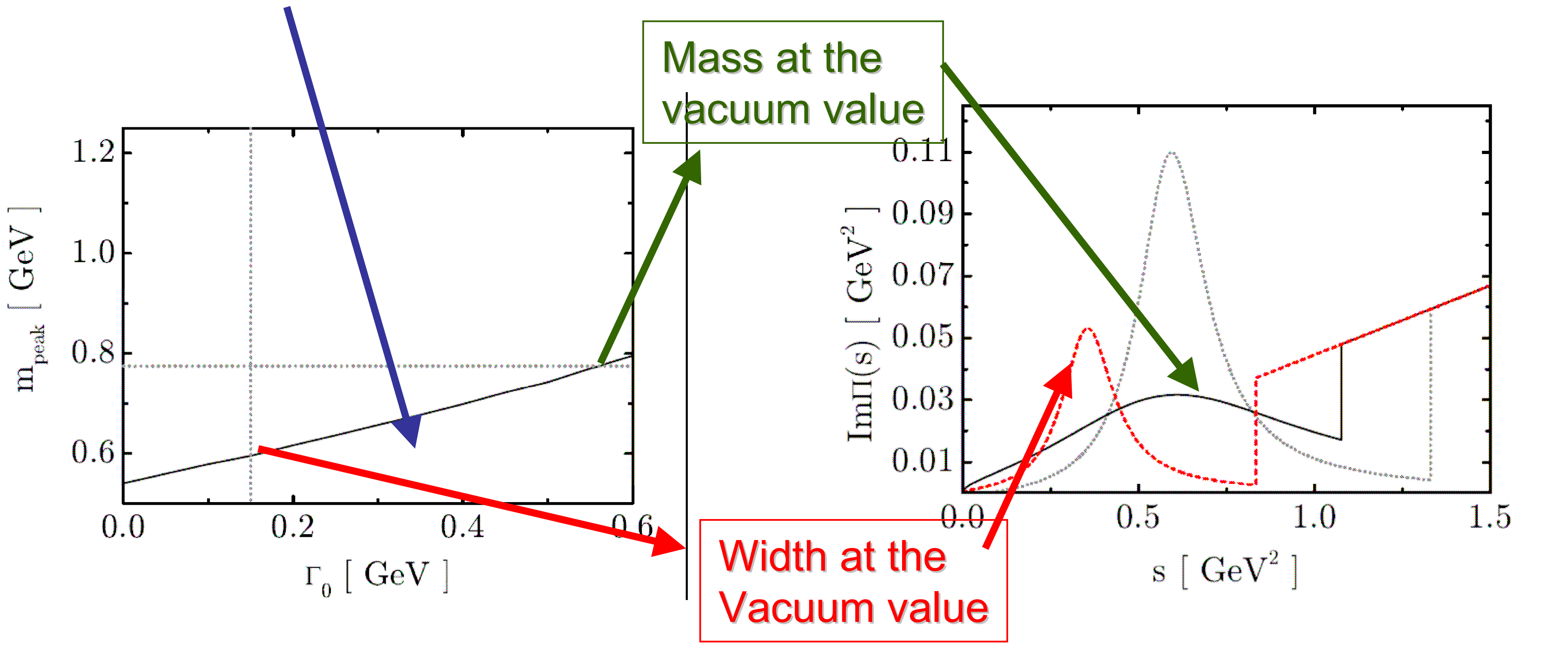
Four quark condensate: $\langle\mathcal{O}_4^V\rangle = \langle\mathcal{O}_4^{\mathrm{sym}}\rangle + \langle\mathcal{O}_4^{\mathrm{br}}\rangle$

$$\langle\mathcal{O}_4^{\mathrm{br}}\rangle = -\frac{81}{112}\left\langle\left(\bar\psi_R\gamma_\mu\lambda^a\tau_3\psi_R\right)\left(\bar\psi_L\gamma^\mu\lambda^a\tau_3\psi_L\right)\right\rangle$$

$$\langle\mathcal{O}_4^{\mathrm{br}}\rangle_{\mathrm{vac}} \approx \frac{9}{7}\langle\bar{q}q\rangle_{\mathrm{vac}}^2$$

Fix all the parameters : mass and width of the rho , condensates from a QCDSR in vacuum

- QCDSR analysis at full restoration (chirally odd four-quark condensate vanishes)
- Extract the (mass, width) of the rho meson at full restoration

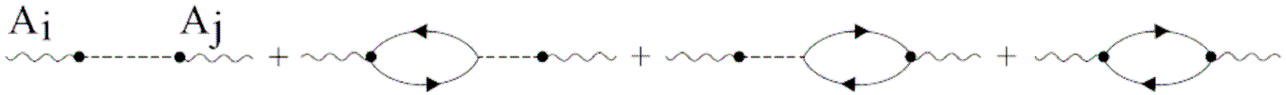


- QCD SR analysis compatible with both a dropping mass and and a broadening
- In all the cases , QCDSR requires more strength below the rho peak
- Sizeable broadening favorized by model calculations (chiral dynamics)

  (pion cloud, resonances)

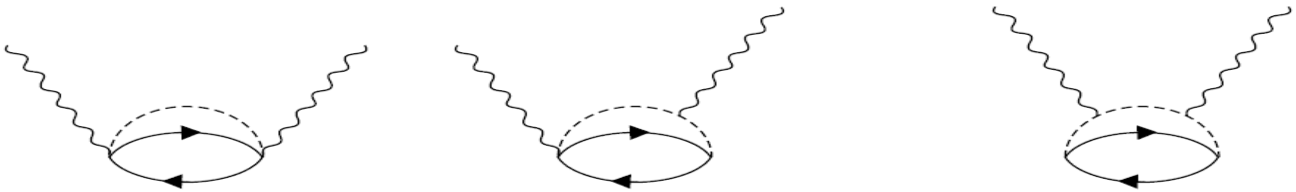
Connection with chiral restoration *(G.C, J. Delorme, M. Ericson)*

Axial correlator at finite density

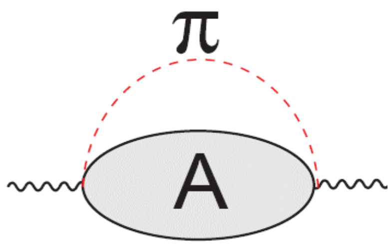


$$\frac{1}{f_\pi^2}A_{ij}(k) = k_ik_jD(k)+2k_ik_j\tilde{\Pi}_0(k)D(k)+\hat{k}_i\hat{k}_j\Pi_L(k)+(\delta_{ij}-\hat{k}_i\hat{k}_j)\Pi_T(k)$$
$$= k_ik_j(1+\tilde{\Pi}_0(k))^2D(k)+\hat{k}_i\hat{k}_j\tilde{\Pi}_0(k)+(\delta_{ij}-\hat{k}_i\hat{k}_j)\Pi_T(k)\ .$$

Vector correlator at finite density: the pion cloud contribution contains a correlator mixing effect at finite density



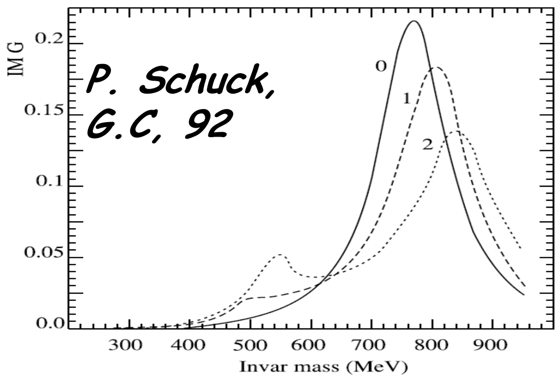
$$K_{ij}(q) = \int \frac{id^4k_1}{(2\pi)^4}\Big(\frac{1}{f_\pi^2}(A_{ij}(k_1)\,D(k_2)\,+\,A_{ij}(k_2)\,D(k_1)) -(1+\tilde{\Pi}_0(k_1))\,(1+\tilde{\Pi}_0(k_2))(k_{1i}\,k_{2j}+k_{1j}\,k_{2i})\,D(k_1)\,D(k_2)\Big)$$



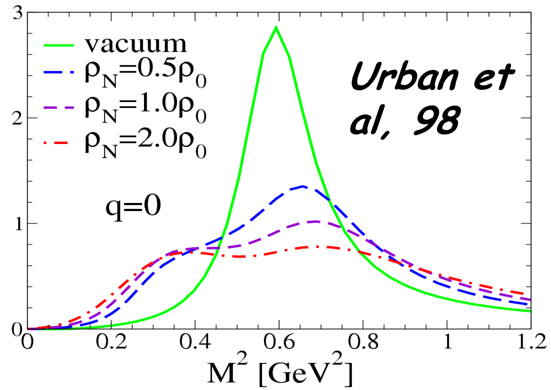
Axial vector mixing: Vector ⟷ Axial vector + In-medium pion

The in-medium rho meson within the ages

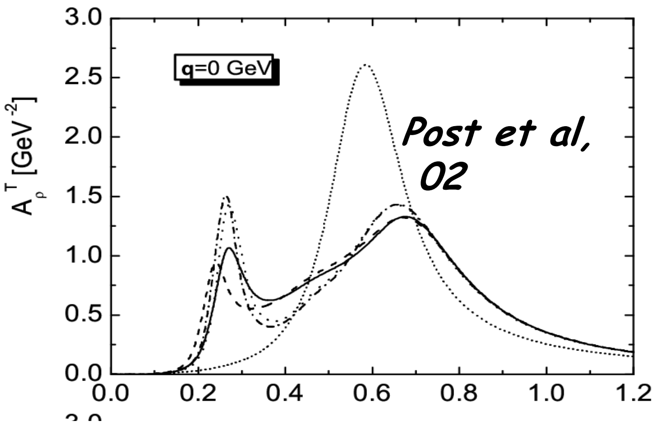
In-medium π cloud



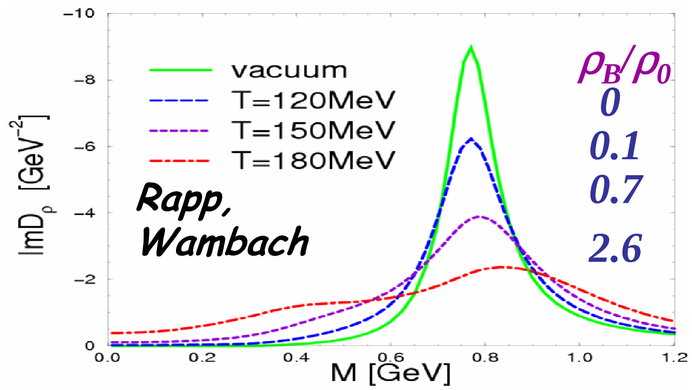
In-medium π cloud+ ρN ⟶ B*



ρN B*

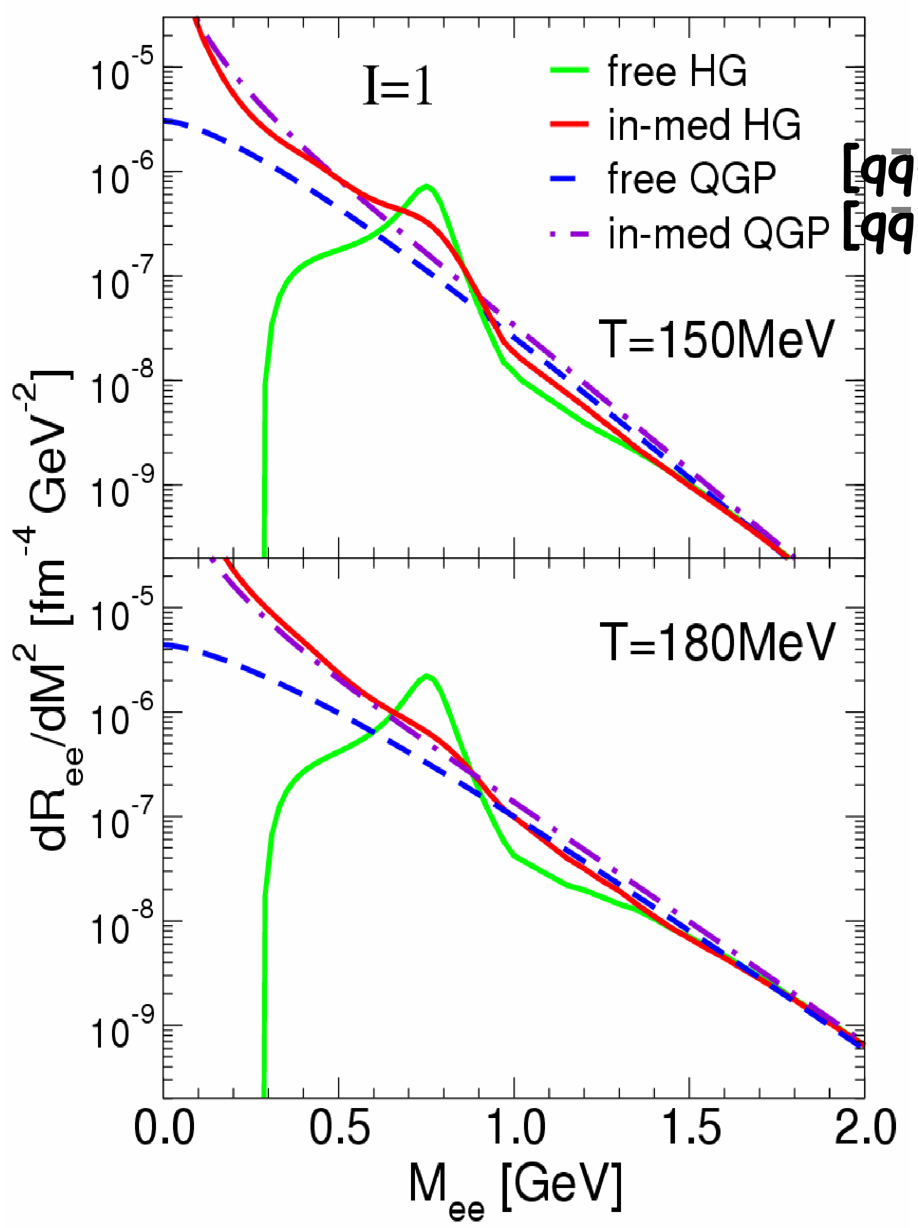


In-medium π cloud+ ρN ⟶ B* +meson gas



Strong broadening/melting of the ρ ⟶ pQCD continuum
Baryon density more important than T

Lowering of the quark-hadron duality threshold as a signature of chiral restoration



[$q\bar{q} \to ee$]

[$q\bar{q} + \mathcal{O}(\alpha_s)$-HTL]

[Braaten,Pisarski+Yuan '90]

- Hard-Thermal-Loop result much enhanced over Born rate
- "matching" of HG and QGP automatic!
- Quark-Hadron Duality at low mass ?!
- Degenerate axial and vector correlators?

QCD PHASES
AND SYMMETRIES

CHIRAL RESTORATION
AND DECONFINEMENT

Thermodynamics and phase structure

We use preferentially a description in terms of intensive variables

Temperature: T

Chemical potential associated with conservation law: baryonic chemical potential: μ

Pressure: $P = P(T, \mu)$ (equation of state)

Why ? - System in thermodynamic equilibrium: P, T, μ uniform (phase coexistence)
- Lattice calculation done with T, μ as control parameters (Grand canonical)
- Particle production in HIC from a thermal source anf fireball evolution constrained by conservation laws

Uniform system

$$dP = \sigma dT + \rho d\mu$$

Density of extensive quantities from the first derivatives of the pressure

$$\sigma = \frac{S}{V} = \left(\frac{\partial P}{\partial T}\right)_{\mu} \qquad \rho = \frac{N}{V} = \left(\frac{\partial P}{\partial \mu}\right)_{T} \qquad \epsilon = \frac{E}{V} = T\sigma + \mu\rho - P$$

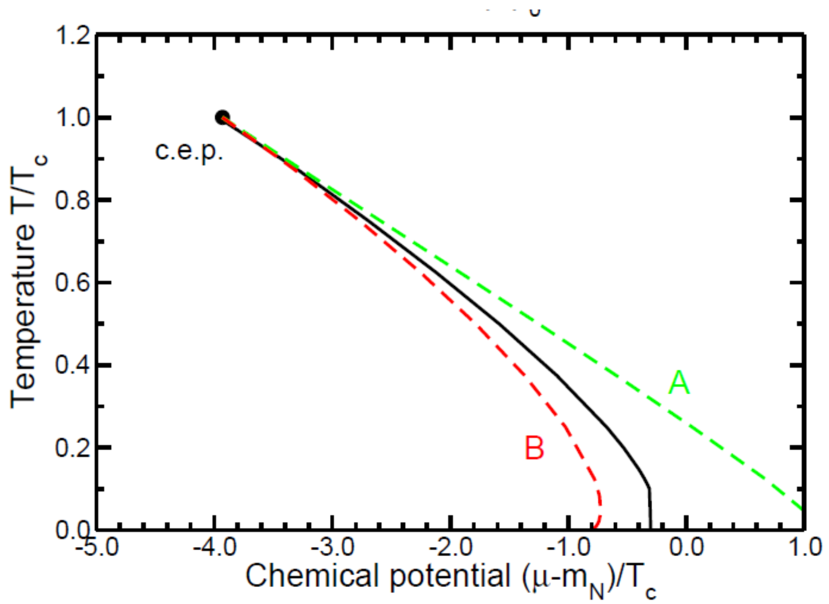
These quantities can differ from one phase to another phase

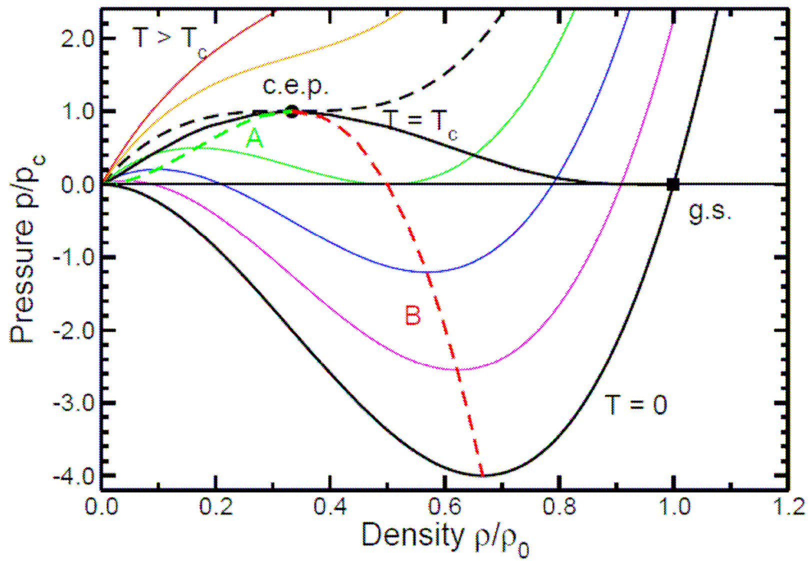
First order phase transition

Given T, μ: coexistence of two phases with two different « mechanical » densities

$$(\rho_1, \epsilon_1) \qquad\qquad (\rho_2, \epsilon_2)$$

Ex: Nuclear Liquid-gas transition





Phase coexistence terminates at a certain critical point (T_c, μ_c) where there is no longer discontinuity in (ρ, ε) but the susceptibilities may diverge

$$C = T\left(\frac{\partial \sigma}{\partial T}\right)_\mu \qquad \sim |T - T_c|^{-\alpha}$$

$$\chi_B = \left(\frac{\partial \rho}{\partial \mu}\right)_T$$

Second order transition
α: critical exponent
(universality class)

QCD phase transition and chiral symmetry

In some systems (spin system, QCD in the chiral limit) it may happen that there is an underlying exact symmetry: different phases with different symmetry realization

- Trivial case, symmetry restored: the chiral transform maps the ensemble on itself
- Spontaneously broken symmetry: chiral transformation creates another ensemble with the same (ε, ρ). Another variable is needed: ORDER PARAMETER

$$\langle H_{\chi SB}\rangle = \int d^3r\, 2m\, \langle \bar{q}q\rangle \Big| \longrightarrow P(T,\mu,m) \longrightarrow \langle \bar{q}q\rangle = -\frac{1}{2}\left(\frac{\partial P(T,\mu,m)}{\partial m}\right)_{T,\mu}$$

- The symmetry breaking transition may very well be of first order: the order parameter, as the other first derivatives of the pressure (σ, ρ), is discontinuous

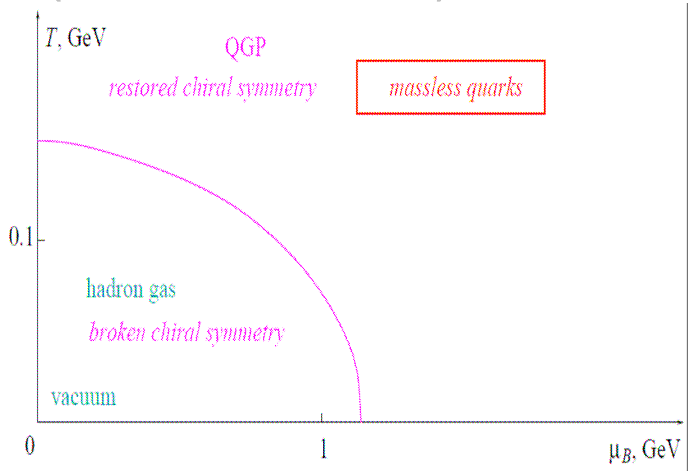
- It may happen that the first derivatives of the pressure (σ, ρ, $\langle \bar{q}q\rangle$) are continuous
  But since the transition connects two states with different symmetry pattern, thermodynamic quantities should exhibit singularities .
  Susceptibilities (second derivatives of the pressure) may diverge

$$C = T\left(\frac{\partial\sigma}{\partial T}\right)_{\mu} \qquad \chi_B = \left(\frac{\partial\rho}{\partial\mu}\right)_T\Big| \qquad \chi_S = \left(\frac{\partial\langle\bar{q}q\rangle}{\partial m}\right)_{T,\mu}\Big| \qquad \sim |T-T_c|^{-\alpha}$$

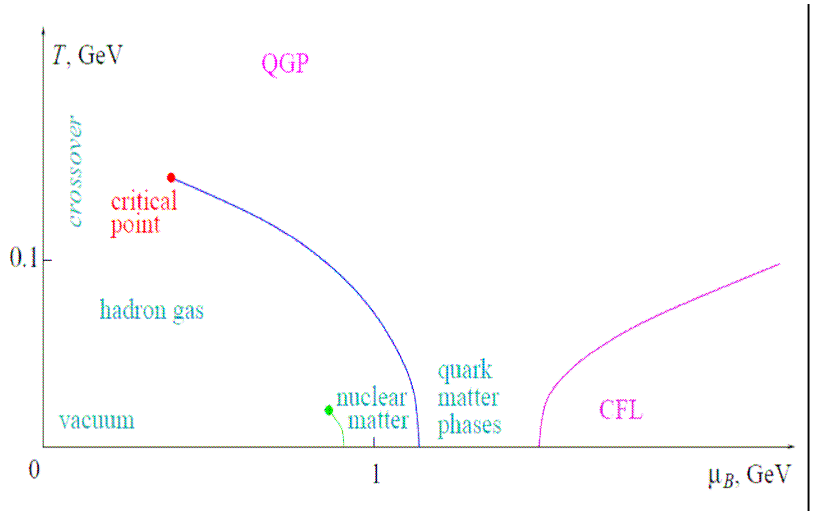
Second order phase transition

Universality class:
SU(2)XSU(2) lσm ~ O(4)
O(4) ferromagnet

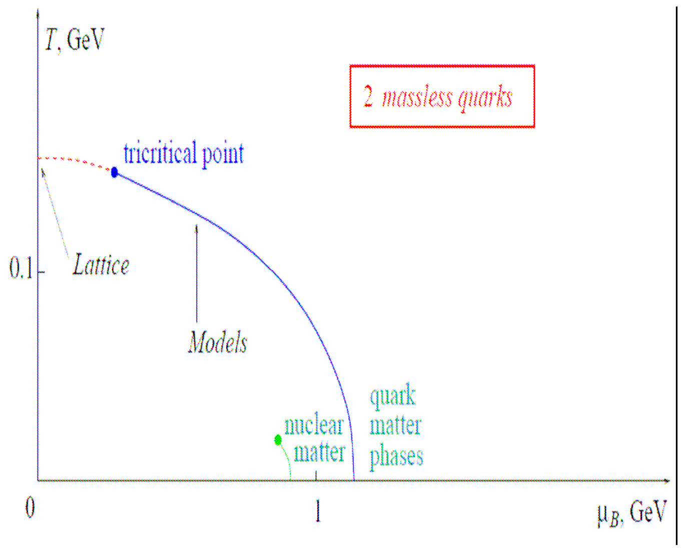
Universitality argument (Pisarski-Wilczeck)

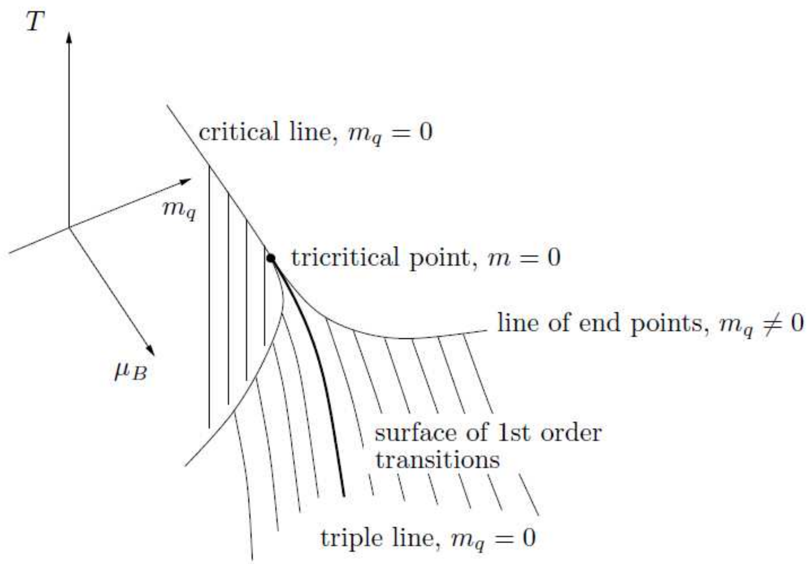


Real QCD: phyical quark masses



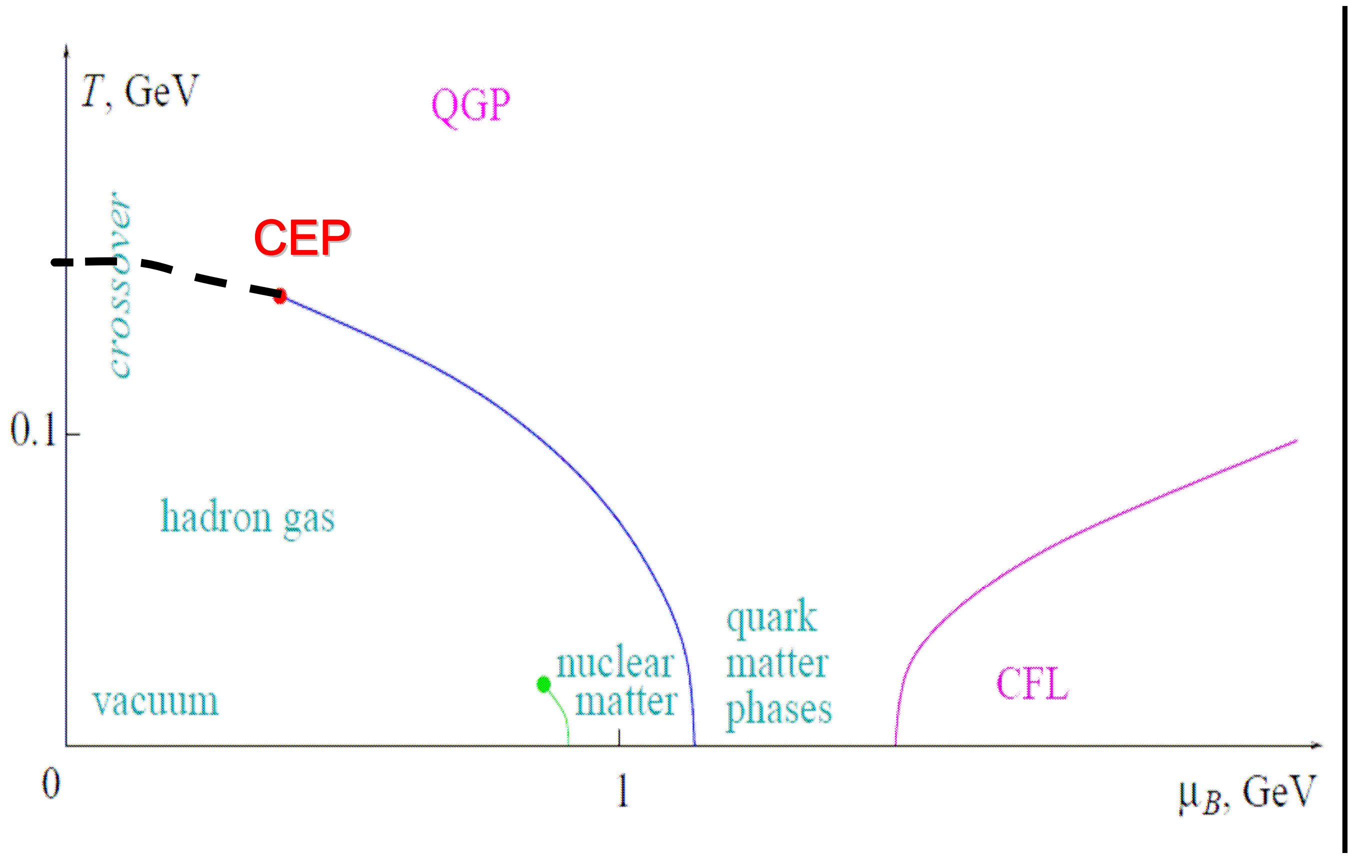
Nf=2 chiral limit





Real QCD: first order transition which terminates at the Critical End Point (CEP)

- Low μ, high T: continuous (although sudden) crossover
- Many similarities with the phase diagram of water



μ= 0 crossover : T=175-185 MeV (Bielefield)

CEP: T_E=162 MeV, μ_E= 360 MeV (Fodor et al)

QCD phase diagram and heavy ion collisions

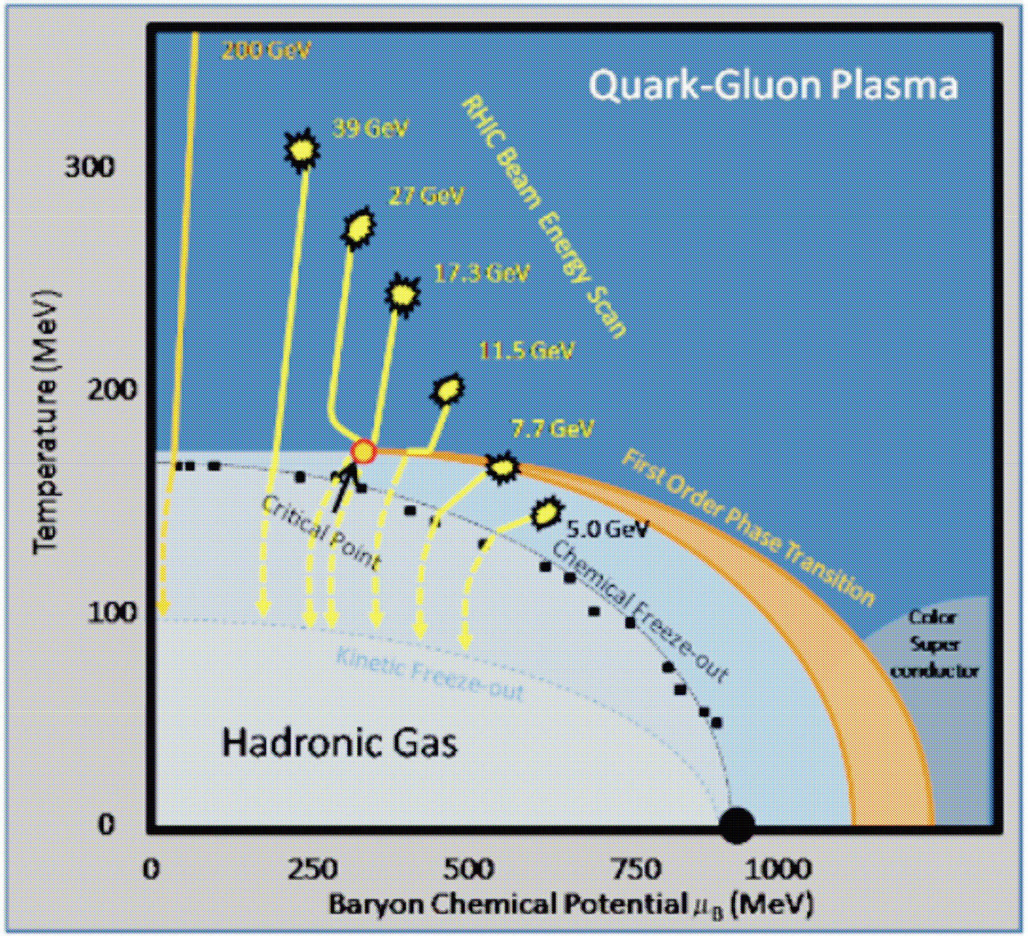


FIG. 1: A schematic of the phase diagram of nuclear matter. The location of the CP is placed within the RHIC BES range. Lattice QCD estimates [17–19] indicate that the CP falls within the interval 250 $<\mu_B<$ 450 MeV. The black closed circles are current heavy-ion experimental calculations of the chemical freeze-out temperature, T_{ch}, and μ_B based on statistical model fits to the measured particle ratios. The yellow curves show the estimated trajectories of the possible collision energies at RHIC.

QCD partition function

Partition function

$$Z = Tr\, e^{-\beta H} = \sum_n \langle n | e^{-\beta H} | n \rangle$$

$$H = xp - L$$

$$p = \partial L / \partial \dot{x}$$

Quantum mechanics $|n\rangle = |x\rangle$ Gauge theory $|n\rangle = \left| \vec{A}(\vec{r}), \psi(\vec{r}) \right\rangle$

Since A_0 has no conjuguate momentum: $-i \frac{\partial \Psi}{\partial A_a^0} = 0$

It follows that Gauss law has to be implemented at the level of physical states

$$\left(D_i^{ab} E_i^b - \rho_a \right) |\Psi\rangle = 0$$

$$D_i^{ab} E_i^b = \partial_i E_i^a - g f_{adb} \vec{A}_d \cdot \vec{E}_b$$

$$\Pi_j^a = -G_0^a = \partial_i A_a^0 + \partial_0 A_a^i - g f_{abc} A_b^0 A_c^i = -E_i^a$$

$$\rho_a = \psi_f^\dagger \frac{\lambda_a}{2} \psi_f$$

And the quantized form of gauge invariance is

$$\Psi \left(\vec{A}, \psi(\vec{r}) \right) = \Psi \left(h \left(\vec{A} + i \vec{\nabla} \right) h^\dagger, h\psi(\vec{r}) \right)$$

h(r) time independent gauge transform

⬇

$$Z = Tr\, e^{-\beta H} = \prod_{\vec{r}} \int \mathcal{D}A(\vec{r}) \int \mathcal{D}\psi(\vec{r}) \quad \langle A(\vec{r}), -\psi(\vec{r}) | e^{-\beta H} \delta \left(D_i^{ab} E_i^b - \rho_a \right) | A(\vec{r}), \psi(\vec{r}) \rangle$$

Euclidean action and center symmetry of QCD

The partition function admits a path integral representation

$$Z(\beta) = \prod_{\vec{r},0<x_4<\beta} \int \mathcal{D}A(\vec{r},x_4) \int \mathcal{D}\psi(\vec{r},x_4) \int \mathcal{D}\bar{\psi}(\vec{r},x_4)\; e^{-S_E}$$

Euclidean action

$$S_E = \int_0^\beta dx_4 \int d^3r\, L_E\left(A_4,\, \vec{A},\, \psi\right)(\vec{r},x_4)$$

with $L_E\left(A_4,\, \vec{A},\, \psi(\vec{r})\right)(\vec{r},x_4) = -\mathcal{L}_M\left(A_0 = -iA_4,\, \vec{A},\, \psi(\vec{r})\right)(\vec{r},x_0 = -ix_4)$

Periodic boundary conditions
(trace operation)

$$A(\vec{r},x_4+\beta) = A(\vec{r},x_4), \qquad \psi(\vec{r},x_4+\beta) = -\psi(\vec{r},x_4)$$

- Gauge invariant L_E : $\quad A_\mu \rightarrow {}^hA_\mu = hA_\mu h^\dagger + ih\partial_\mu h^\dagger, \qquad \psi \rightarrow {}^h\psi = h\psi$
- Gauge invariant action
(Periodic gauge transform) $\quad {}^hA_\mu(\vec{r},x_4+\beta) = {}^hA_\mu(\vec{r},x_4), \qquad {}^h\psi(\vec{r},x_4+\beta) = -{}^h\psi(\vec{r},x_4)$
- Transformation periodic up to a global (constant) f $\quad h(\vec{r},x_4+\beta) = f\, h(\vec{r},x_4)$

$$f \in Z(3) \Rightarrow f = z\,I \qquad z = e^{2i\pi n/3} \qquad n = 1,2,3$$

$$ {}^hA_\mu(\vec{r},x_4+\beta) = f\,{}^hA_\mu(\vec{r},x_4)f^\dagger \equiv {}^hA_\mu(\vec{r},x_4)$$

$$ {}^h\psi_\mu(\vec{r},x_4+\beta) = -z\,{}^h\psi_\mu(\vec{r},x_4)$$

- The pure gauge action has the center symmetry Z(3)
- Z(3) broken by quarks

The Polyakov loop as an order parameter for deconfinement

Pure gauge: put a static (infinitely heavy) static quark Q at point R

Quark-glue interaction $$S_Q = \int_0^\beta dx_4 \int d^3r \left(gt_a \, \delta(\vec{r} - \vec{R})\right) (-iA_4^a)(\vec{r}, x_4) = -ig \int_0^\beta dx_4 \, A_4(\vec{R}, x_4)$$

The partition function is $$Z_Q(\beta, \vec{R}) = \int \mathcal{D}\left[A, \psi, \bar{\psi}\right] e^{-S_E(pure\, gauge)} \, L(\vec{R})$$

Polyakov loop: $$L(\vec{R}) = \frac{1}{N_c} Tr_c \, exp\left[ig \int_0^\beta dx_4 \, A_4(\vec{R}, x_4)\right]$$

→ $$\left\langle L(\vec{R}) \right\rangle = \frac{1}{Z(Glue)} \int \mathcal{D}\left[A, \psi, \bar{\psi}\right] e^{-S_E(pure\, gauge)} \, L(\vec{R}) = \frac{Z_Q}{Z(Glue)} = e^{-\beta F_Q(\vec{R})}$$

- The Polyakov loop is gauge invariant but NOT Z(3) invariant $L(\vec{R}) \rightarrow z\, L(\vec{R})$
- If color is confined (low T), the free energy: $F_Q = +\infty$ $\Phi = \langle L \rangle = 0$
- If color is not confined (high T) $\Phi \neq 0$ $\Phi(T = \infty) = 1$

The Polyakov loop is an order parameter for the center Z(3) symmetry associated with confinement/deconfinement for pure glue

The Polyakov loop as an order parameter for deconfinement in pure gauge theory

QCD partition function $$Z = Tr\, e^{-\beta H} = \sum \langle n | \, e^{-\beta H} \, | n \rangle = \int [d\psi dA] \; e^{-S_E}$$

Euclidean QCD action $$S_E = \int_0^\beta dx_4 \int d^3 r \, L_E(\psi, A) \qquad \beta = 1/T$$

The time x_0 is replaced by the imaginary time x_4=- i x_0
The field A_0 is replaced by A_4=- i A_0

Define the Polyakov loop $$L(\vec{R}) = \frac{1}{N_c} Tr_c \; exp\left[ig \int_0^\beta dx_4 \, A_4(\vec{R}, x_4) \right]$$

Pure gauge: put a static (infinitely heavy) quark Q at point R $$\left\langle L(\vec{R}) \right\rangle = e^{-\beta F_Q(\vec{R})}$$

- If color is confined (low T), the free energy: $F_Q = +\infty$ $\quad \Phi = \langle L \rangle = 0$
- If color is not confined (high T) $\quad \Phi \neq 0 \quad \Phi(T = \infty) = 1$

The Polyakov loop is an order parameter associated with confinement/deconfinement for pure glue

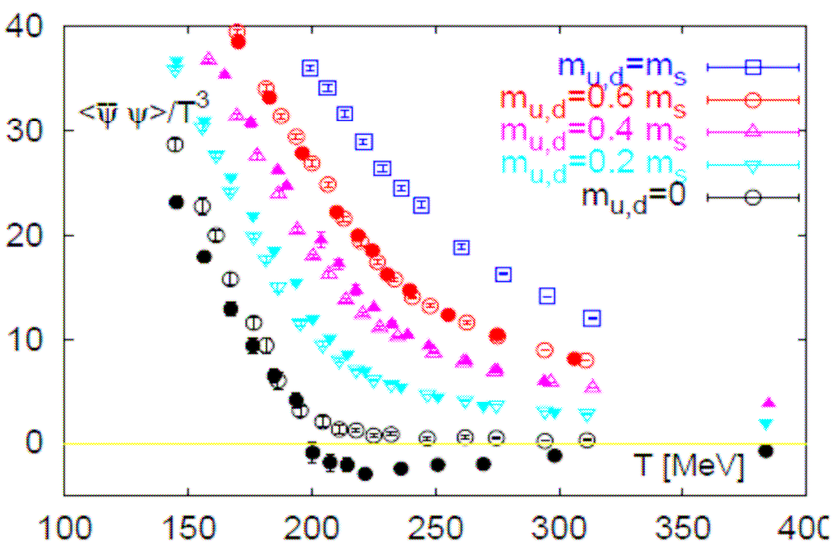
The associated group symmetry is the center Z(3) $$f = e^{2i\pi n/3}\, I, \qquad n = 1, 2, 3$$

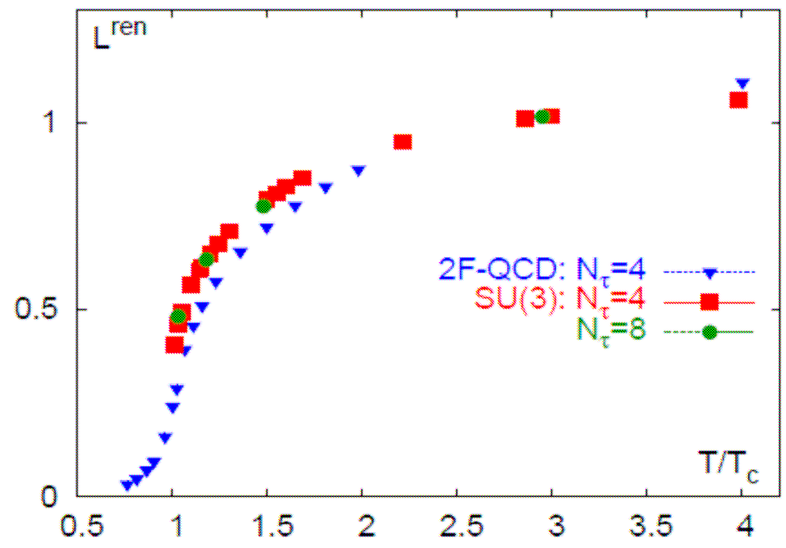
Real QCD

Chiral symmetry is not exact: although small , m_u and m_d~5 MeV are finite.
The quark condensate is no longer an order parameter but remains a good indicator for the rapid crossover for chiral restoration

Center Symmetry is not exact in presence of light quarks
The Polyakov Loop is no longer an order parameter but remains a good Indicator for the rapid crossover for deconfinement (although no precise criterion exists)

These statements can be tested on the lattice at zero chemical potential





But strong difficulties of lattice QCD at finite μ

The PNJL model

Aim: study simultaneously chiral restoration and deconfinement in the whole (T,μ) plan

The vacuum possesses a quark condensate (NJL): parameters fixed on vacuum
The vacuum possesses a condensate of « Wilson lines »

Quarks coupled to a background gauge field associated with the Polyalov loop

$$\mathcal{L} = \mathcal{L}_{NJL} - i\,q^{\dagger} A_4 q - U(\Phi, T)$$

$$\Phi = \frac{1}{N_c} Tr_c\, exp\left[ig \int_0^{\beta} dx_4\, A_4(\vec{R}, x_4)\right]$$

Grand potential

$$\Omega(T,\mu;M,\Phi) = U(\Phi,T) + \frac{(M-m)^2}{2G_1} - 2N_f N_c \int \frac{d^3p}{(2\pi)^3} E_p + \Omega_{QP}(T,\mu;M,\Phi)$$

$$\Omega_{QP}(T,\mu;M,\Phi) = -2N_f \int \frac{d^3p}{(2\pi)^3} \Big[ln\left(1 + 3\Phi e^{-\beta(E_p-\mu)} + 3\Phi e^{-2\beta(E_p-\mu)} + e^{-3\beta(E_p-\mu)}\right)$$
$$+ln\left(1 + 3\Phi e^{-\beta(E_p+\mu)} + 3\Phi e^{-2\beta(E_p+\mu)} + e^{-3\beta(E_p+\mu)}\right)\Big]$$

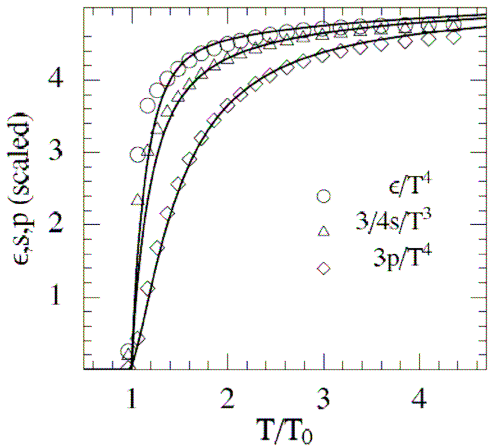
Polyakov loop and constituent quark mass obtained from

$$\frac{\partial \Omega}{\partial M} = 0$$
$$\frac{\partial \Omega}{\partial \Phi} = 0$$

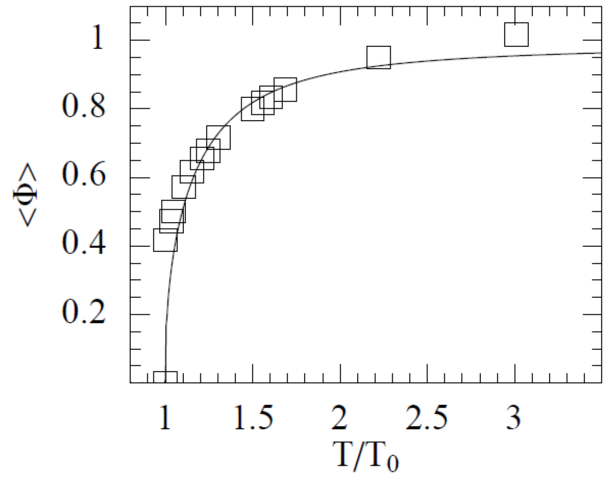
$$E_p = \sqrt{p^2 + M^2}$$

Pure Gauge: Effective potential for Poliakov loop U(Φ,T) fixed by comparison with pure gauge lattice data

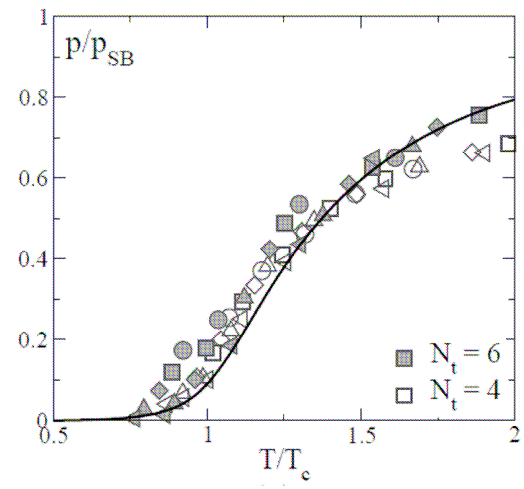
$$P(T) = -U(\Phi, T)$$

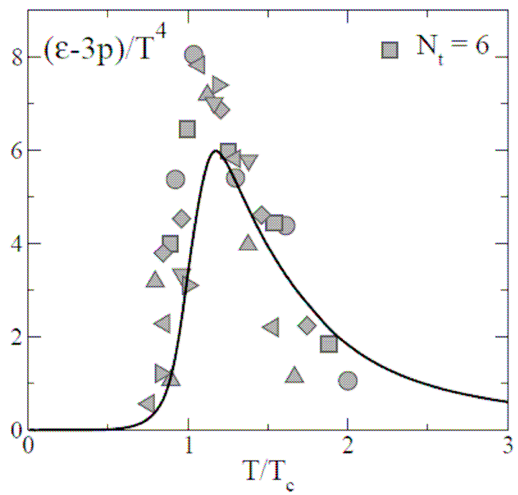


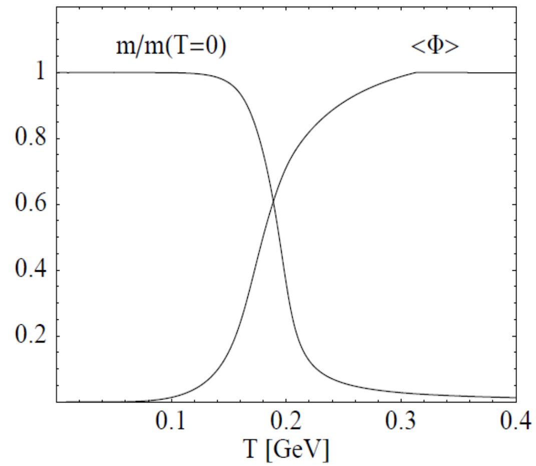
(Ratti et al)



Inclusion of quarks: Comparison with lattice data at zero chemical potential







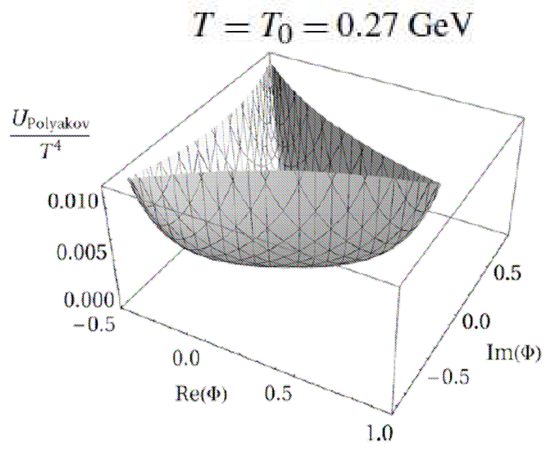
Deconfinement and chiral restoration (almost) coincide

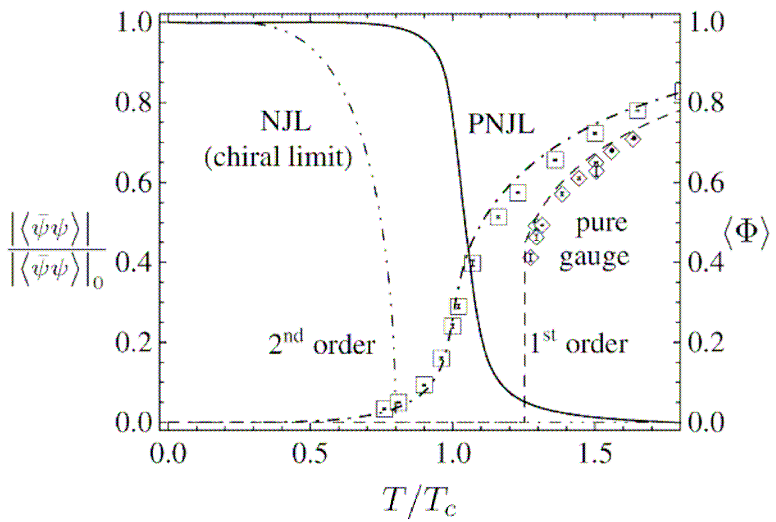
Improved potential

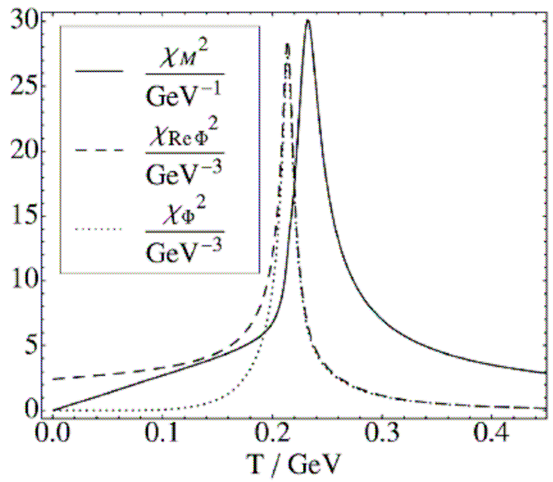
(Rossner et al)

$$\frac{\mathcal{U}(\Phi,\Phi^*,T)}{T^4} = -\frac{1}{2}a(T)\Phi^*\Phi + b(T)\ln\left[1-6\Phi^*\Phi+4\left(\Phi^{*3}+\Phi^3\right)-3(\Phi^*\Phi)^2\right]$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \quad \text{and} \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3.$$



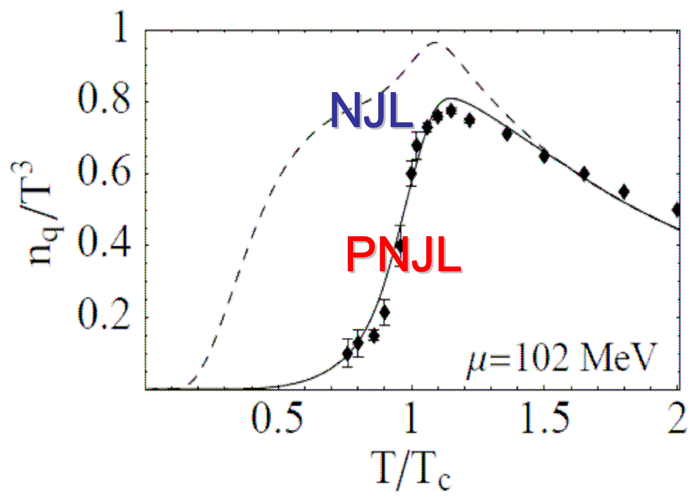




Finite chemical potential

Quark number density

$$f_\Phi^+(E_p) = \frac{\left(\bar\Phi + 2\Phi e^{-\beta(E_p-\mu)}\right) e^{-\beta(E_p-\mu)} + e^{-3\beta(E_p-\mu)}}{1+3\left(\bar\Phi + \Phi e^{-\beta(E_p-\mu)}\right) e^{-\beta(E_p-\mu)} + e^{-3\beta(E_p-\mu)}}$$



High T

$\langle\Phi\rangle \to 1$ $\qquad f_\phi^+(E_p) = 1/\left(e^{(E_p-\mu)/T}+1\right)$

Low T

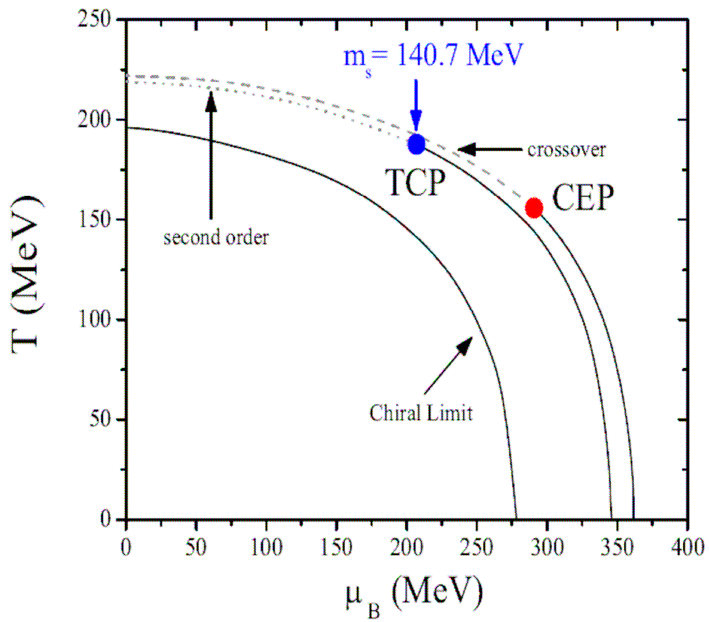
$\langle\Phi\rangle = 0$ $\qquad f_\phi^+(E_p) = 1/\left(e^{3(E_p-\mu)/T}+1\right)$

Polyakov loop considerably decreases the net quark number at low T

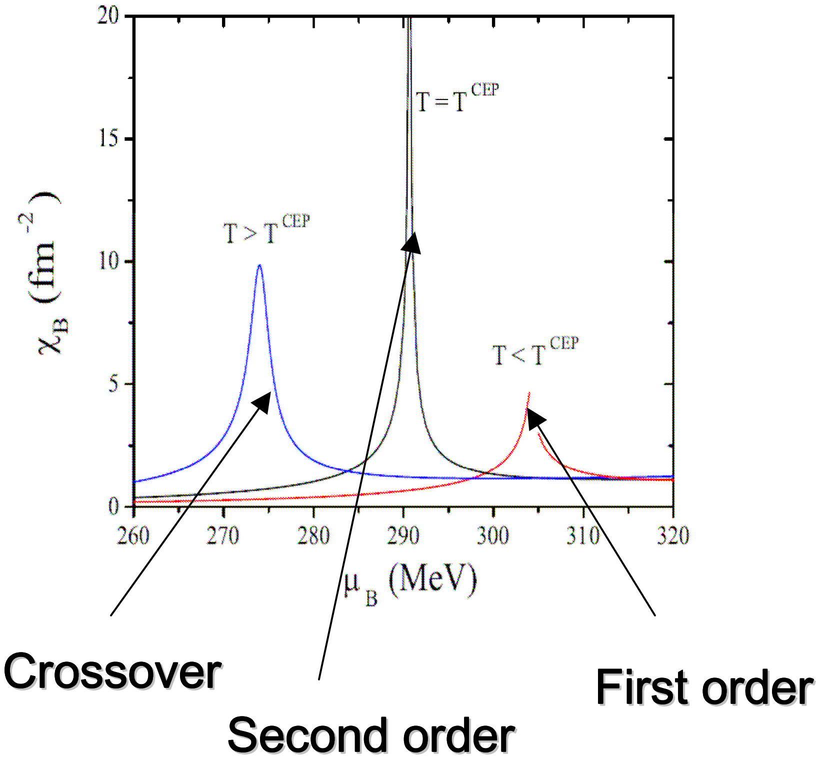
PNJL mimics three quarks clustering: « statistical confinement »

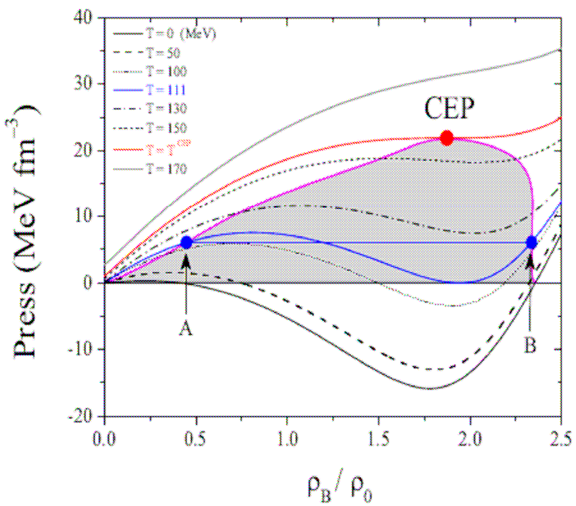
Recent PNJL calculation *(Coimbra- Lyon)*

PNJL phase diagram

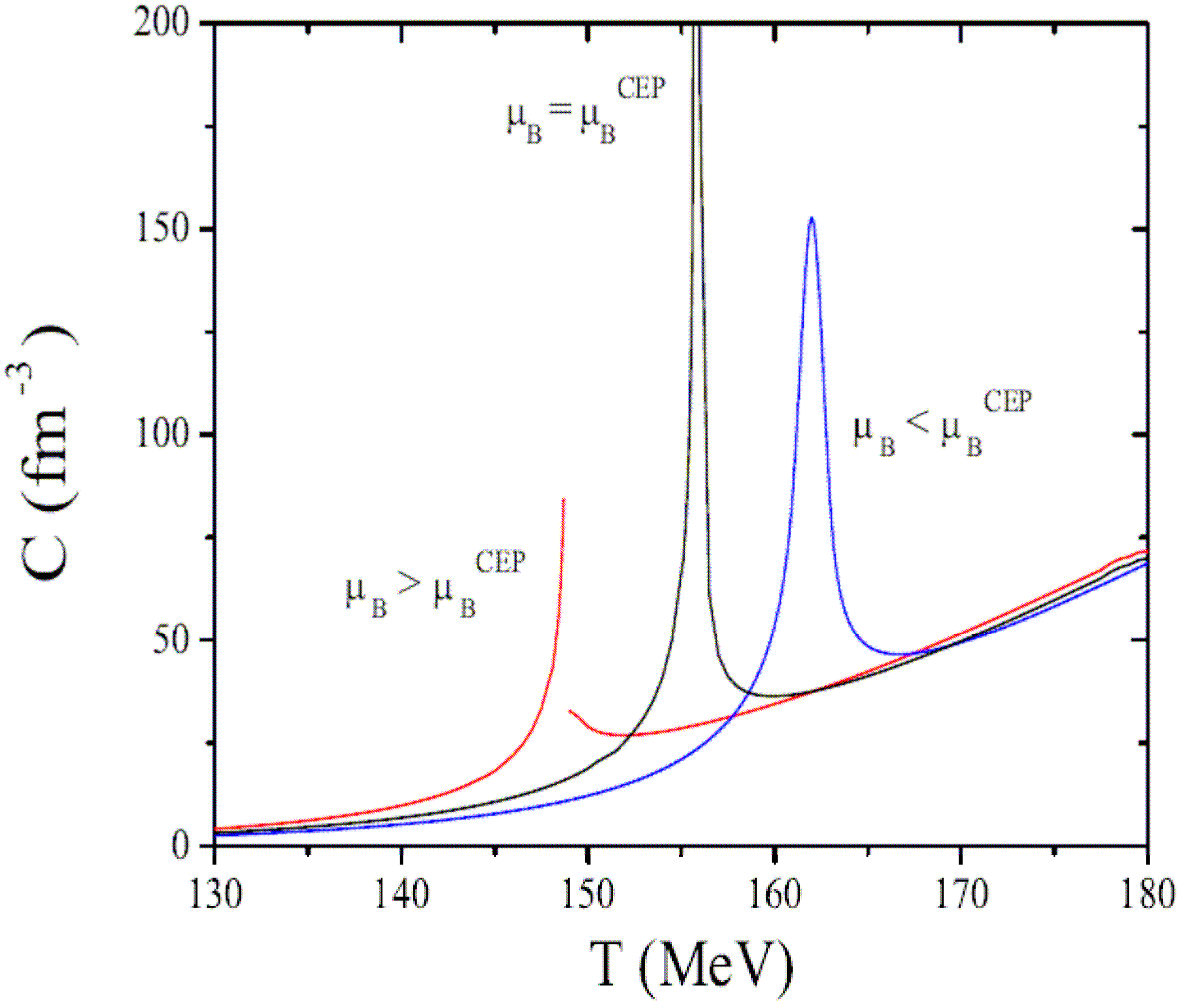


Baryon number susceptibility





Heat capacity



SUMMARY

QCD is gauge field theory based on invariance under LOCAL SU(3)c transformation

Only color singlet hadrons exist with M ~ 1 GeV and R ~ 1 fm (but m_π=140 MeV)
This is color confinement

In the light quark sector QCD is classically scale invariant. Scale invariance is broken by quantum fluctuations

All the physical observables scales as O = C Λ_{QCD} ,,fundamental scale of QCD

Existence of a gluon condensate; The « binding energy » of the QCD vacuum due to non perturbative quantum fluctuations is 0.5 GeV.fm^{-3}

The nucleon plausibly looks like a Y shaped string ended by constituent quarks

In the light quark sector there is an almost exact global chiral symmetry
-Vector symmetry: isospin multiplet
-Axial symmetry spontaneously broken:
--- No parity partners with opposite parity
--- Goldstone bosons: pion. Low energy pion weakly interact (Chi PT)
--- Order parameters: pion decay constant, quark condensate
-Chiral symmetry should be progressively restored by increasing temperature and/or baryonic density (rho meson spectral function from NA60)

QCD phase diagram: from lattice and models:

- Chiral restoration: first order line terminated by a critical end point (CEP)
- Low baryonic chemical potential: crossover
- Quark condensate (Polyakov loop) indicator of chiral restoration (deconfinement)
- Lattice , PNJL: chiral restoration and deconfinement coincide
- Experimental search for the CEP: RHIC energy scan, FAIR/CBM