“The search for the symmetry energy in nuclear dynamics”

1. “Symmetry energy”:
   – What is it? Who cares?
2. “Nuclear dynamics”:
   – What do you mean by nuclear dynamics? Why do you care about it?
3. “Search”:
   – Search what? How to search?

JC 2010, Lacanau (FRA),
Sep 27th-Oct 2nd, 2010

Giuseppe Verde, INFN, Catania, Italy
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Symmetry Energy: who cares?

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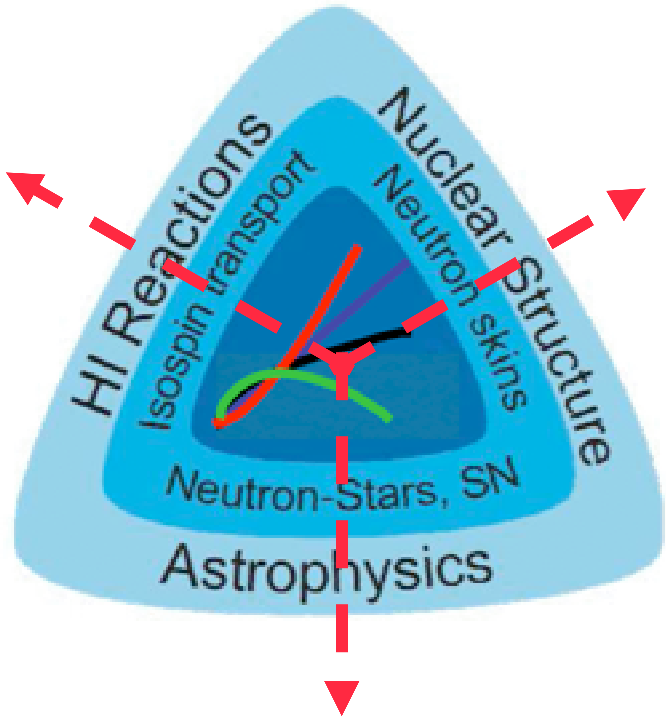
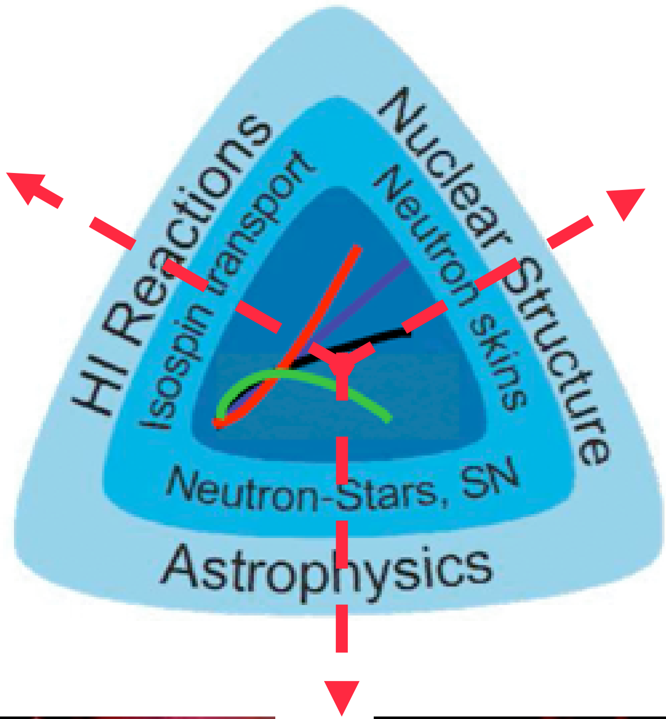
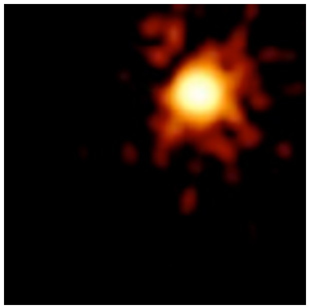


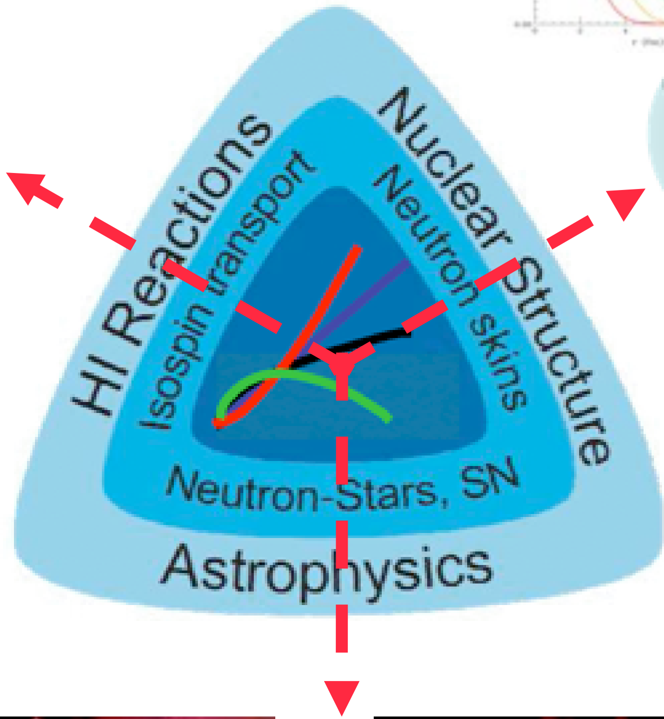
Symmetry Energy: who cares?

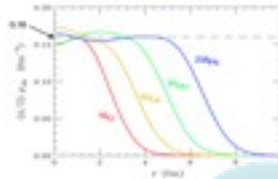




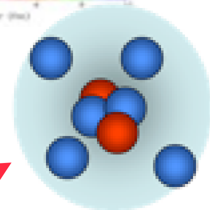
Supernovae, neutron stars

Symmetry Energy: who cares?



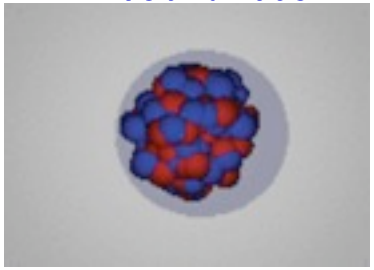


skins

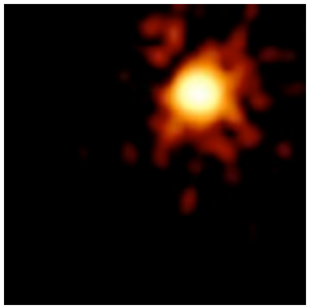


halos

resonances



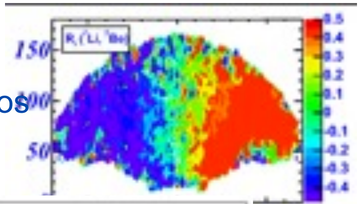
GDR & PYGMY RESONANCE



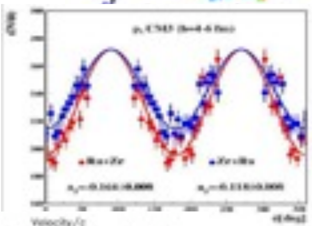
Supernovae,
neutron stars

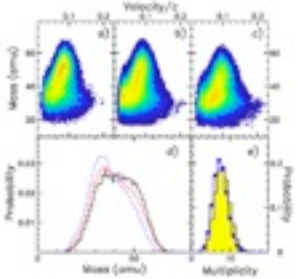
Symmetry Energy: who cares?

Lassa @ MSU
Imbalance ratios

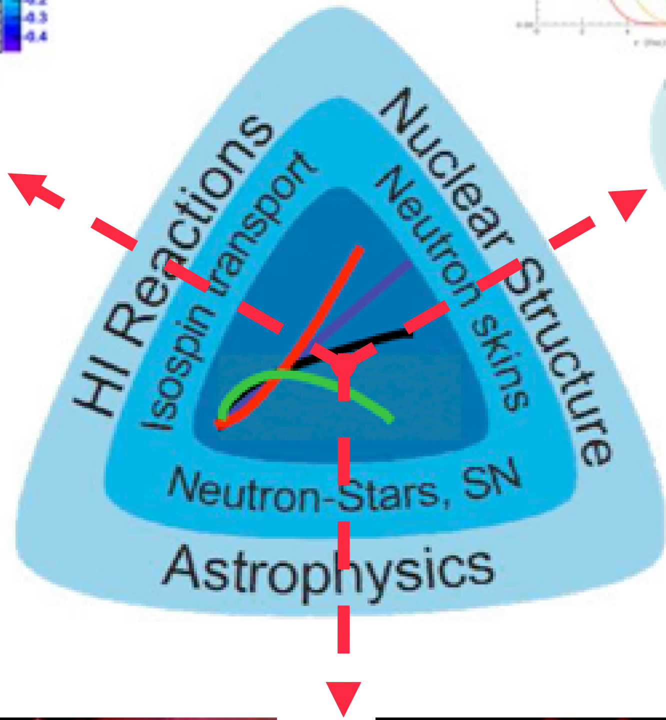


FOPI @ GSI
Elliptic flow

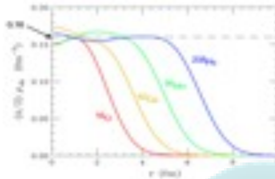




Chimera @ LNS
Competitiion Inc. Fusion / DIC



skins



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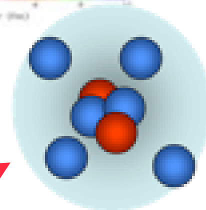
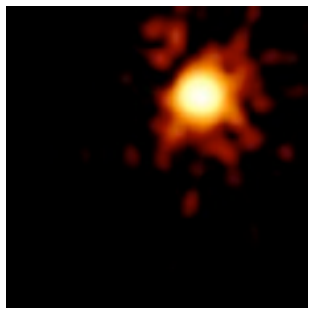


resonances

GDR & PYGMY RESONANCE

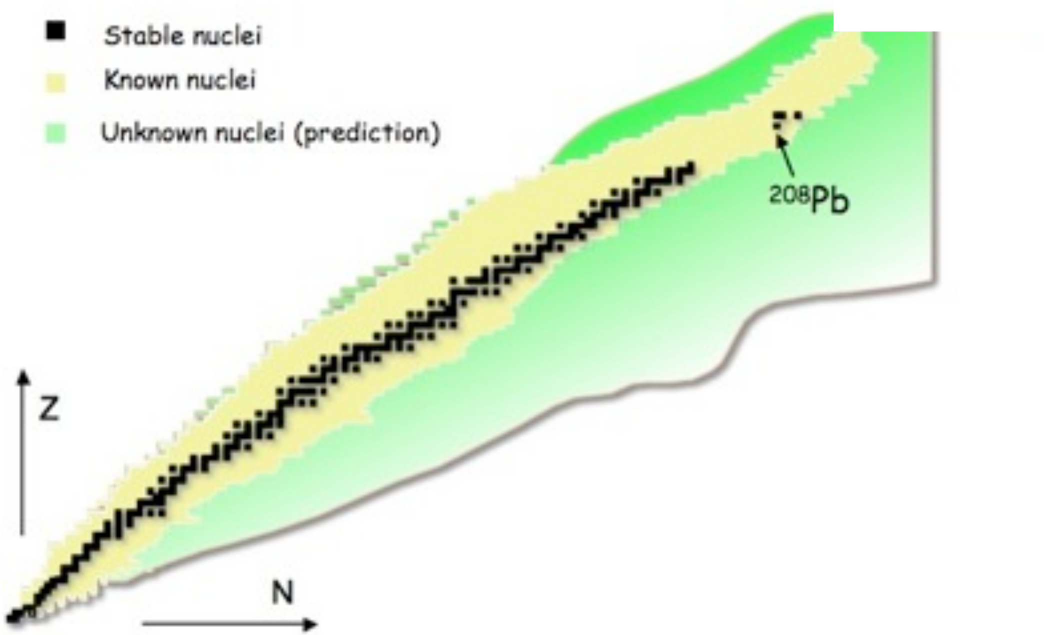


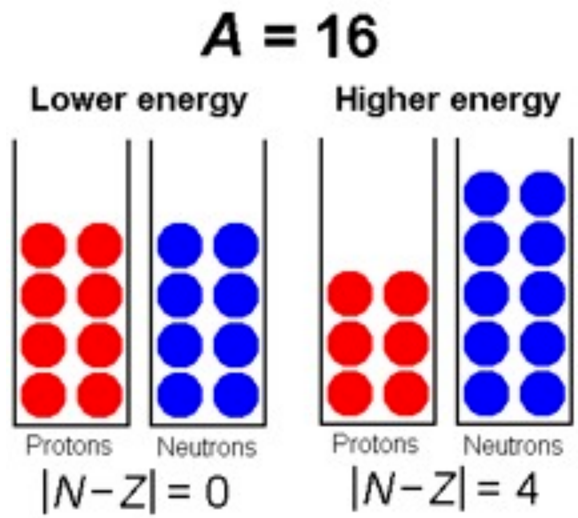
Supernovae,
neutron stars

Symmetry energy in finite nuclei

Bethe-Weiszacker

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C \frac{Z(Z-1)}{A^{1/3}} + a_{sym}\frac{(N-Z)^2}{A} + \ldots$$





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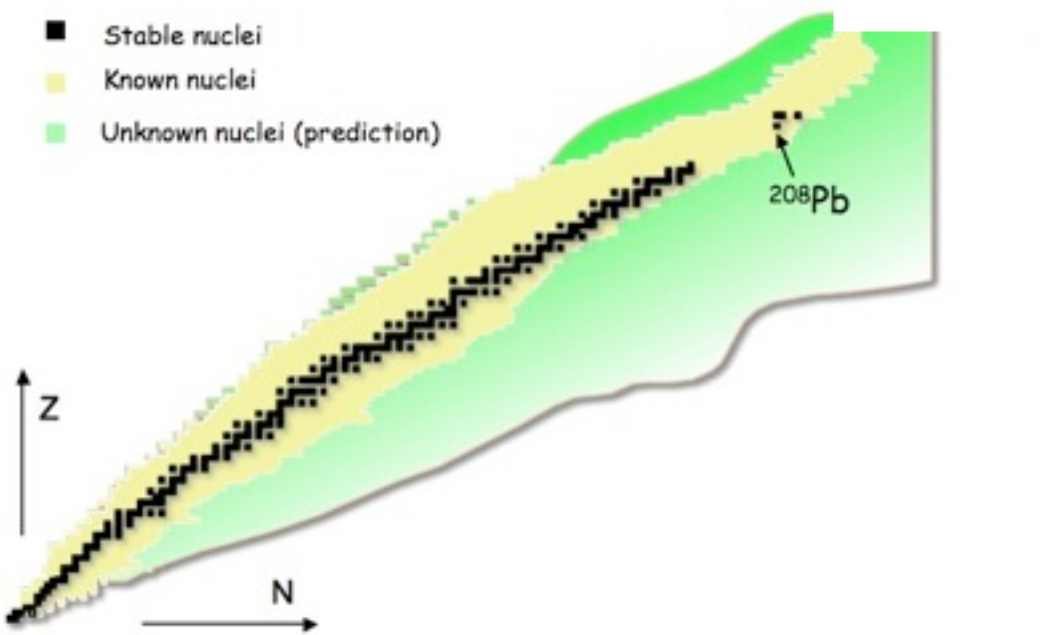

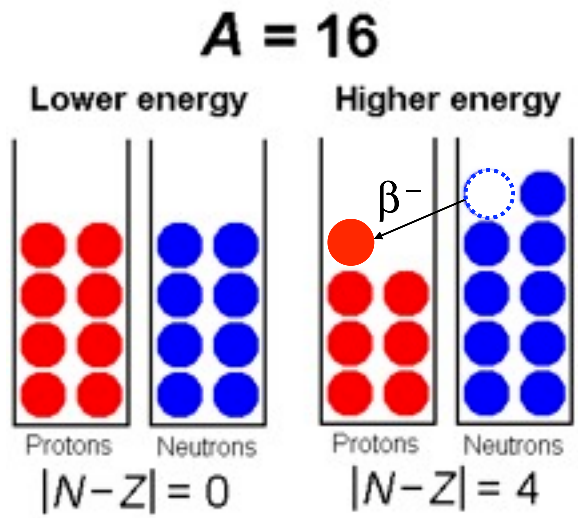

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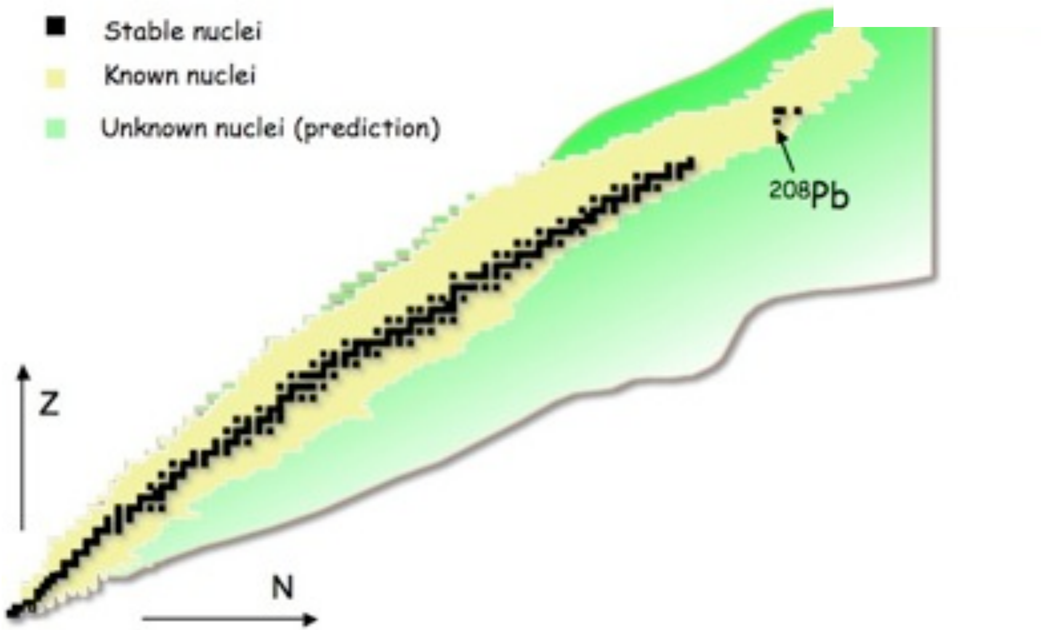
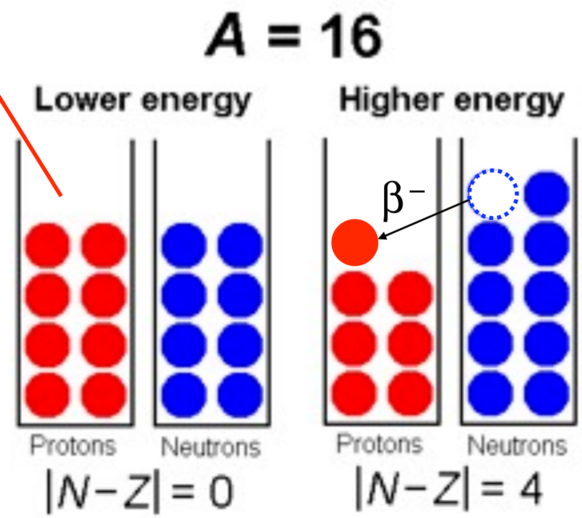




From nuclei to nuclear matter

From finite nuclei… $\rho=\rho_0=0.16\ \mathrm{fm}^{-3}$ $T=0$

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C \frac{Z(Z-1)}{A^{1/3}} + a_{sym} \frac{(N-Z)^2}{A} + \ldots$$

… to infinite nuclear matter… $\rho\neq\rho_0$ and $T>0$

$$E(A,Z) \quad ==> \quad E(\rho,\delta,T)=0$$

Isospin asymmetry

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

→

Finite nuclei, heavy-ion collisions:

$$\delta = \frac{N-Z}{N+Z} = \frac{N-Z}{A}$$

The EOS of asymmetric nuclear matter

$\rho = \rho_n + \rho_p$ isoscalar density

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$ neutron/proton isovector density asymmetry

$$E(\rho, \delta) =$$

EOS of Symmetric nuclear matter ($\delta=0$)

+

Asymmetry term

The EOS of asymmetric nuclear matter

$\rho = \rho_n + \rho_p$ isoscalar density

$\delta = \dfrac{\rho_n - \rho_p}{\rho_n + \rho_p}$ neutron/proton isovector density asymmetry

$$E(\rho, \delta) =$$

EOS of Symmetric nuclear matter (δ=0)

+

Asymmetry term

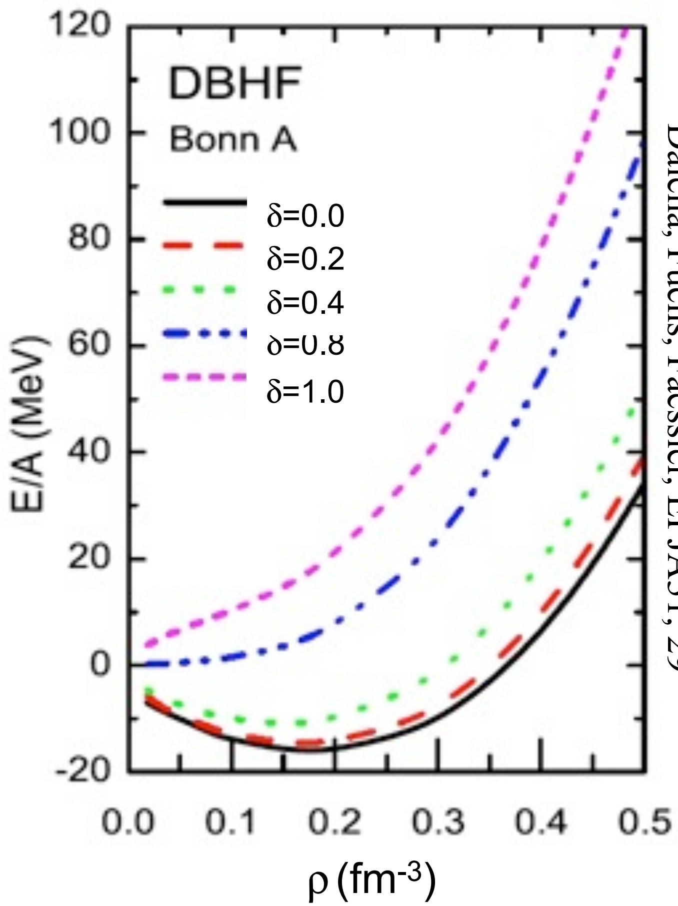
1. How does it depend on neutron/proton asymmetry δ?
2. How does it depend on density ρ?

The EOS of asymmetric nuclear matter

Symmetric NM + "asymmetry term"

$$E(\rho,\delta) \approx E(\rho,\delta=0) + ASY - Term$$

$$\left(\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}\right)$$



Dalena, Fuchs, Faessler, EPJA31, 29

The EOS of asymmetric nuclear matter

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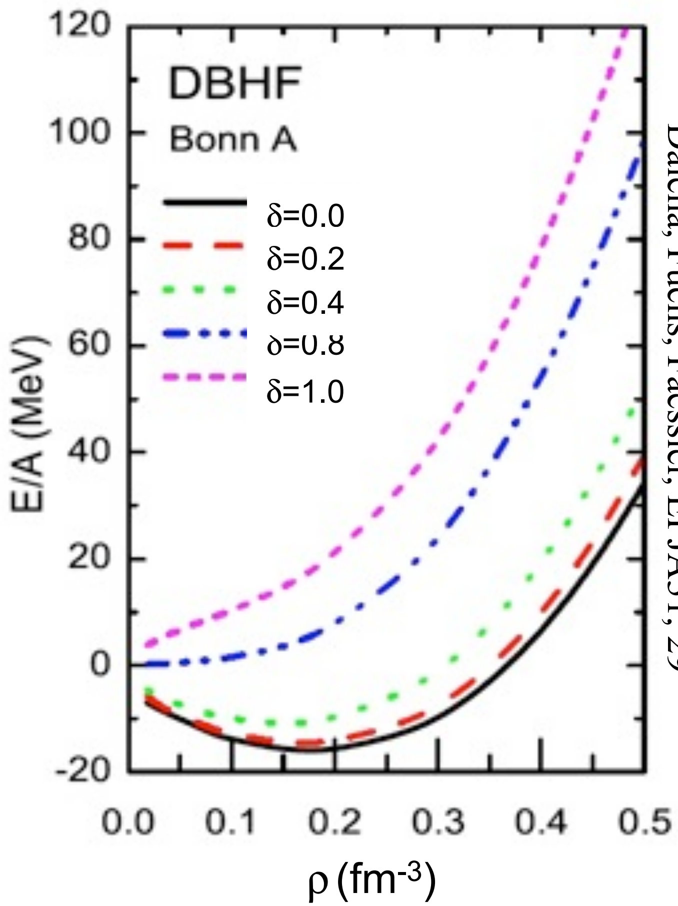


Dalena, Fuchs, Faessler, EPJA31, 29

$$ASY - Term = f(\rho,\delta)$$

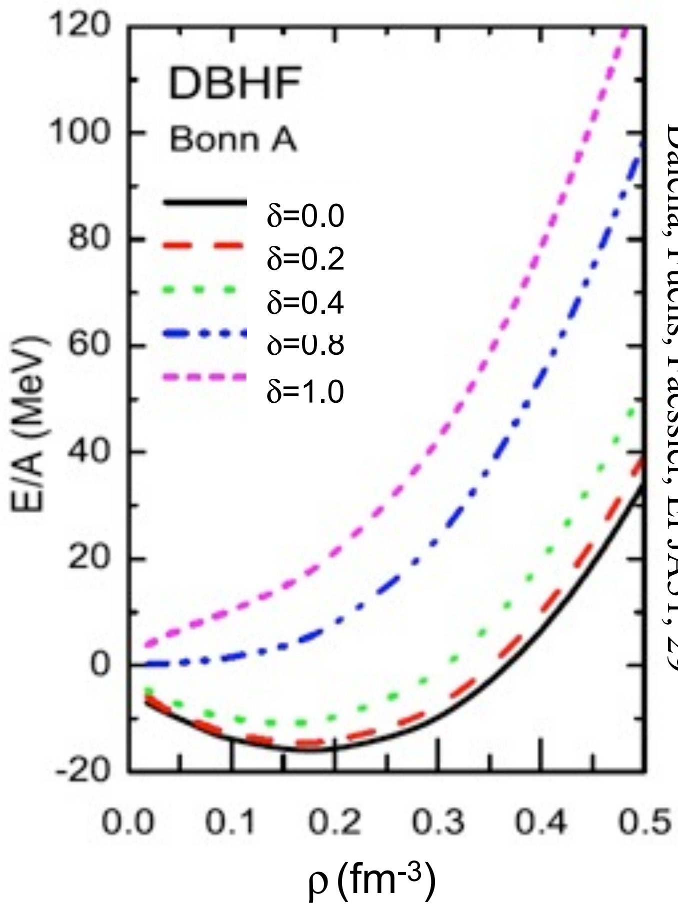
$$= E_{sym}(\rho) \cdot \delta^2$$

The EOS of asymmetric nuclear matter

Symmetric NM + "asymmetry term"

$$E(\rho,\delta) \approx E(\rho,\delta=0) + E_{sym}(\rho)\cdot\delta^2$$

$$\left(\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}\right)$$



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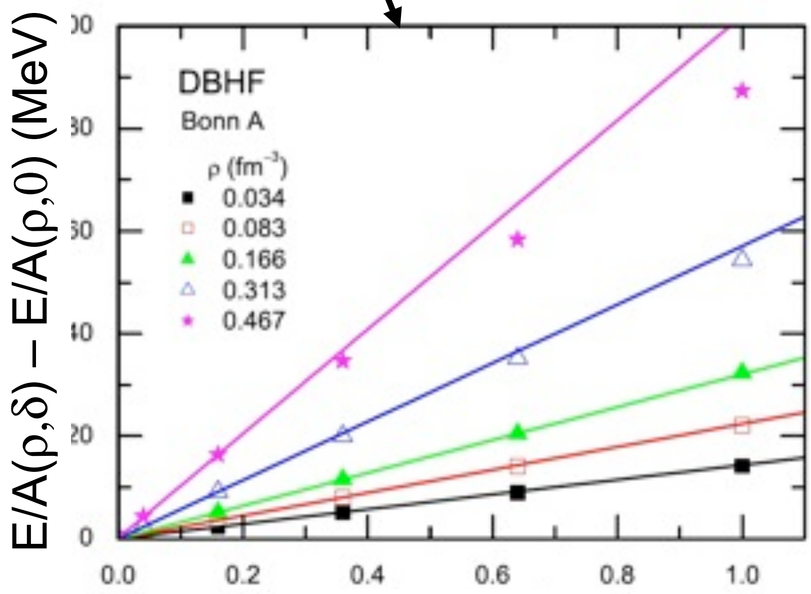


Dalena, Fuchs, Faessler, EPJA31, 29

$$\delta^2 = \left(\frac{\rho_n - \rho_p}{\rho_n + \rho_p}\right)^2$$

Almost all models agree on δ^2-scaling… up to $\rho \sim 3\cdot\rho_0$!

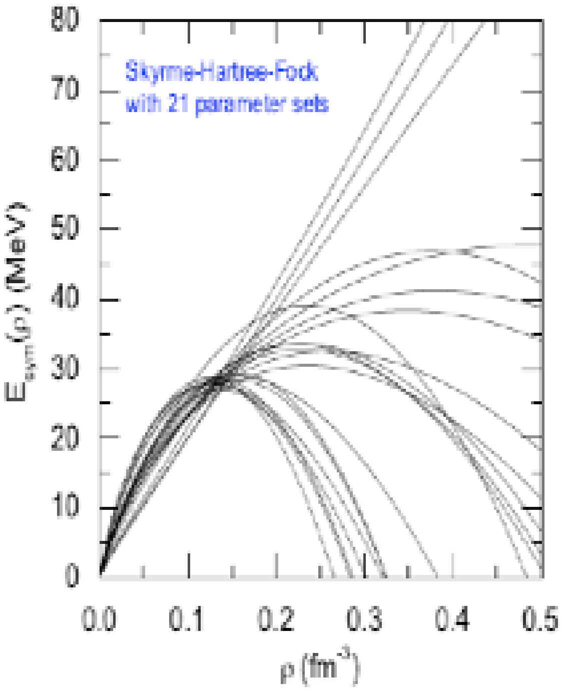
Density dependence of the asymmetry term

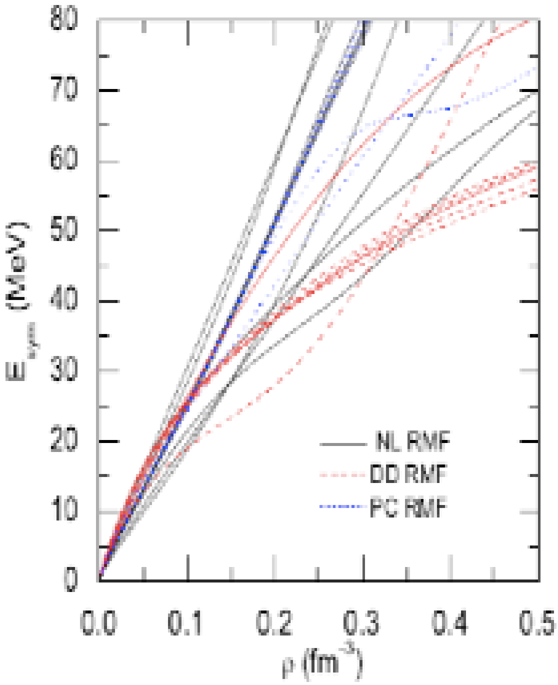
$$E(\rho,\delta) \approx E(\rho,\delta=0) + \boxed{E_{sym}(\rho)} \cdot \delta^2$$

$$\left(\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}\right)$$

???

B.A. Li et al., Phys. Rep. 464, 113 (2008)

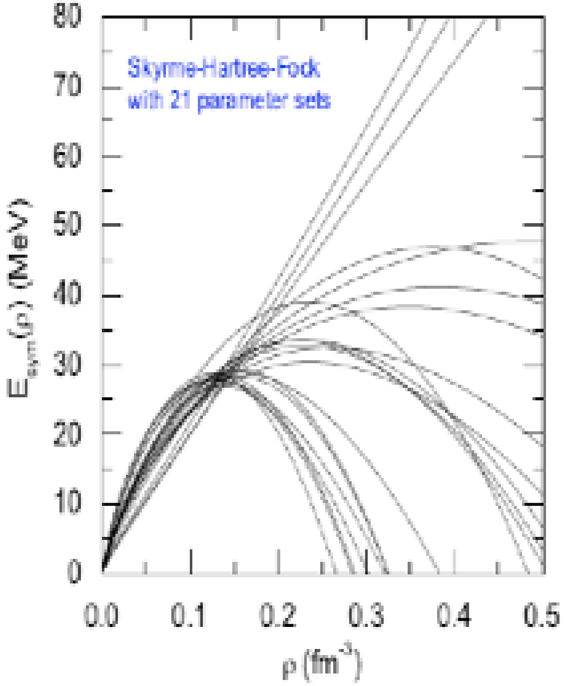




Many approaches… large uncertainties

Microscopic many-body, phenomenological, variational

Slope and curvature of $E_{sym}(\rho)$



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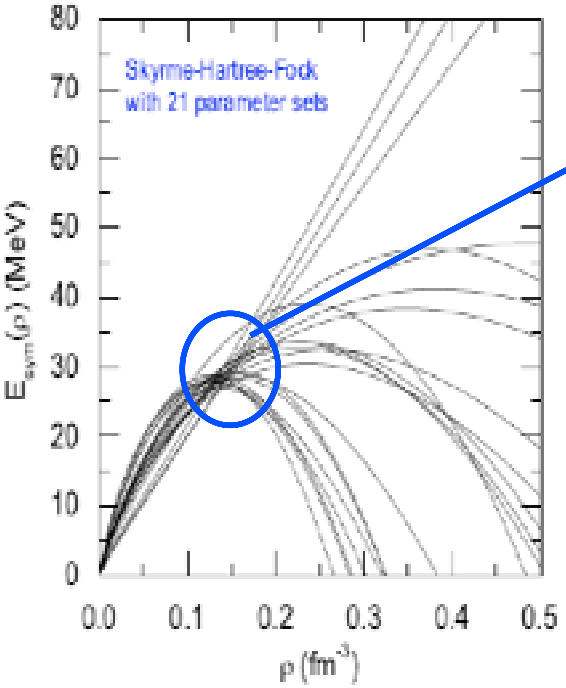
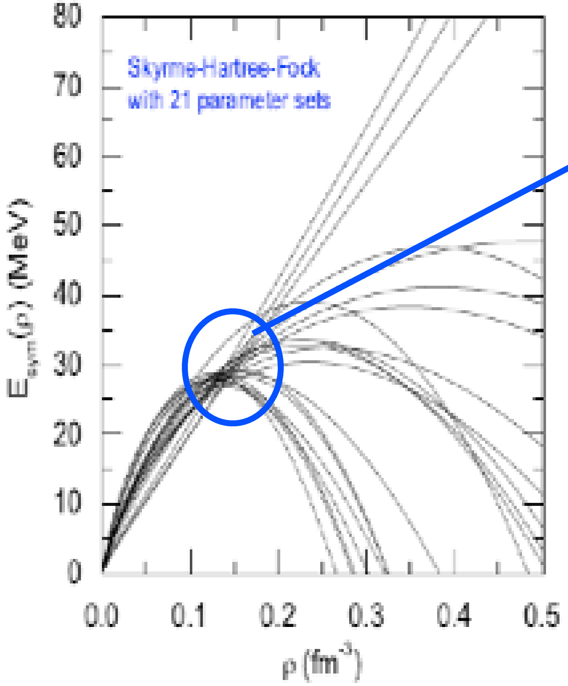


$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 +$$

Slope and curvature of $E_{sym}(\rho)$



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$$S_0 = E_{sym}(\rho_0)$$

strength

$$K_{sym} = 9\rho_0^2 \left|\frac{\partial^2 E_{sym}(\rho)}{\partial \rho^2}\right|_{\rho_0}$$

curvature

$$L = 3\rho_0 \left|\frac{\partial E_{sym}(\rho)}{\partial \rho}\right|_{\rho_0} = \left(\frac{3}{\rho_0}\right) p_0$$

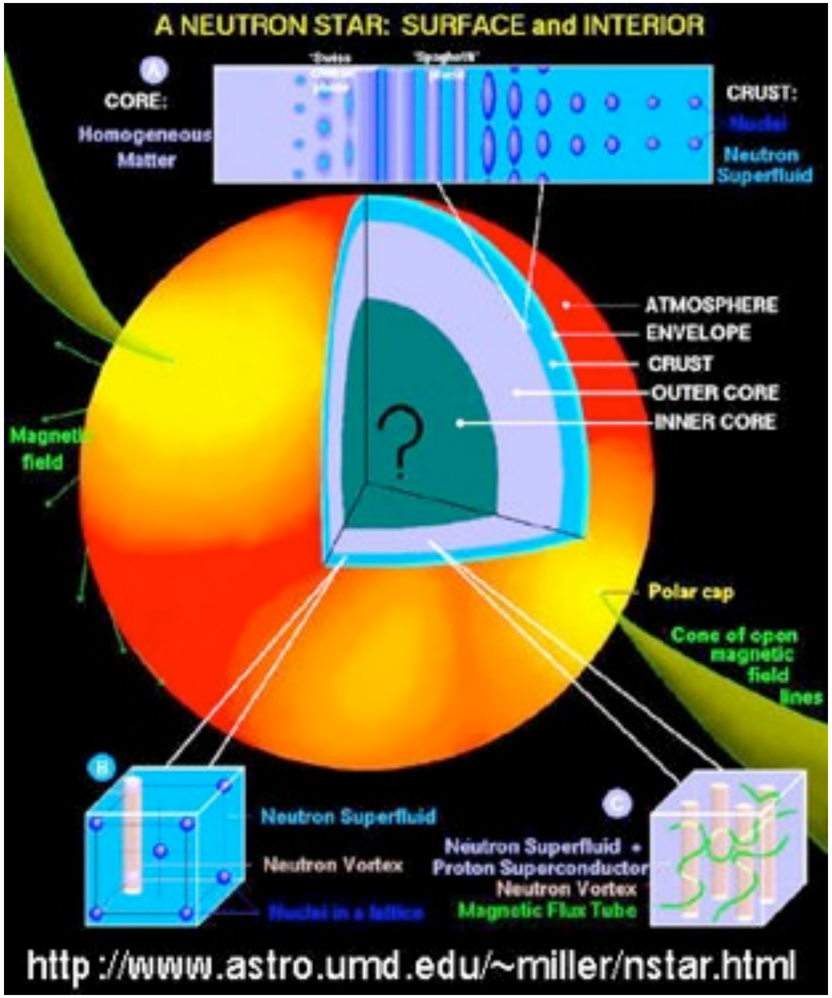
slope

S_0, L and K_{sym} relevant to neutron stars, neutron skins, nuclear collective motion (GMR, GDR, PDR), …

Who cares about $E_{sym}(\rho)$?

- Nuclear physicists
  - Isovector isospin dependent in-medium NN interaction, neutron skins, halos, nuclear many-body problems with isospin dof, EoS, giant and pygmy resonances, …
- Astrophysicists
  - Neutron stars, supernovae explosions, …

Neutron stars



- Radii, Moments of inertia
- Light glitches
- Pressure from pure baryonic matter at $\rho=\rho_0$
- Frequencies of crustal vibrations
- Composition and thickness of inner crust
- URCA processes
- Phases within the star

Large variations of nuclear densities ρ and large isospin asymmetries δ

How to produce density gradients of asymmetric nuclear matter?

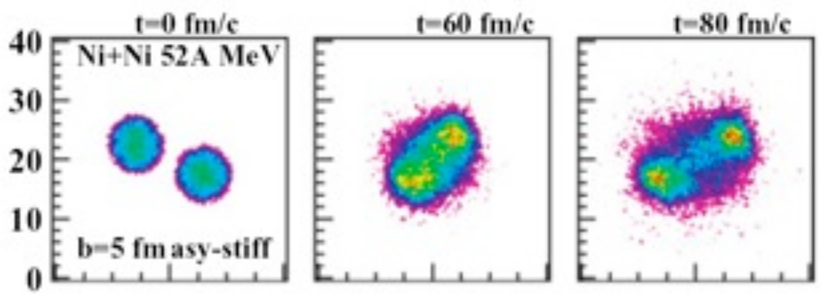
Goal: *study the density dependence of the symmetry energy*

$$E_{sym}(\rho)$$

“on earth”, under laboratory-controlled conditions

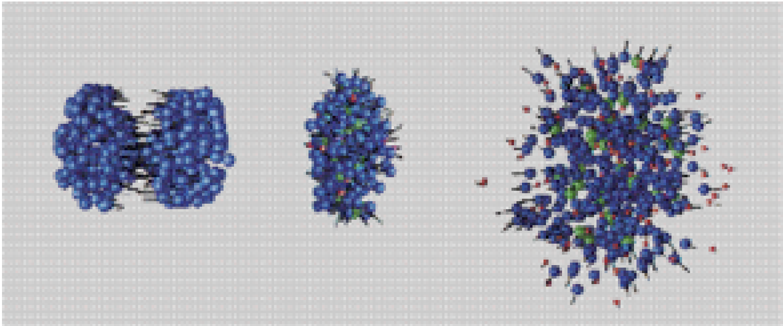
Heavy-ion collisions (HIC) and $E_{sym}(\rho)$

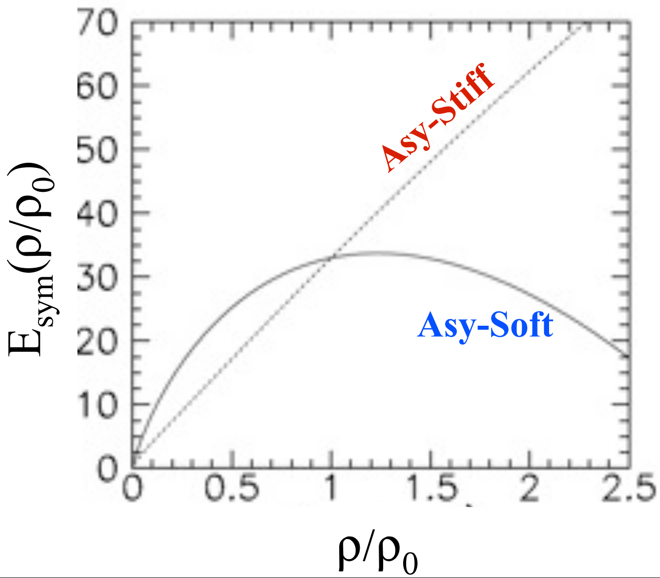
Intermediate energies: E/A=20-100 MeV



SMF - Baran, Colonna, Di Toro, Greco

High energies: E/A>200 MeV





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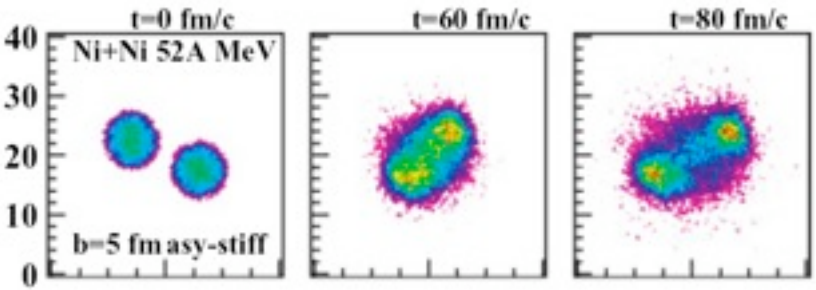
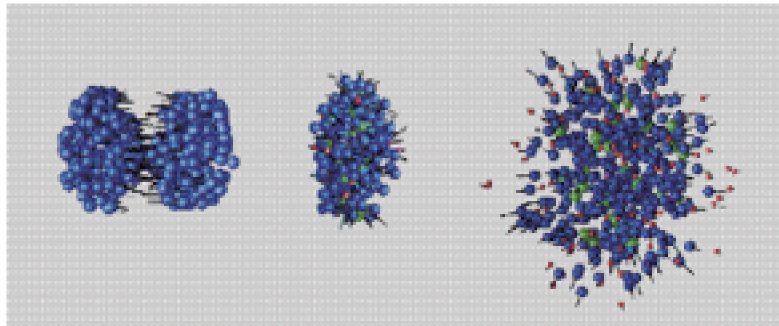


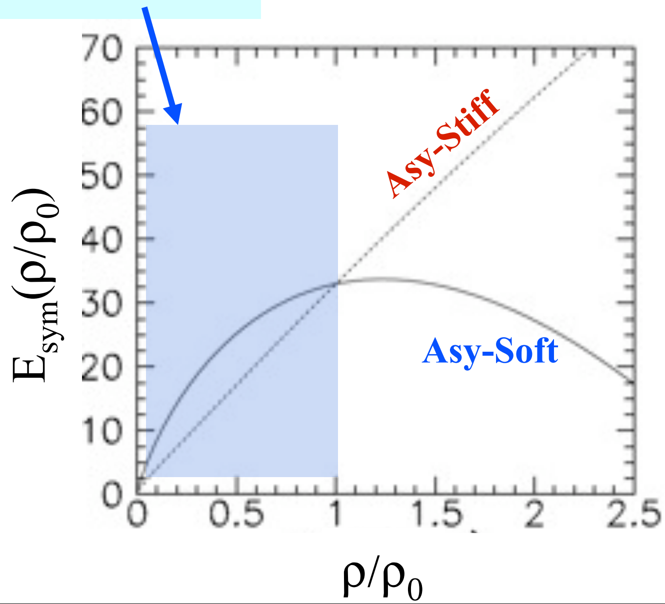
SMF - Baran, Colonna, Di Toro, Greco

Ganil, Eurisol, Frib, Lns, Nscl,
Spiral2, Tamu, …

High energies: E/A>200 MeV



Low density



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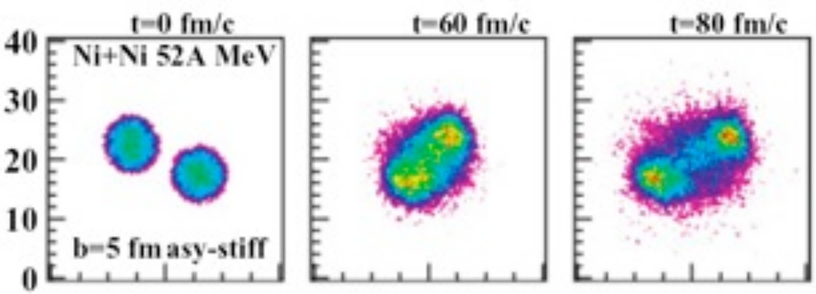

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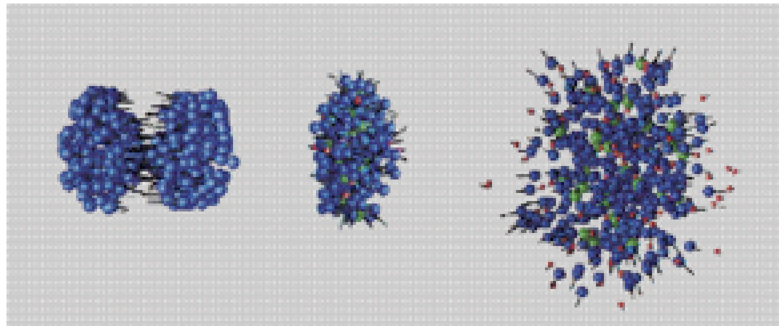
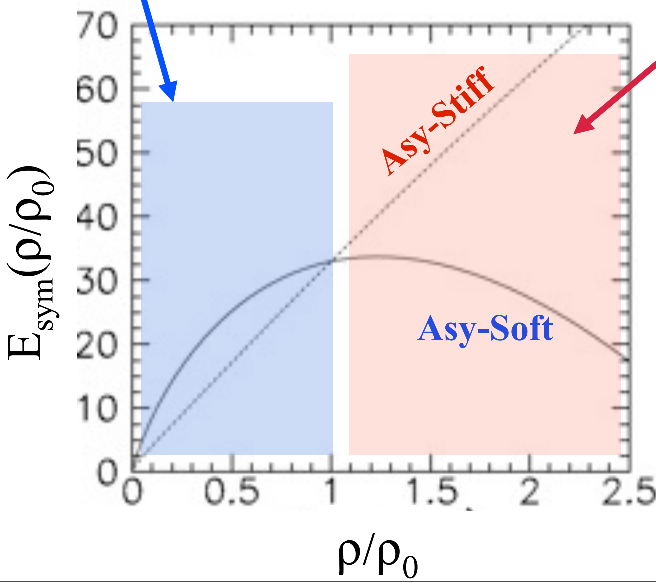


CSR, GSI/Fair, FRIB, Riken, …

Low density



High density

Density gradients and space-time scales in HIC

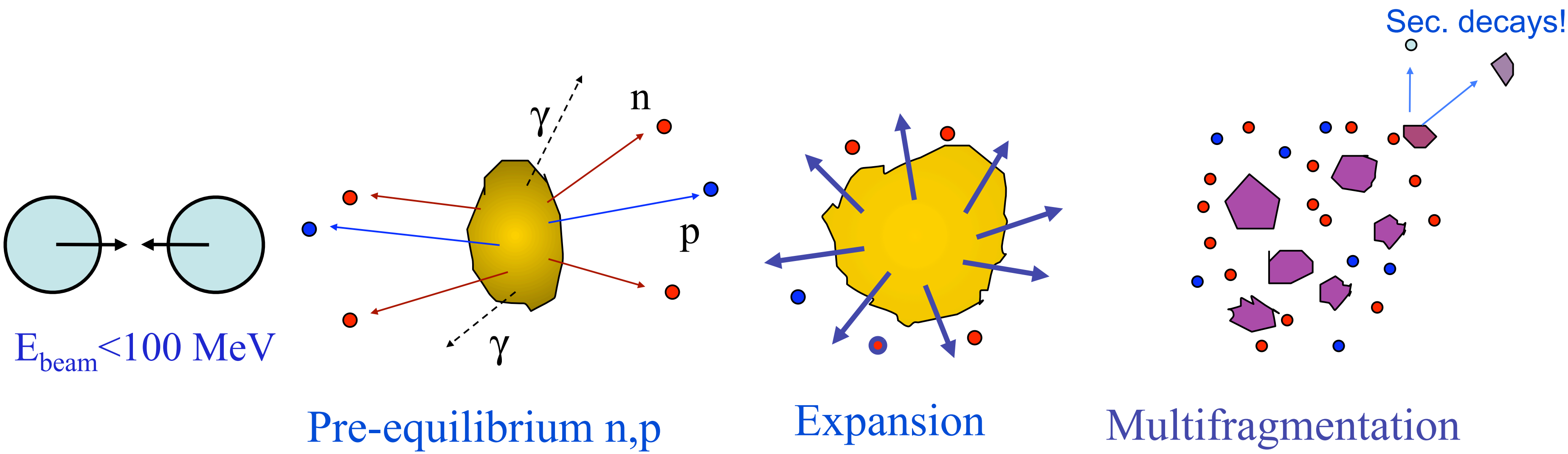
Gradients in

- Space
  - Typical distance scales as short as 1 fm
    "Femtoscopy"
- Time
  - Typical times as small as 10^{-21}s
    "Zepto-physics?"

Transient systems quickly evolving with time

Difficult task

HIC - Intermediate energies



- Measured spectra = convolution of different emitting sources and processes (ex: secondary decays)
- Hierarchy in particle emission: different particles at different stages

Difficulty: No barometers, no thermometers, no “femtoscopes”…

Probing the symmetry energy

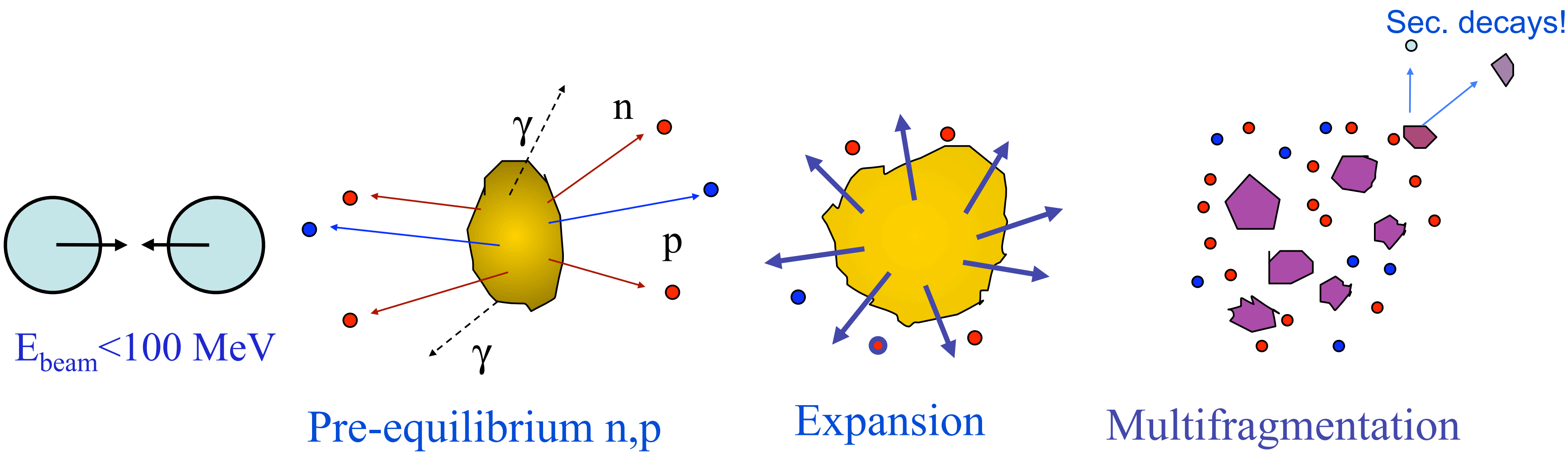
- Probes at *Intermediate* energies (E/A<100 MeV): *sub-saturation* density Asy-EoS ($\rho<\rho_0$)
- Probes at *Medium* energies (E/A>200 MeV): *supra-saturation* density Asy-EoS ($\rho>\rho_0$)

Saturation density $\rho_0 \sim 0.16\ fm^{-3}$

Experimental tools

- Choice of beam and target nuclei
- Detectors
- Observables
- How to turn measurements into conclusions about $E_{sym}(\rho)$

Choosing beam and target nuclei



$$E(\rho,\delta) = E(\rho,\delta=0) + E_{sym}(\rho)\times\delta^2$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

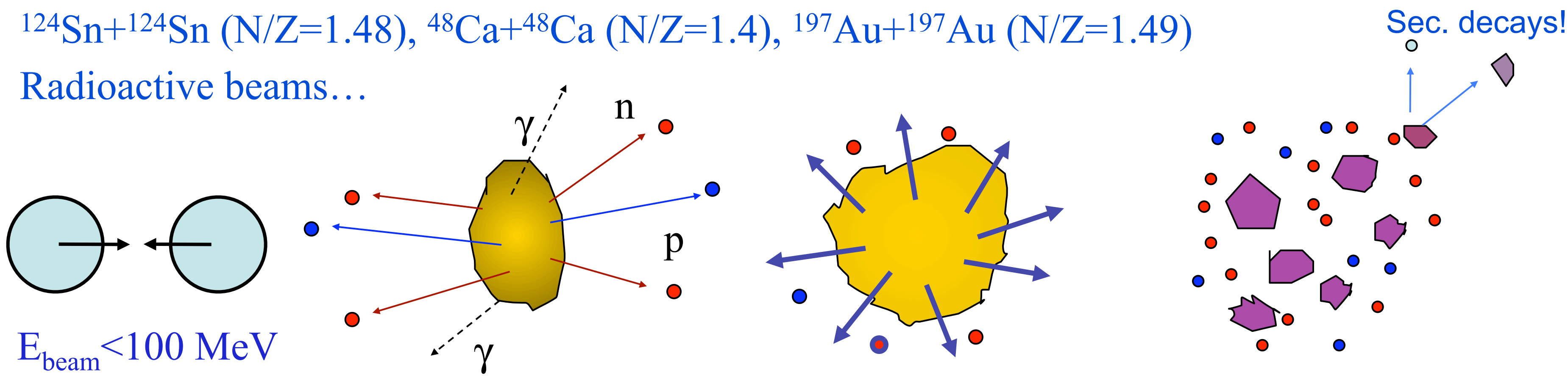
$$\rho \le \rho_0 \approx 0.16\, fm^{-3}$$

Choosing beam and target nuclei

Large N/Z to enhance effects of $E_{sym}(\rho)$

$^{124}Sn + ^{124}Sn$ (N/Z=1.48), $^{48}Ca + ^{48}Ca$ (N/Z=1.4), $^{197}Au + ^{197}Au$ (N/Z=1.49)

Radioactive beams…



Pre-equilibrium n,p   Expansion   Multifragmentation

$$E(\rho,\delta) = E(\rho,\delta=0) + E_{sym}(\rho)\times\delta^2$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$\rho \le \rho_0 \approx 0.16\, fm^{-3}$$

A trick: measuring relative effects

Effects induced by the symmetry energy remain generally small ☹

Ratios: A/B as "amplifiers" ☺

Reaction 1: low N/Z

Ex: $^{40}Ca+^{40}Ca$

Reaction 2: high N/Z

Ex: $^{48}Sn+^{48}Sn$

$$\frac{Obs(Reaction1)}{Obs(Reaction2)}$$

Neutron/proton ratios

$$\frac{Y(n)}{Y(p)} \qquad \frac{Obs(n)}{Obs(p)}$$

Isobar ratios ≈ n/p

$$\frac{Y(t)}{Y(^3He)} \qquad \frac{Obs(^7Li)}{Obs(^7Be)}$$

Turn measurements into $E_{sym}(\rho)$

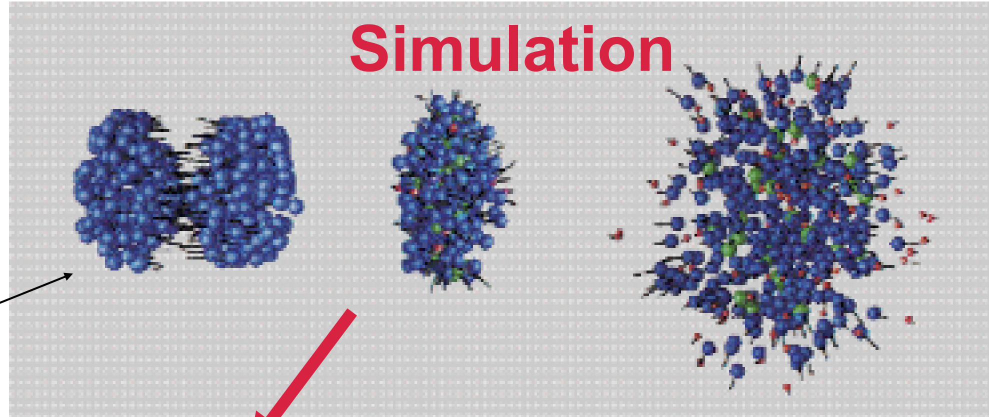
Choose $E_{sym}(r)$

$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial q}$$

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}.$$

$H=T+V(E_{sym})$

Input



Calculate spectra, distributions…

Measure spectra, distributions…

Experimental data

Turn measurements into $E_{sym}(\rho)$

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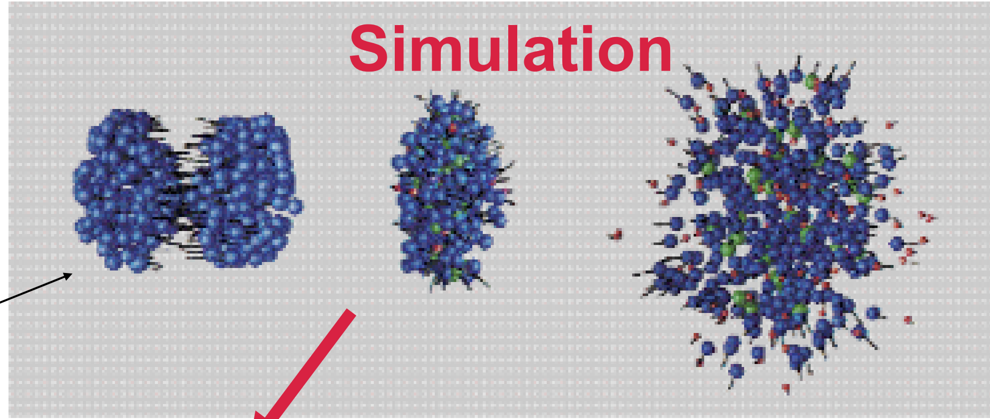

Calculate spectra, distributions…

Comparison?

Measure spectra, distributions…

Experimental data

Turn measurements into $E_{sym}(\rho)$

Choose $E_{sym}(r)$

$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial q}$$

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}.$$

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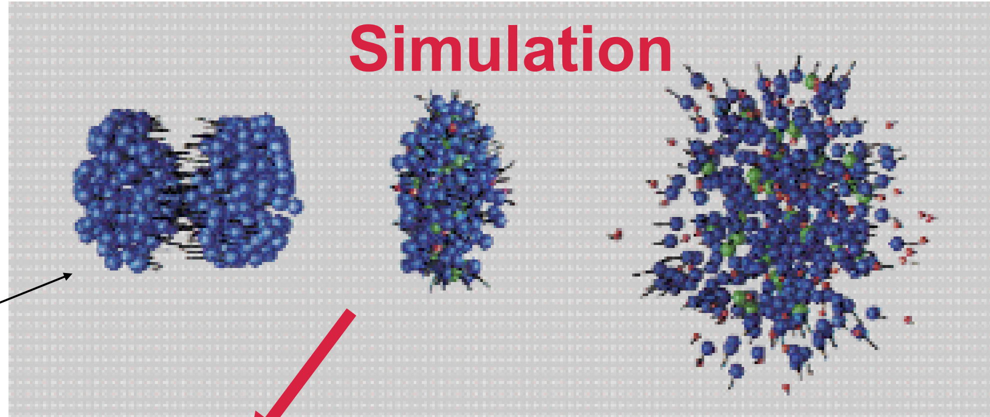
Input



Calculate spectra, distributions…

NO…

Comparison?

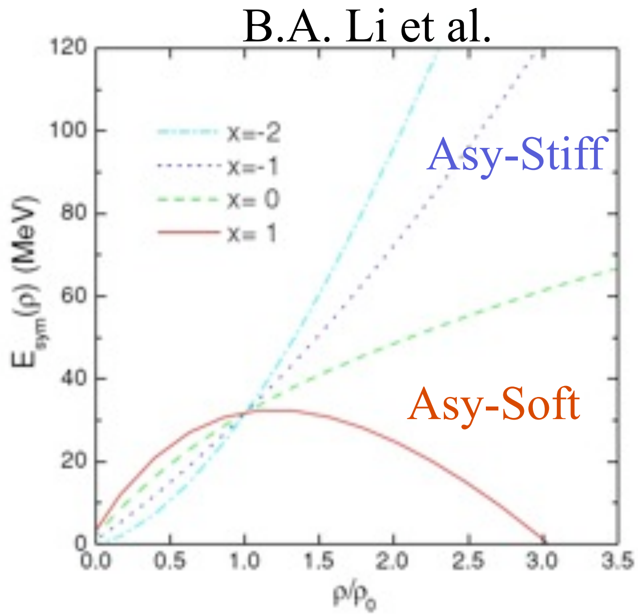
YES!

Measure spectra, distributions…

make progress…!

Experimental data

Typical $E_{sym}(\rho)$ parameterizations



$$E_{sym}(\rho) = E_{sym}^{kin}(\rho) + E_{sym}^{pot}(\rho) = a \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3} + b \cdot \left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

$\gamma=2$ ~ Super stiff

$\gamma=0.3$ ~ Super

Want to know more about transport models?

Go google search ☺

"QMD subatech Aichelin"

"IBUU04 Bao-An Li Lie-Wen Chen"

"BNV SMF Baran Colonna Di Toro"

"AMD Akira Ono"

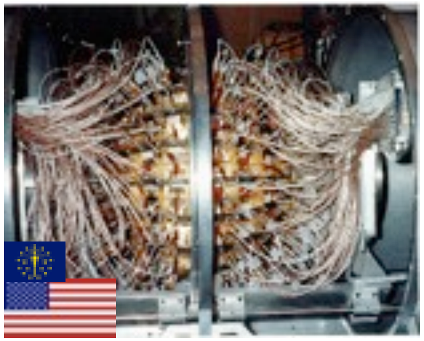
B. A. Li et al., Phys. Rep. 464, 113 (2008)

Detectors

- 4π coverage:
  - Impact parameter, reaction plane, ...
- High granularity
  - Several fragments per event (M>10)
- Angular resolution (θ,ϕ)
- Isotopic resolution & neutron detection
- Large dynamic ranges (E_{kin}/A=0.5-100 MeV)
  - preamplifiers, shapers, DAQ, …

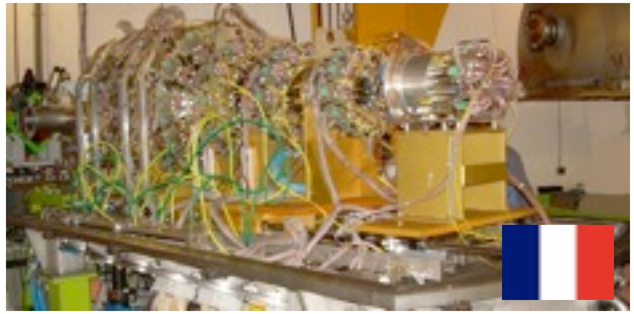
Worldwide postcards

Isis

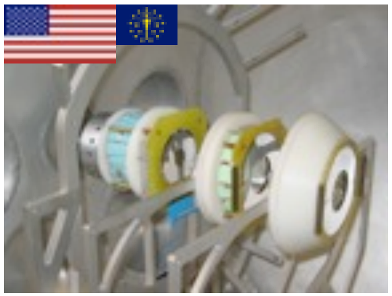


Lassa

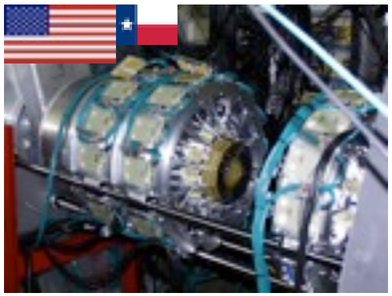
Indra



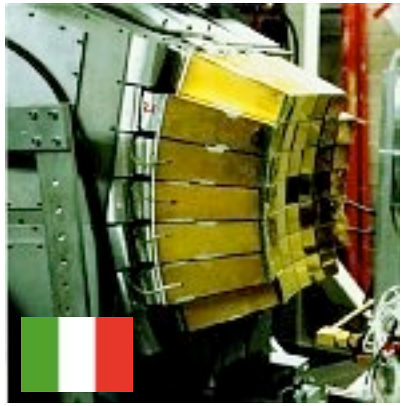
First



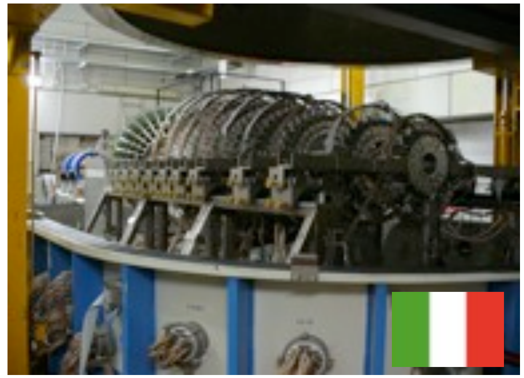
NIMROD



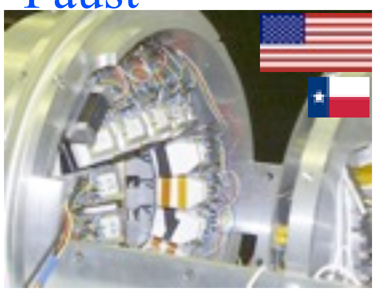
Multics



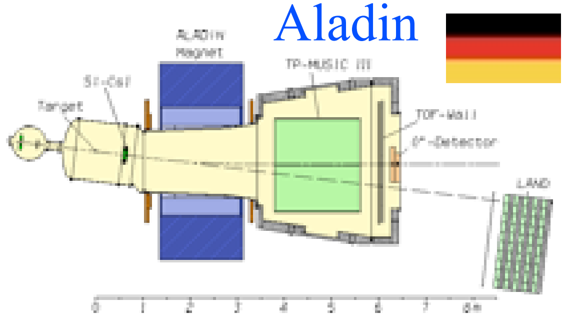
Chimera



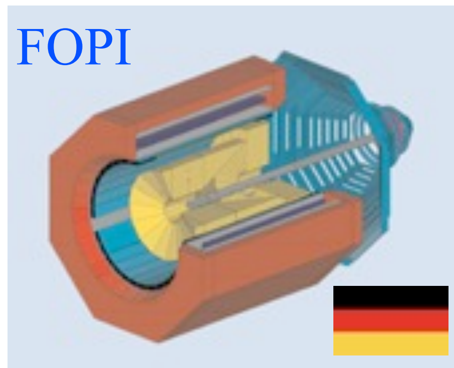
Faust



Aladin

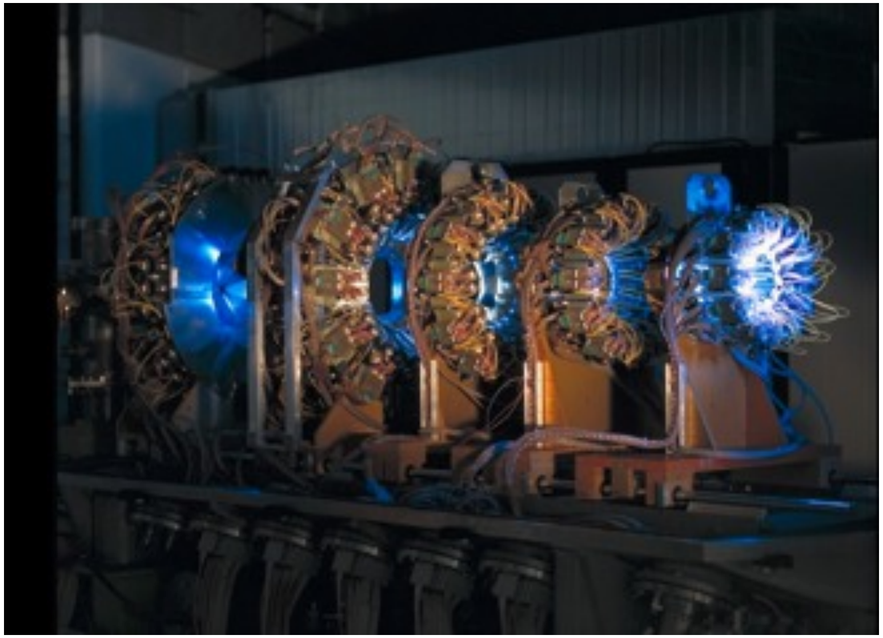


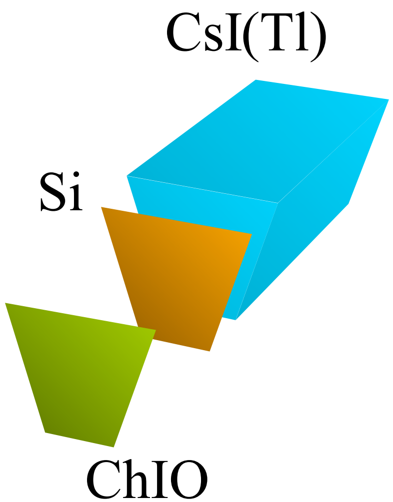
FOPI

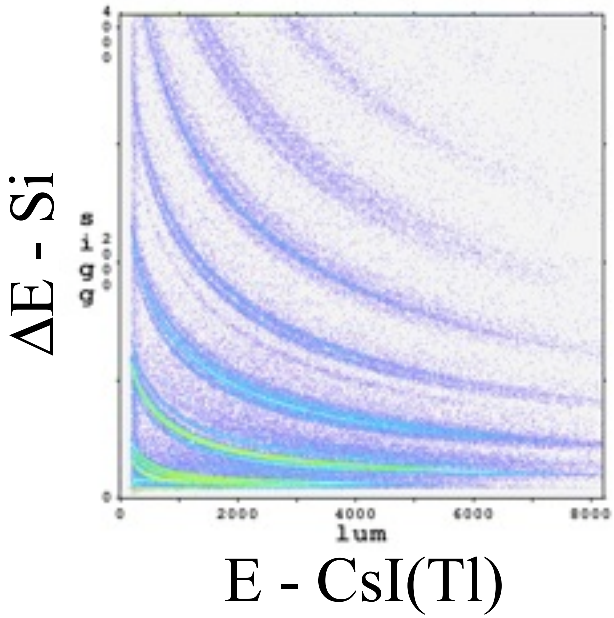


Indra @ GANIL

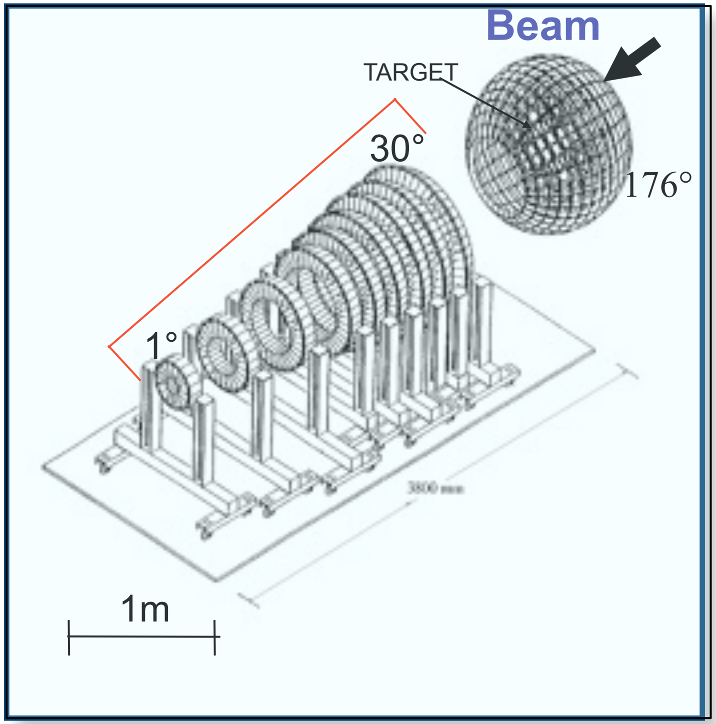
J. Pouthas et al., NIMA357, 418 (1995)







Chimera @ INFN LNS



1192 Si-CsI(Tl) Telescopes

35 rings in the range 1° ≤ θ ≤ 176°

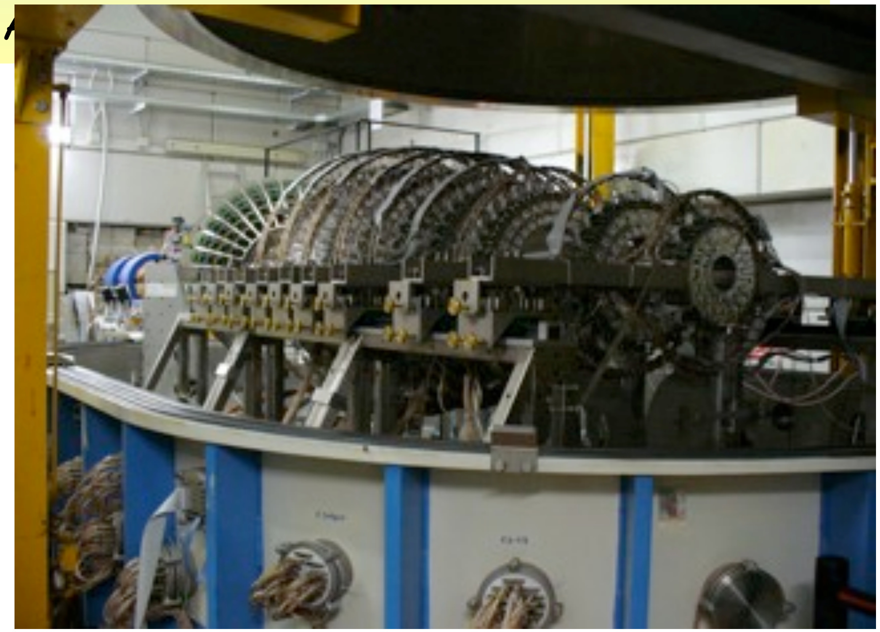
High granularity and efficiency up to 94% 4π

Z identification up to beam charge (ΔE-E)

Z and A identification by ΔE-E up to Z ≤ 9

Z and A identification in CsI up to Z ≤ 4

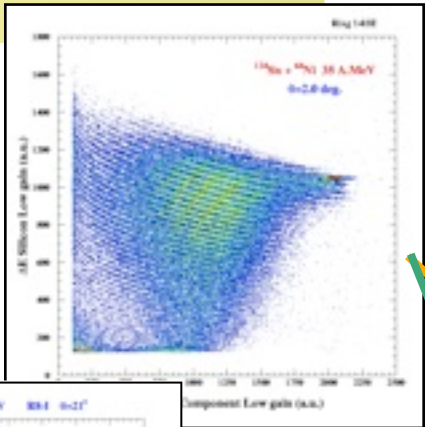
Mass identification with low energy threshold (<0.3



Particle identification with Chimera

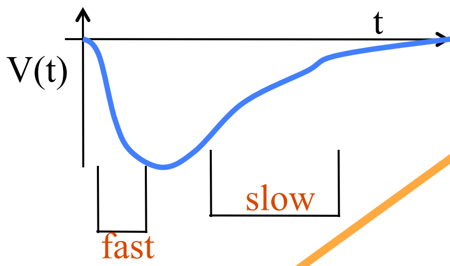
ΔE(Si)-E(CsI)

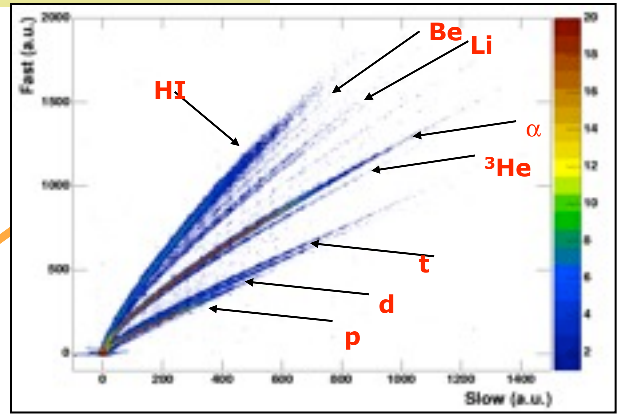
Charge Z for particles punching throught the Si detector

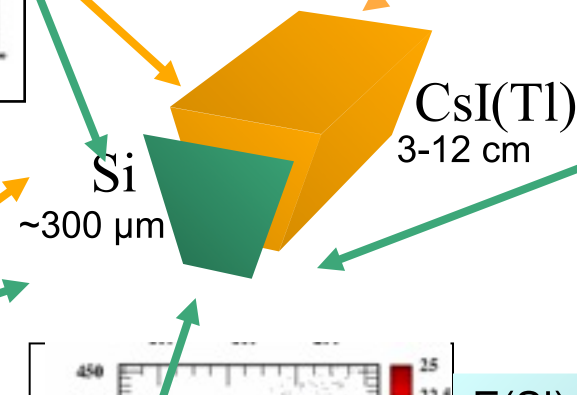


PSD in CsI(Tl)

Z and **A** for light charged particles

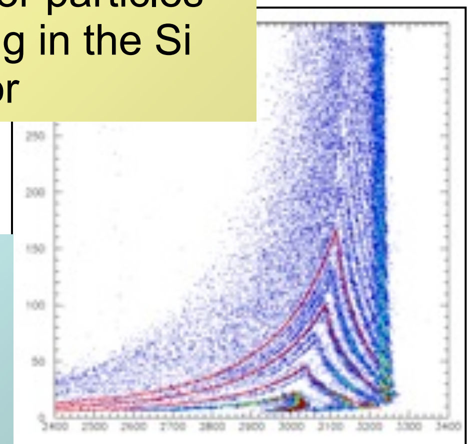


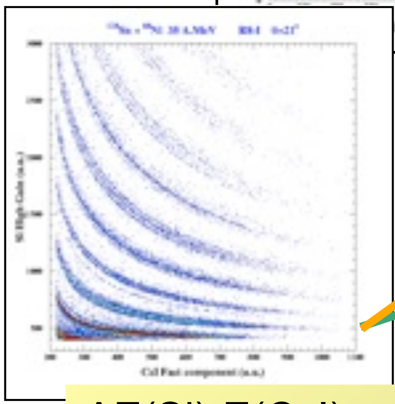




ΔE(Si)-ToF

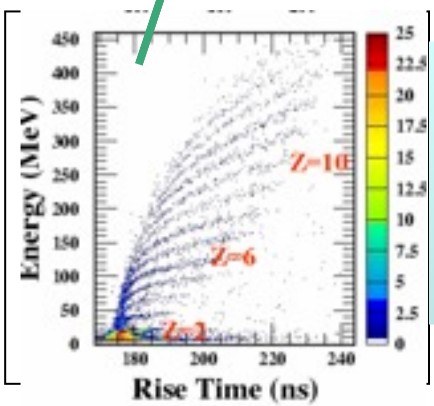
Mass for particles stopping in the Si detector





ΔE(Si)-E(CsI)

Charge Z and A for light ions (Z<9) punching throught the Si detector



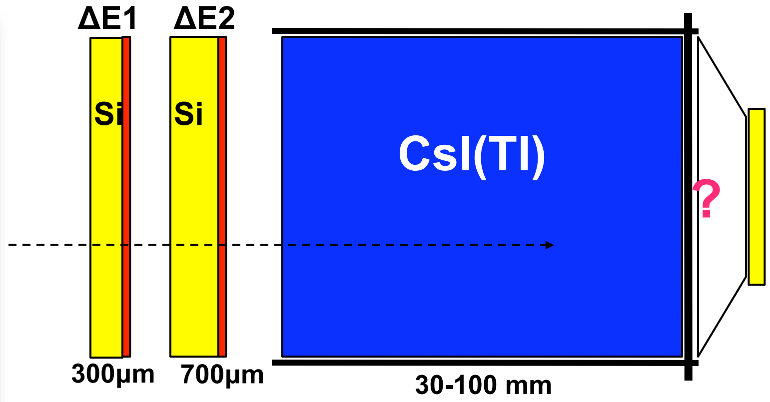
E(Si)-Rise time

Charge Z for particle stopping in Si detectors (**NEW**)

FAZIA - Four-π A-Z Identification Array

Detector:

- **Telescope: 2 ΔE Silicons + 1 CsI(Tl)**
- **ΔE2 used as a silicon det and as a photodiode for CsI(Tl)**
- **Silicon strips flexibility for angular resolution**
- ***Pulse Shape* on ΔE1 Silicon: low thresholds**



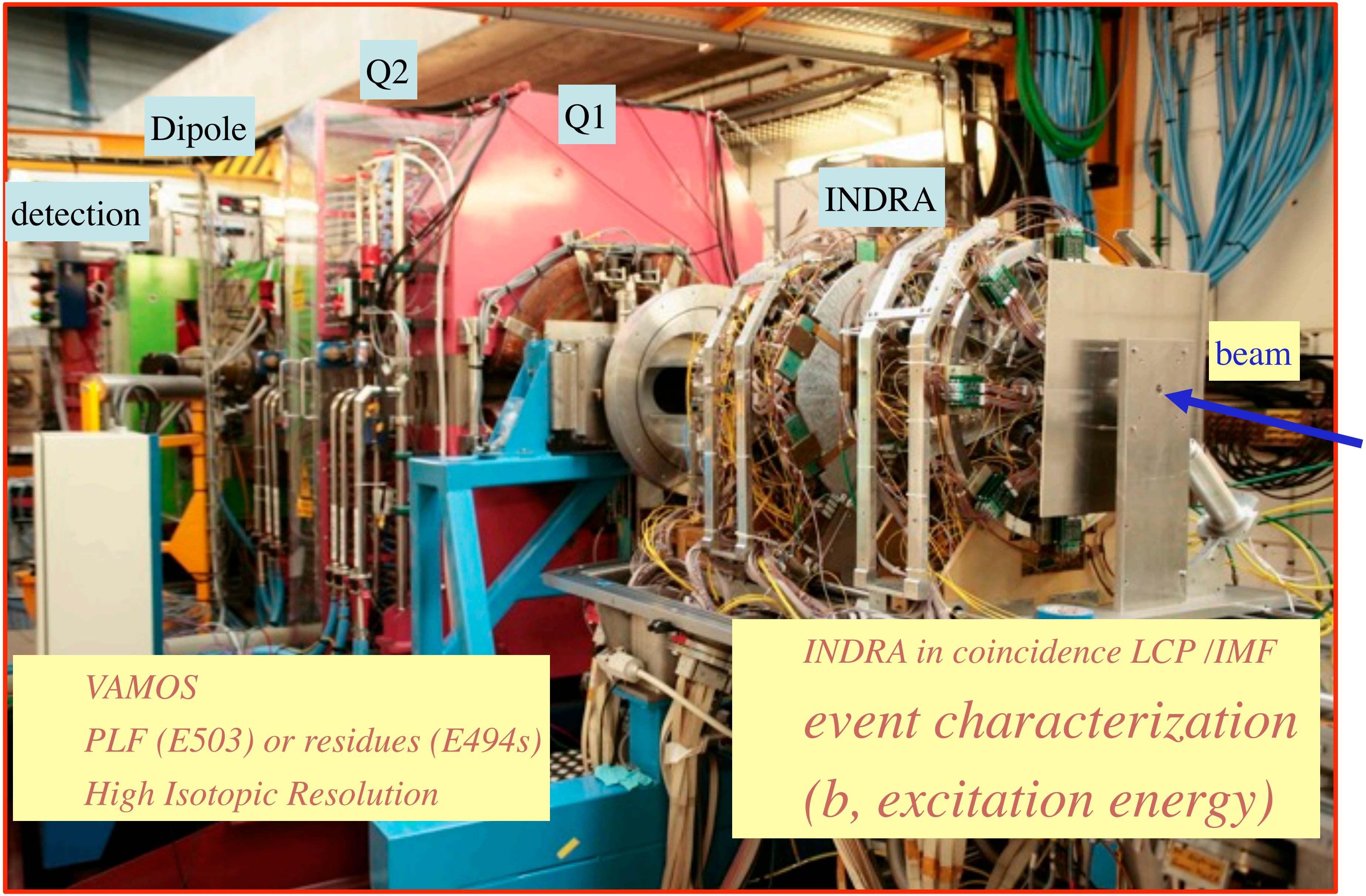
Very high isotopic resolution

Coupling different detectors

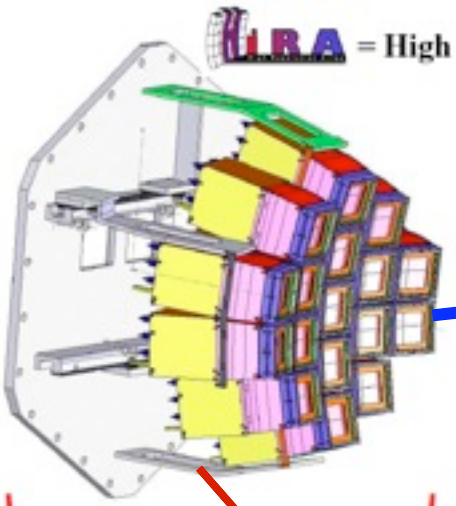
$Det_1 + Det_2 + Det_3 + \ldots$

$= 4\pi$ detectors

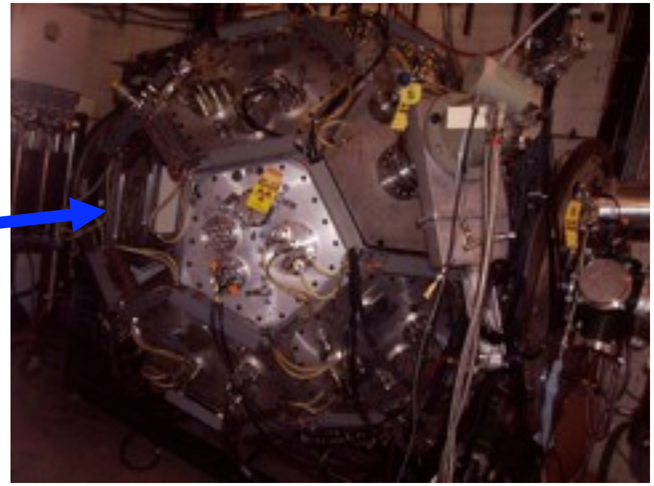
Coupling Indra+Vamos @ GANIL (2007)



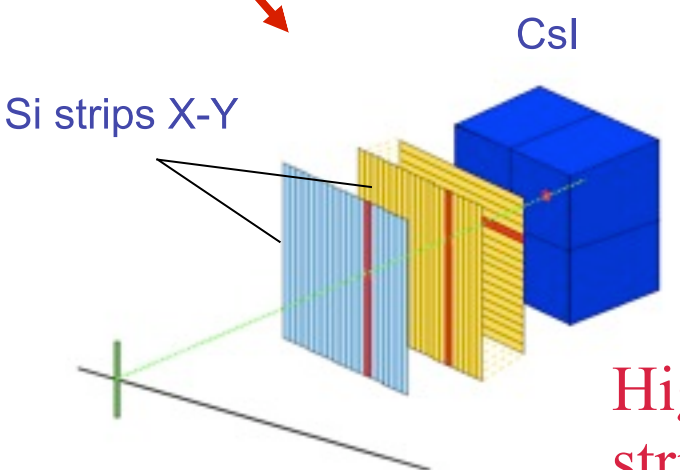
Coupling 4π + silicon strip arrays



NSCL - MSU



4π for event characterization



High angular resolution (x-y silicon strips) for correlations, “femtoscopy”, isotopic resolution

Neutron/proton experiments

M. Famiano et al., PRL97, 052701 (2006)

n detectors

Liquid scintillators

p detectors

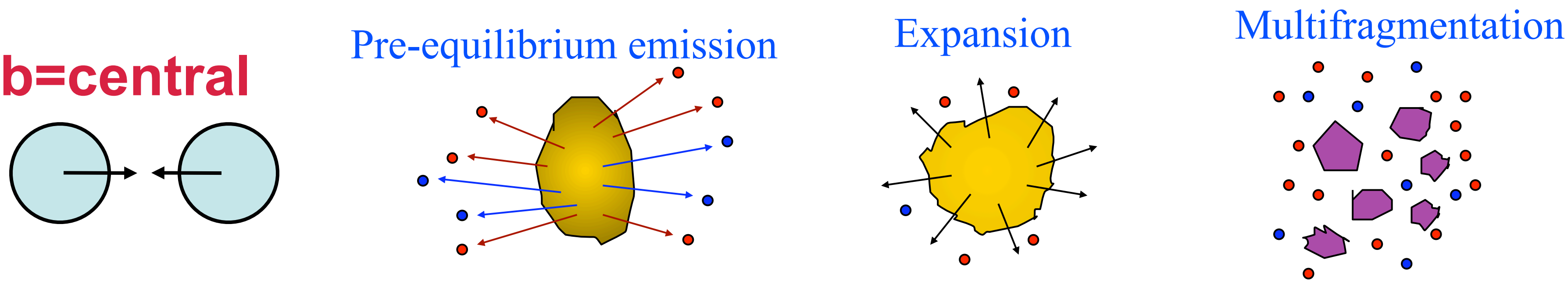
DE-E (Lassa)

- **Difficult experiments**
  - neutron efficiency
  - neutron cross-talk…
  - neutron background (shadow bars, n-γ discrimination, …)
- **…but worth the effort**

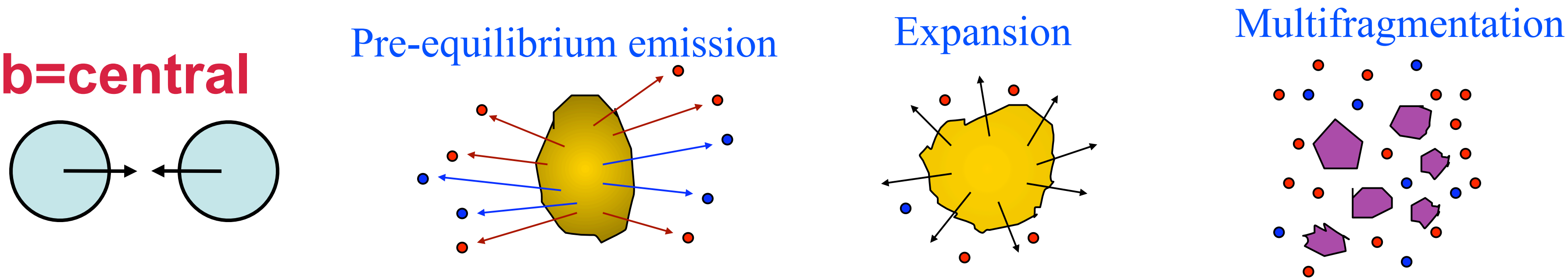
n/p observables the most sensitive probes of E_{sym}

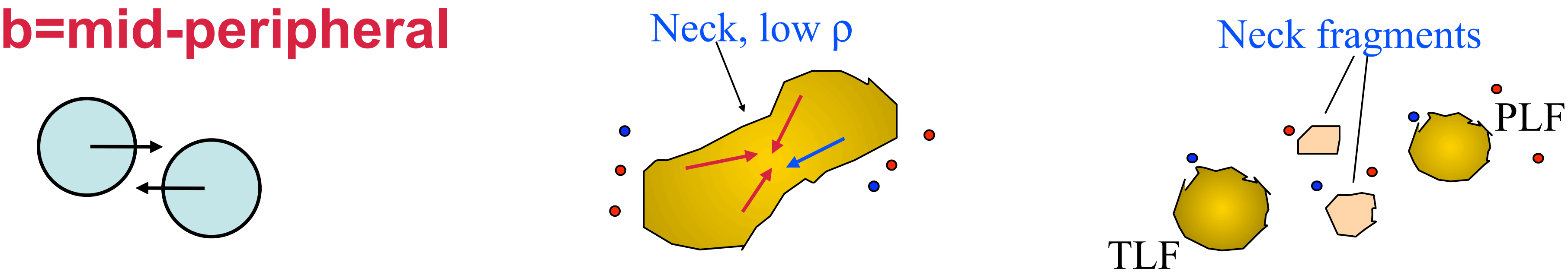
HIC at intermediate energies: $E_{sym}(\rho)$ at $\rho<\rho_0$

HIC at intermediate energies: $E_{sym}(\rho)$ at $\rho<\rho_0$



HIC at intermediate energies: $E_{sym}(\rho)$ at $\rho<\rho_0$





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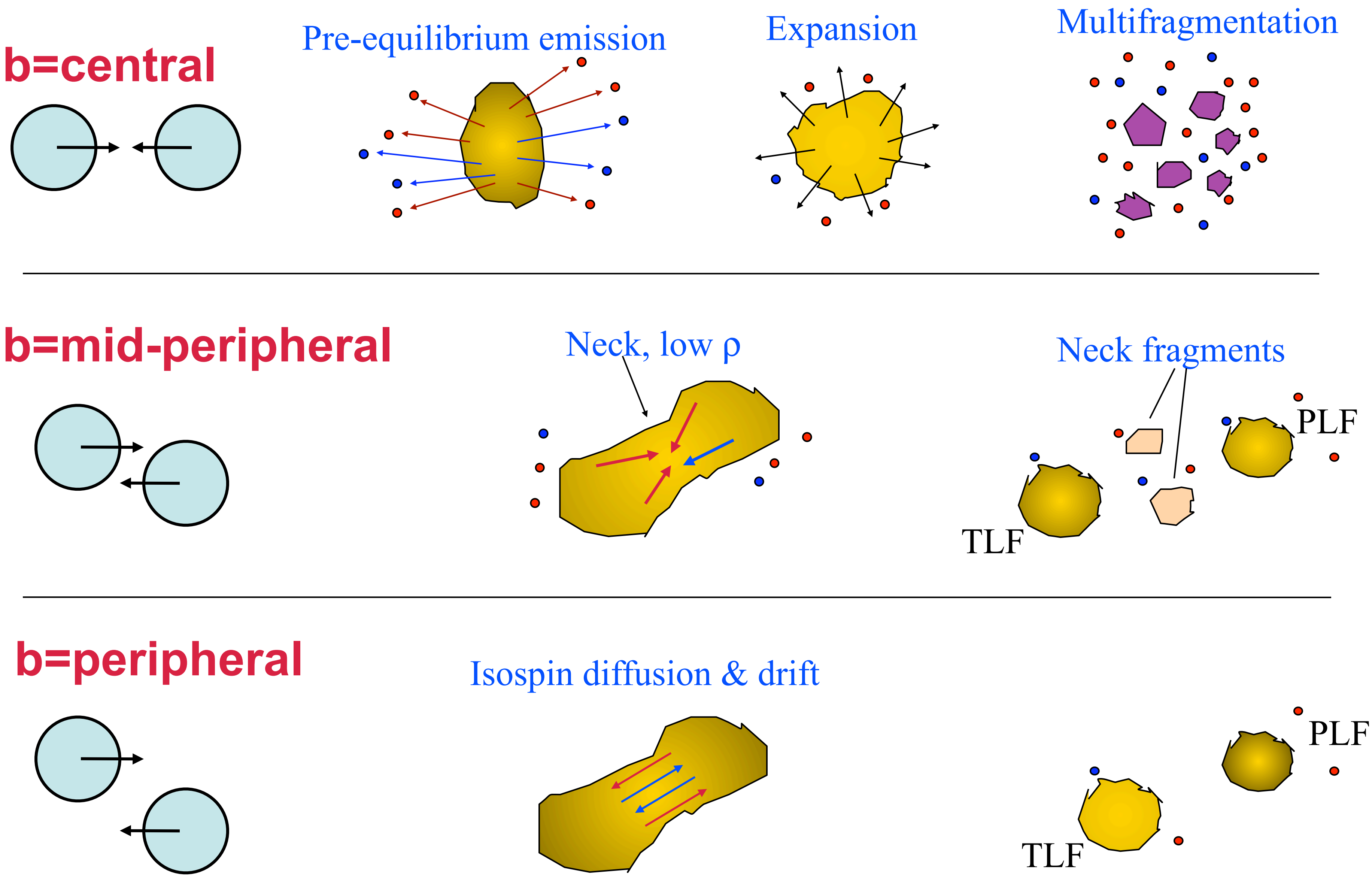

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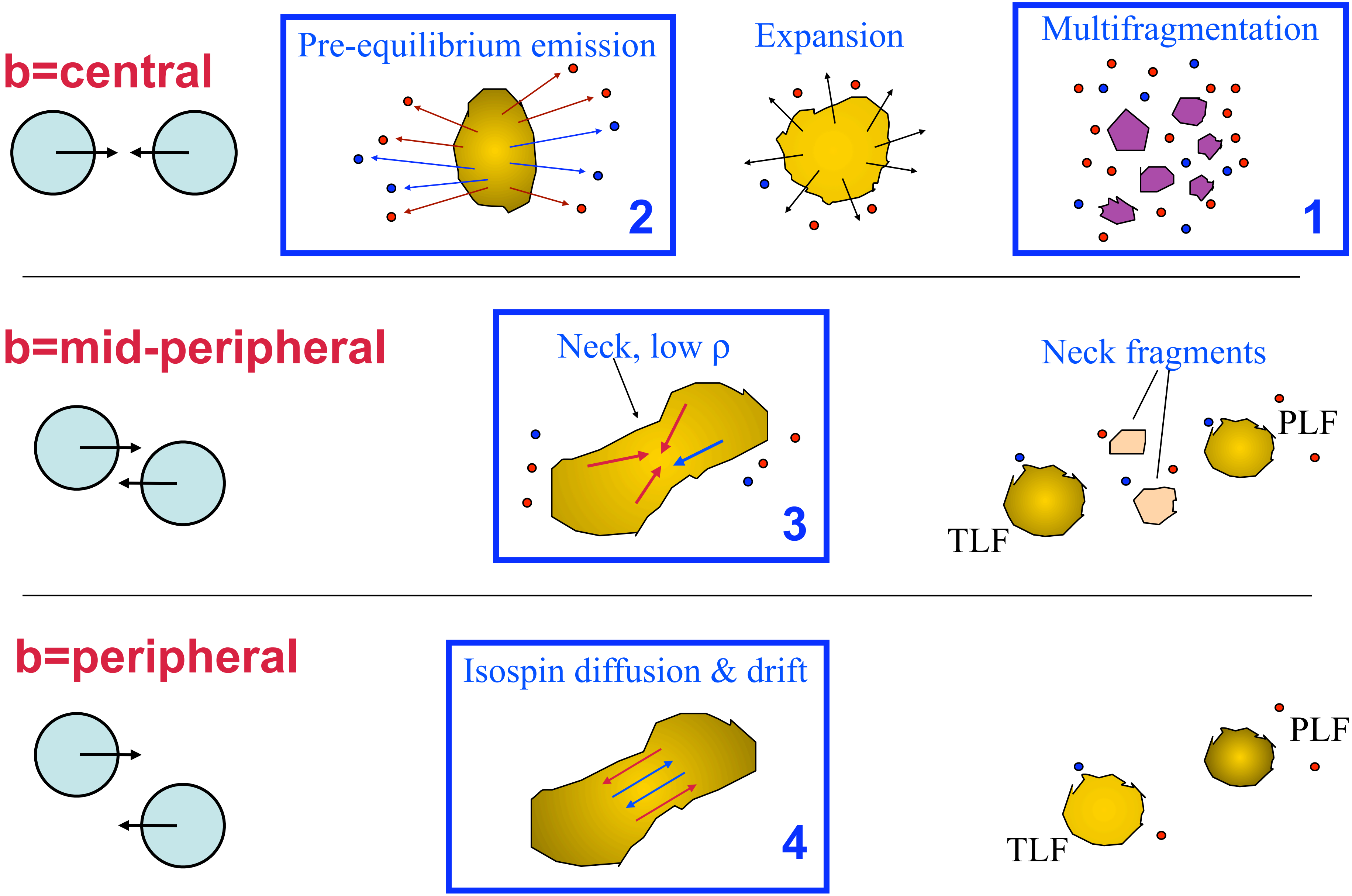
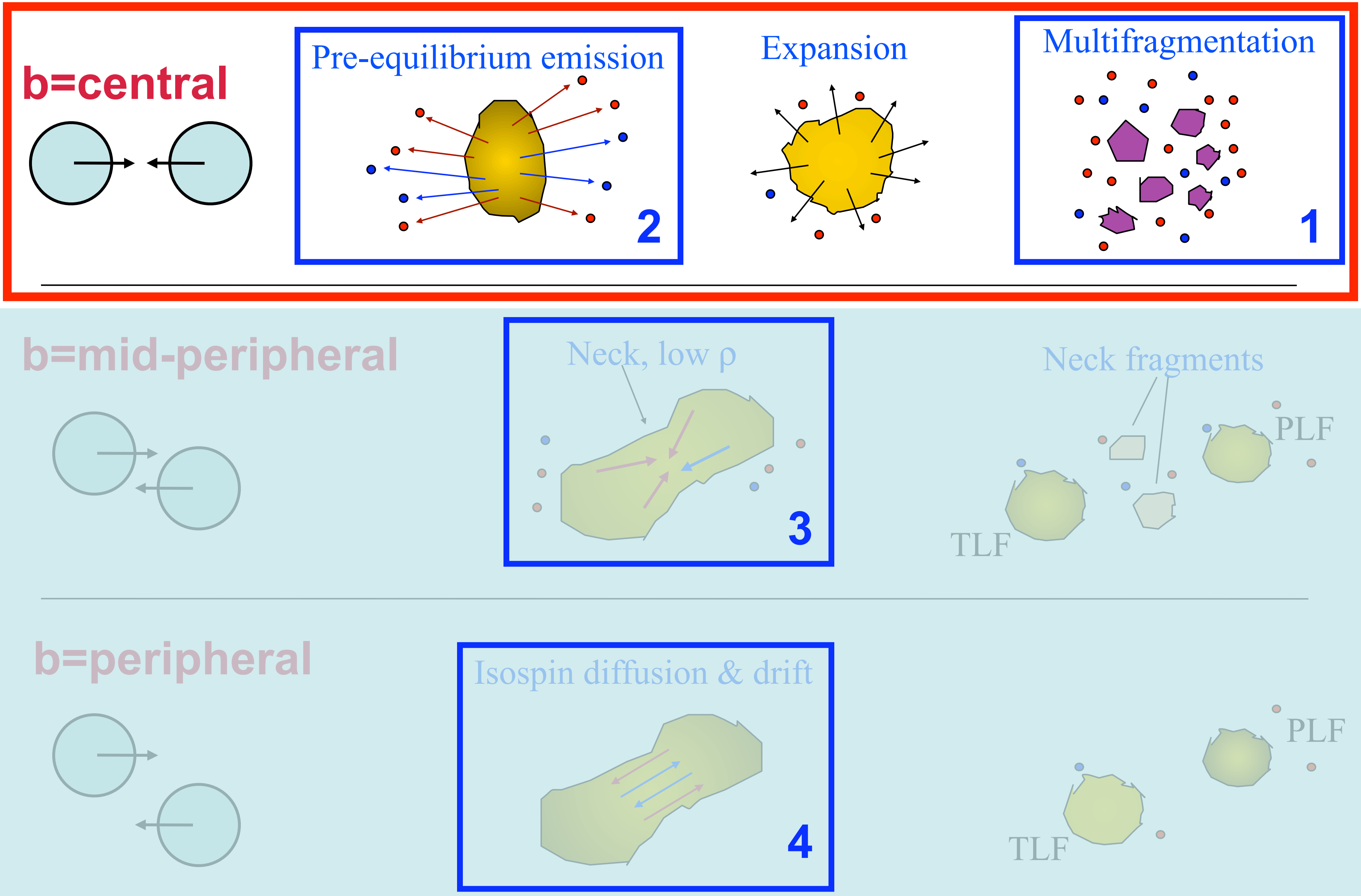
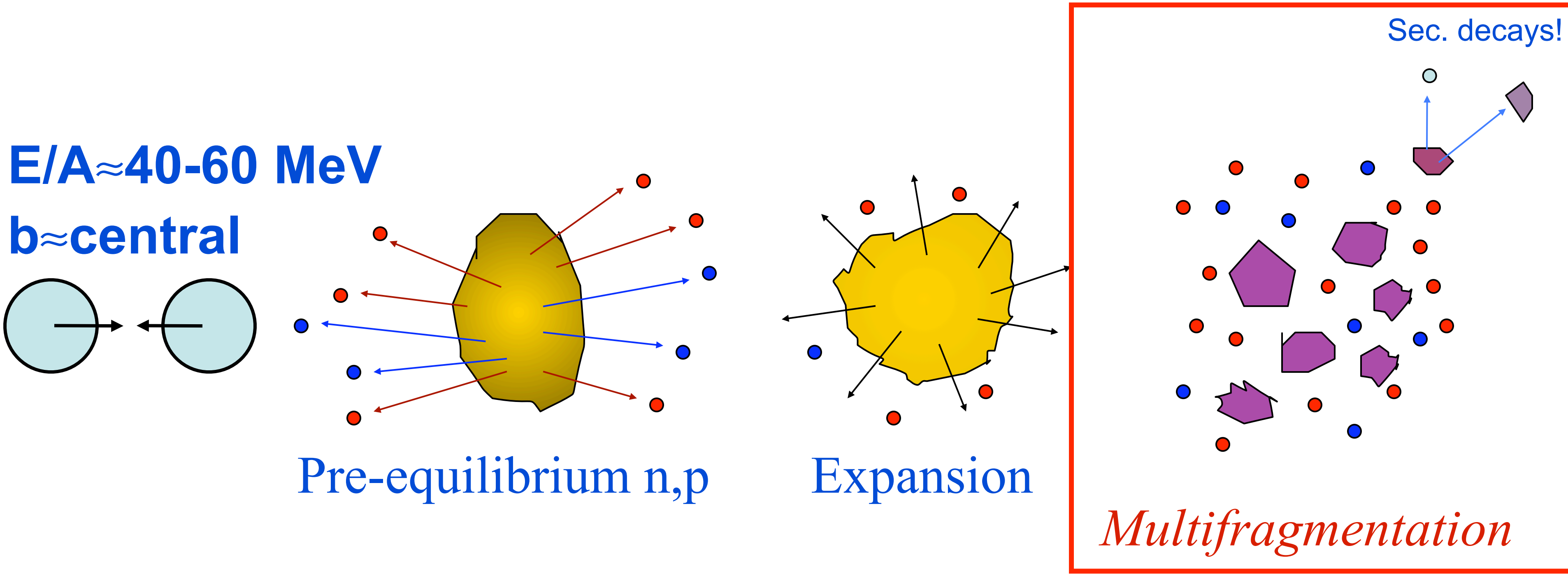


HIC at intermediate energies: $E_{sym}(\rho)$ at $\rho<\rho_0$

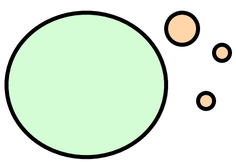


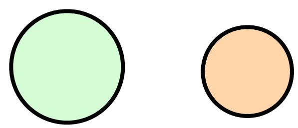
Multifragmentation



Vocabulary:

Copious emission of intermediate mass fragments (IMF, 3<A<30) in the same event

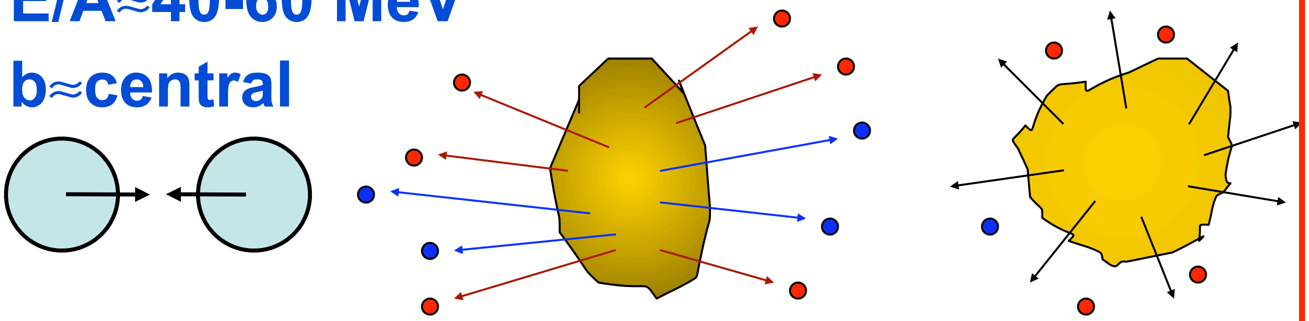
≠Fusion 

≠Fission 

Multifragmentation

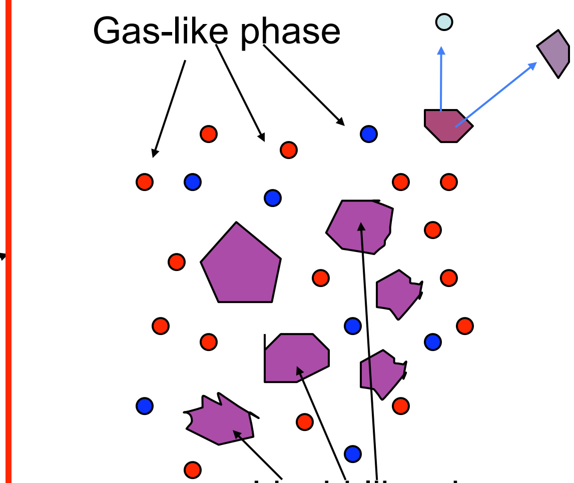
E/A≈40-60 MeV

b≈central



Pre-equilibrium n,p

Expansion



Multifragmentation

Vocabulary:

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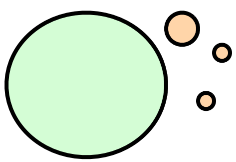
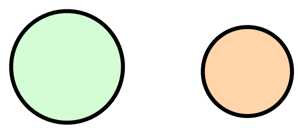
≠Fusion 

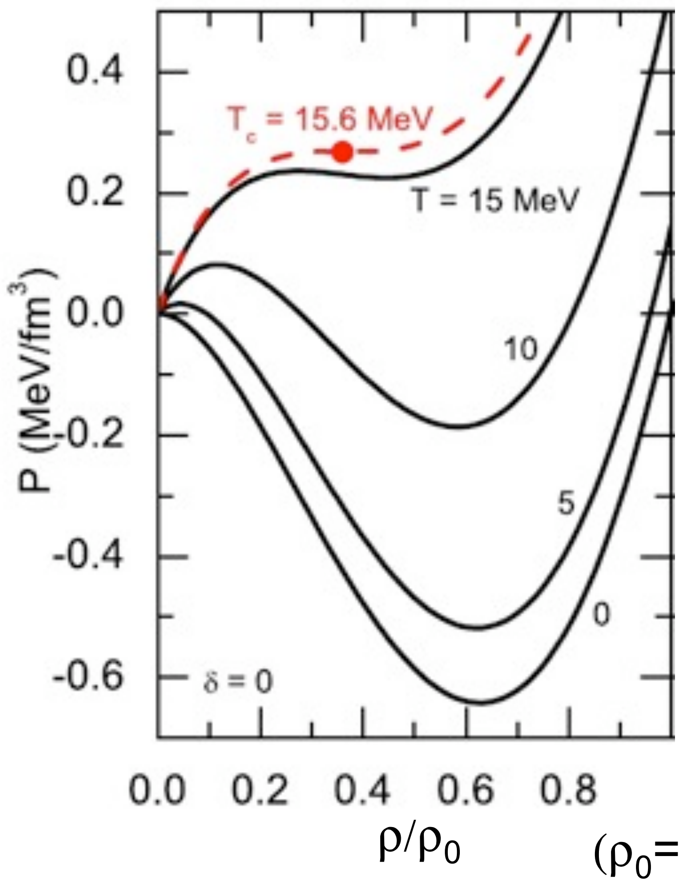
≠Fission 

N/Z effects in multifragmentation

- Isoscaling
- Isospin fractionation
- Links to the density dependence of the symmetry energy $E_{sym}(\rho)$

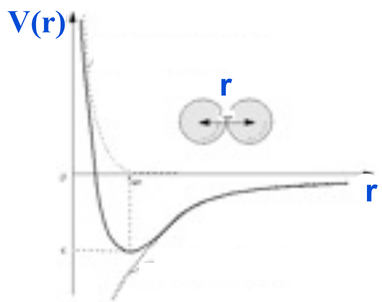
Why multifragmentation?

Nuclear Phase Diagram



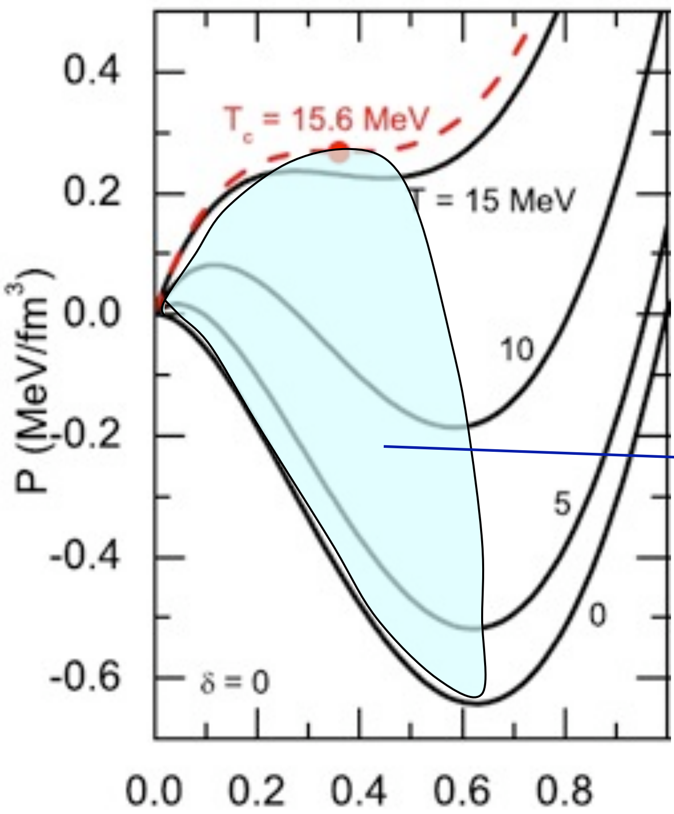
$(\rho_0 = 0.16\ \mathrm{fm}^{-3})$

nuclear interaction - Van der Waals-like



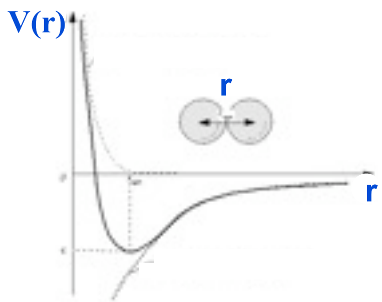
Why multifragmentation?

Nuclear Phase Diagram



ρ/ρ_0 $(\rho_0=0.16\ fm^{-3})$

nuclear interaction - Van der Waals-like



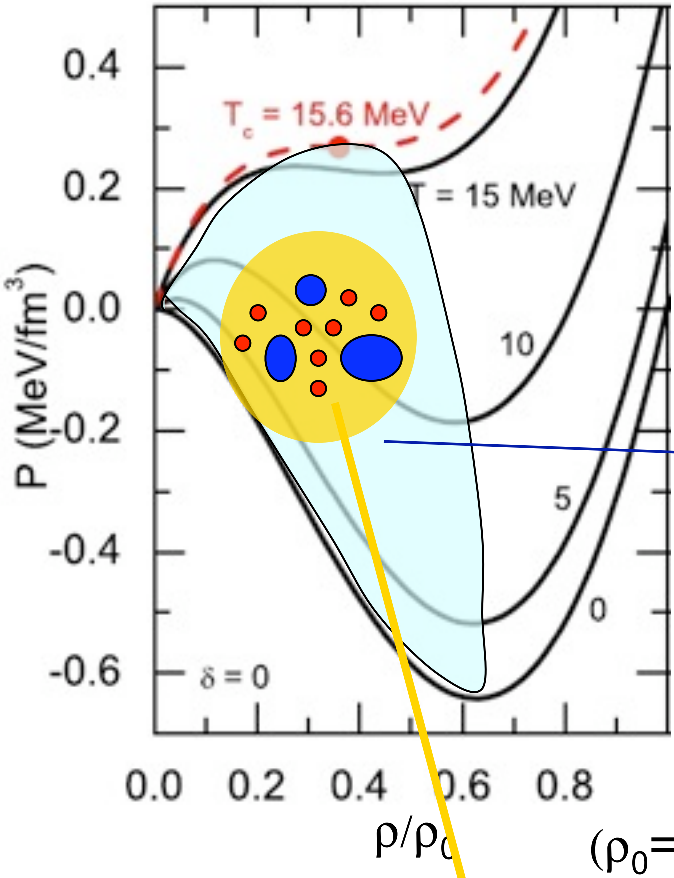
HIC take you into the mechanical instability region

$$(\partial P/\partial \rho) < 0$$

Growth of density fluctuations – Fragments are formed

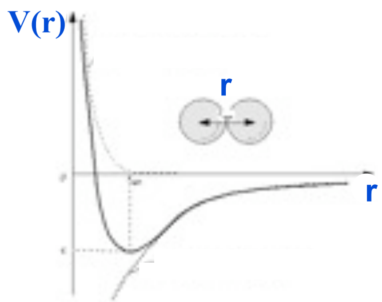
Why multifragmentation?

Nuclear Phase Diagram



(ρ_0=0.16 fm^{-3})

nuclear interaction - Van der Waals-like



HIC take you into the mechanical instability region

$$(\partial P/\partial \rho) < 0$$

Growth of density fluctuations – Fragments are formed

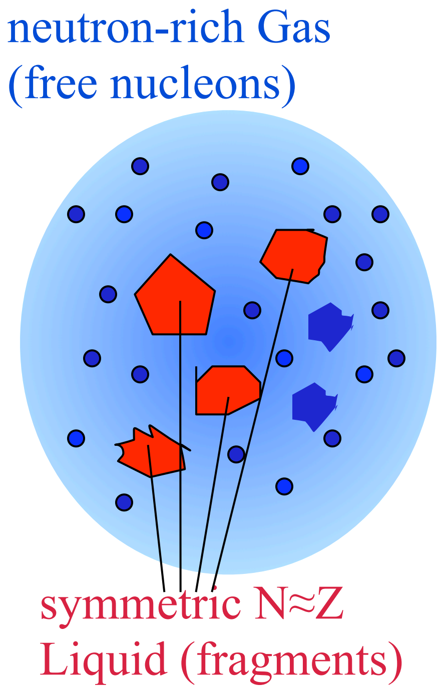
Liquid-gas phase coexistence

Free nucleons (GAS) + Complex fragments (LIQUID DROPS)

Isospin fractionation or distillation

$$\frac{\rho_n - \rho_p}{\rho_n + \rho_p} \neq 0$$

Coexistence region liquid-gas



N>Z neutron rich

N≈Z symmetric

RMF at finite T

Mueller & Serot, PRC52, 2072 (1995)

Neutron-rich gas

&

N/Z-symmetric liquid

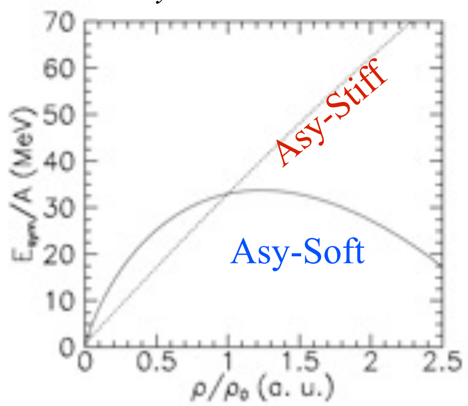
Isospin Fractionation and $E_{sym}(\rho)$

BNV model calculations:
Mean-field with $E_{sym}(\rho)$

M. Di Toro et al., arXiv:1003.2957v1 [nucl-th]

Sn+Sn, E/A=50 MeV, Central collisions

E_{sym} vs (ρ/ρ_0)



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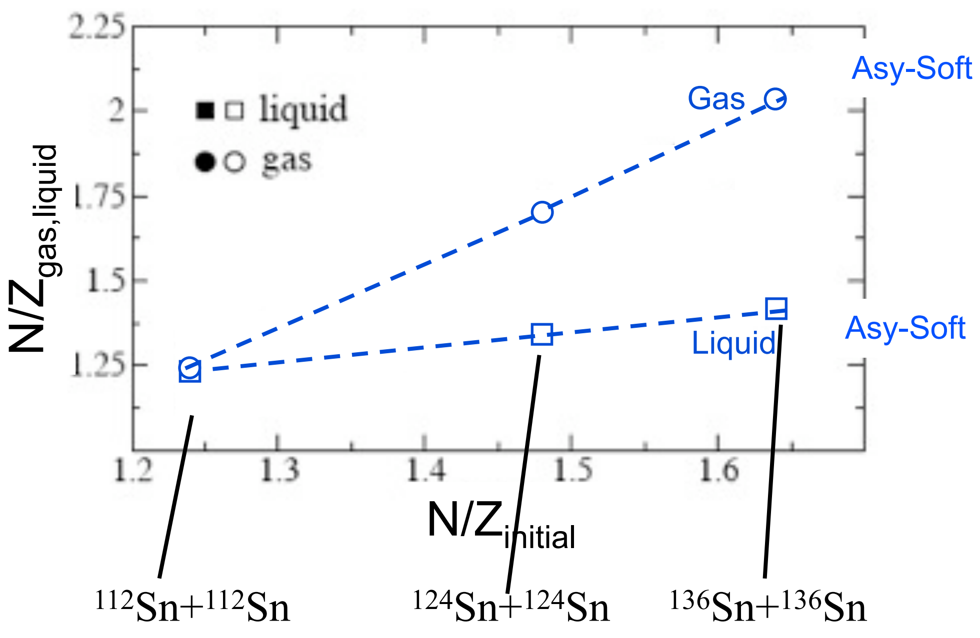

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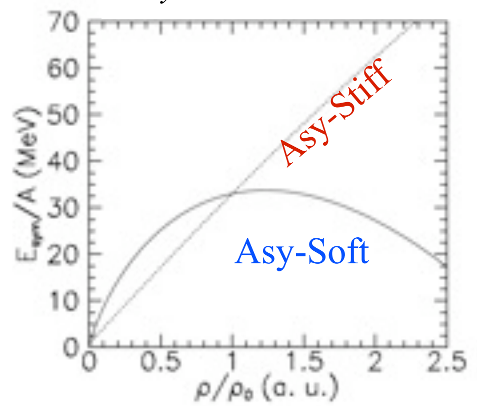

$N/Z_{Gas} > N/Z_{Liquid}$

Isospin Fractionation

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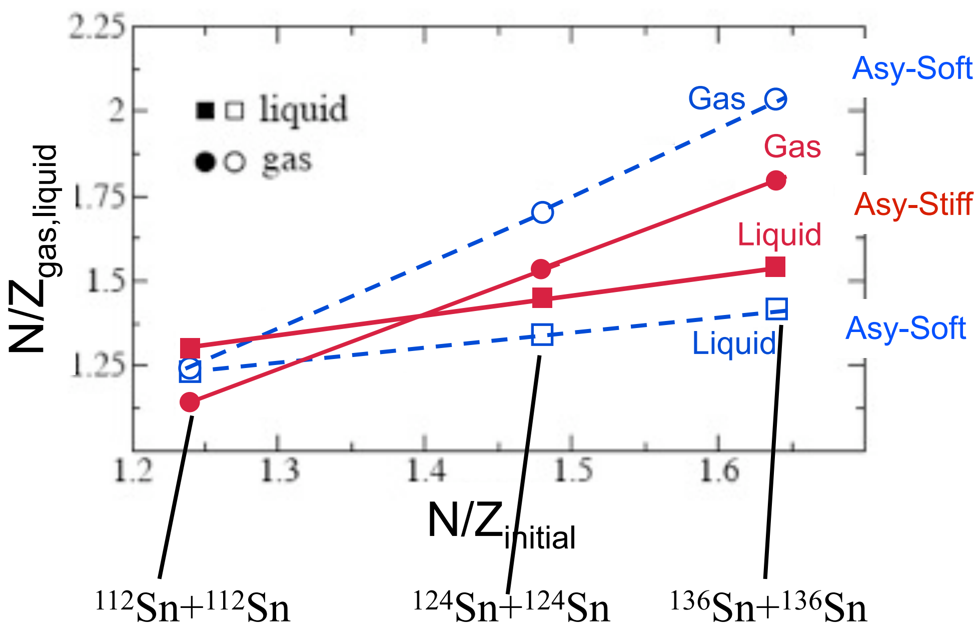
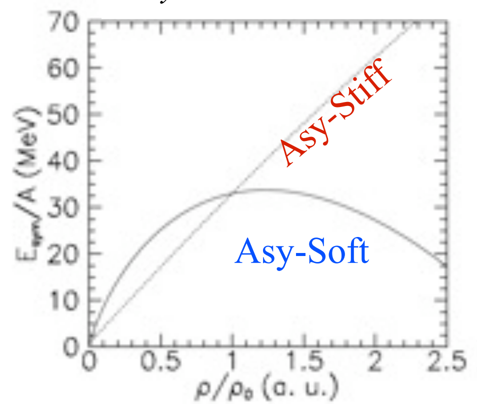


E_{sym} vs (ρ/ρ_0)



Sensitive to $E_{sym}(\rho)$

Soft > Stiff

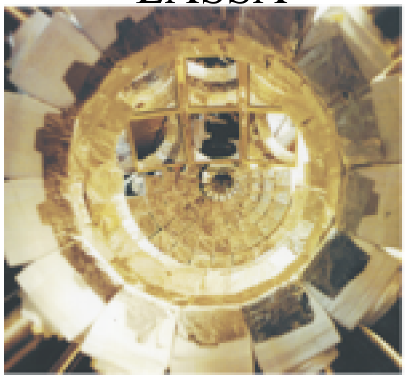
$N/Z_{Gas} > N/Z_{Liquid}$
Isospin Fractionation

In a real experiment

- Observe multifragmentation
- Measure the isotopic composition of
  - complex fragments (A>4) - “liquid”
  - free neutrons and protons - “gas”
- Do we see isospin fractionation?
  - $(N/Z)_{free\text{-}nucleoen} > (N/Z)_{fragments}$?
- If yes, link it to the symmetry energy

Isotopic composition of fragments

LASSA

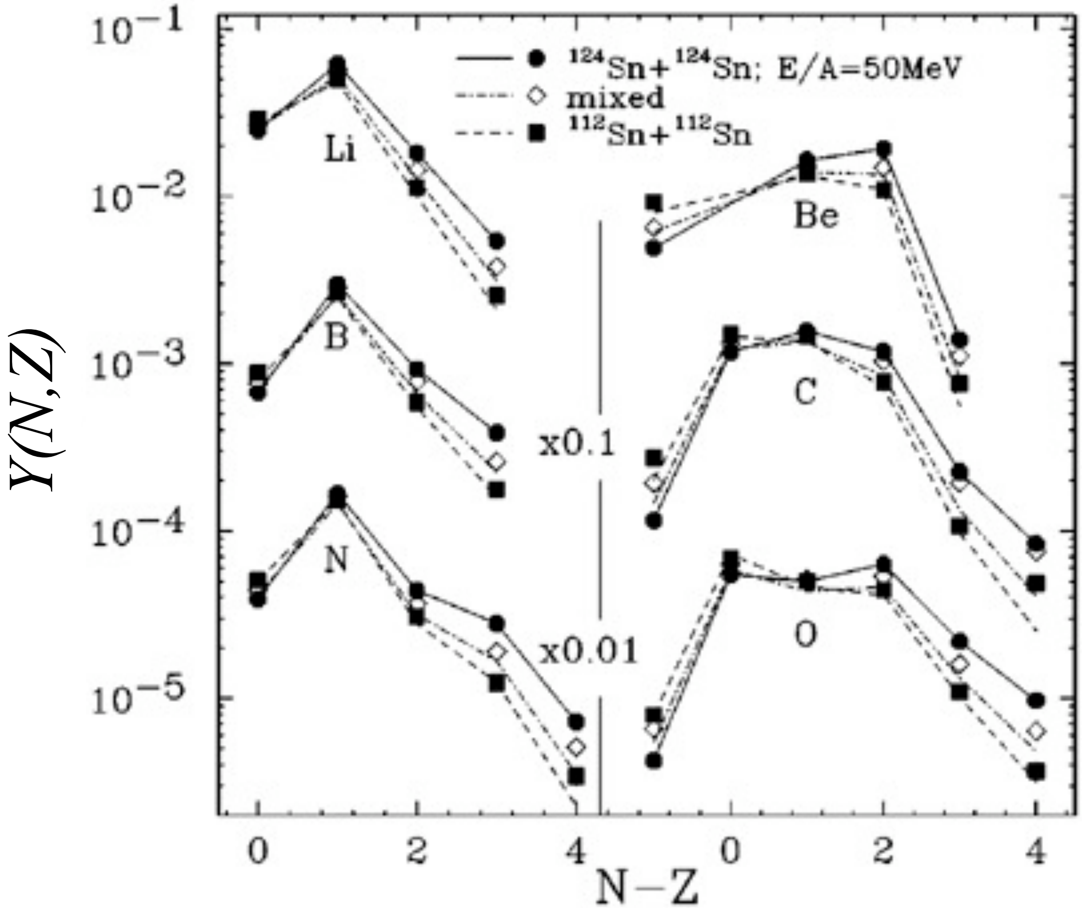


High isotopic resolution

$^{112}Sn+^{112}Sn$, $^{112}Sn+^{124}Sn$, $^{124}Sn+^{124}Sn$

N/Z = 1.24 1.36 1.48

E/A=50 MeV, **Central**

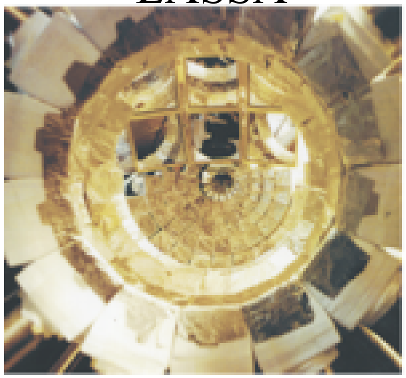


Measure the yields of fragments for each value of Z=3-8

Y(N,Z) vs N-Z

Isotopic composition of fragments

LASSA



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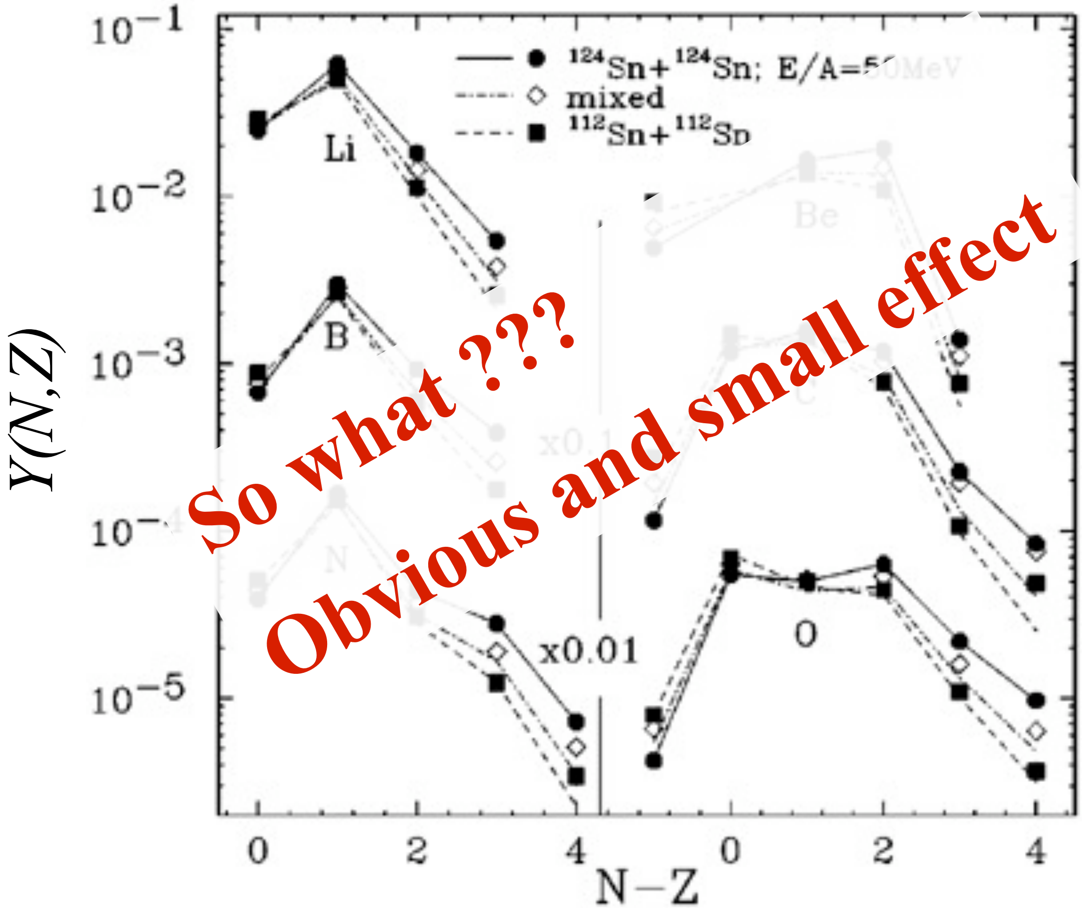
N/Z = 1.24 1.36 1.48

E/A=50 MeV, **Central**

Measure the yields of fragments for each value of Z=3-8

Y(N,Z) *vs* *N-Z*

Larger N/Z reaction (124+124) produces more n-rich fragments



Isotope ratios and isoscaling

$^{112}Sn+^{112}Sn$ vs $^{124}Sn+^{124}Sn$ E/A=50 MeV

Amplify isotopic effects with ratios

$$R_{12}(N,Z) = \frac{Y^{124+124}(N/Z)}{Y^{112+112}(N/Z)}$$

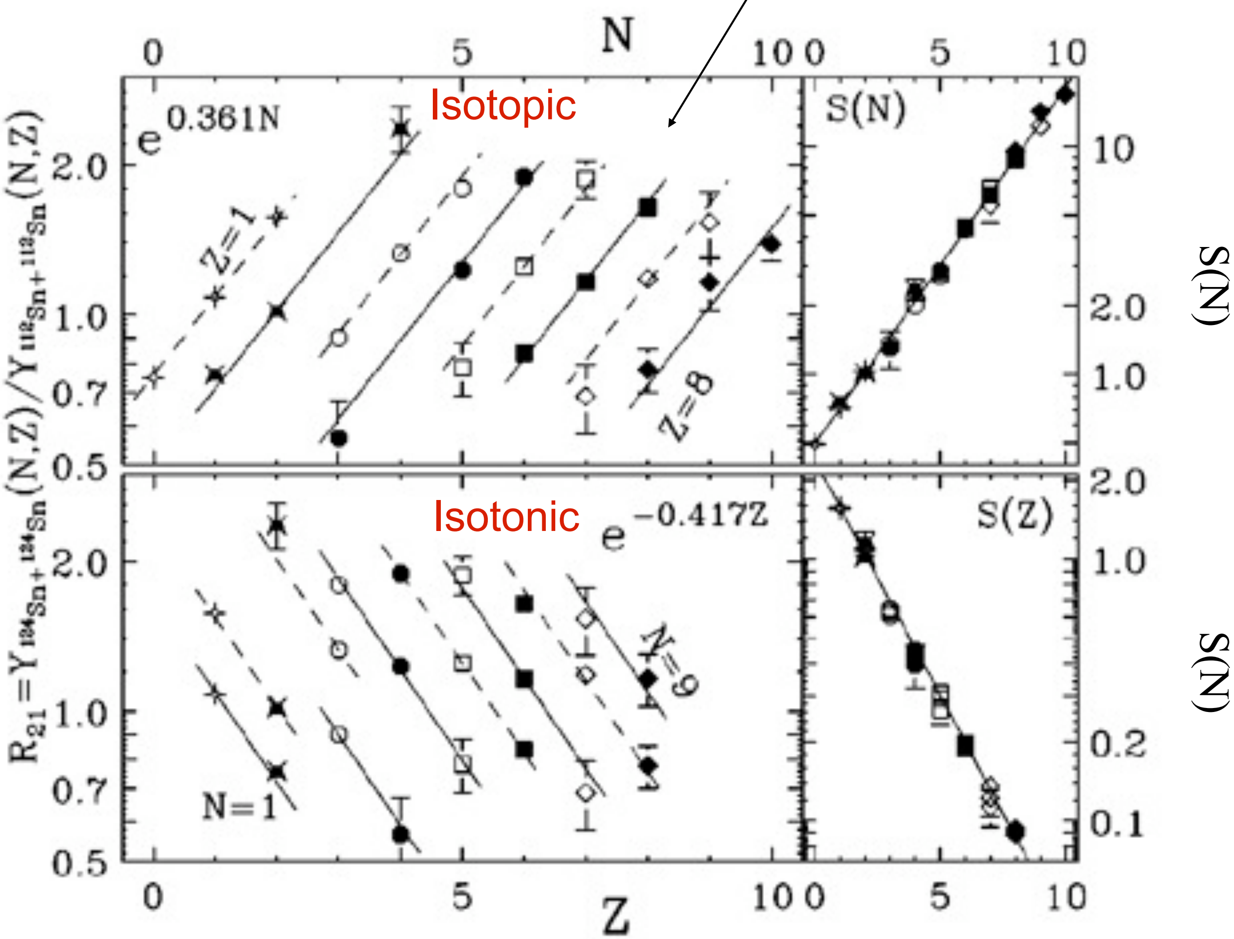
Tsang et al, PRL86, 5023 (2001)

Isotope ratios and isoscaling

$^{112}Sn+^{112}Sn$ vs $^{124}Sn+^{124}Sn$ E/A=50 MeV

Amplify isotopic effects with ratios

$$R_{12}(N,Z)=\frac{Y^{124+124}(N/Z)}{Y^{112+112}(N/Z)}\propto \exp(\alpha\cdot N+\beta\cdot Z)$$



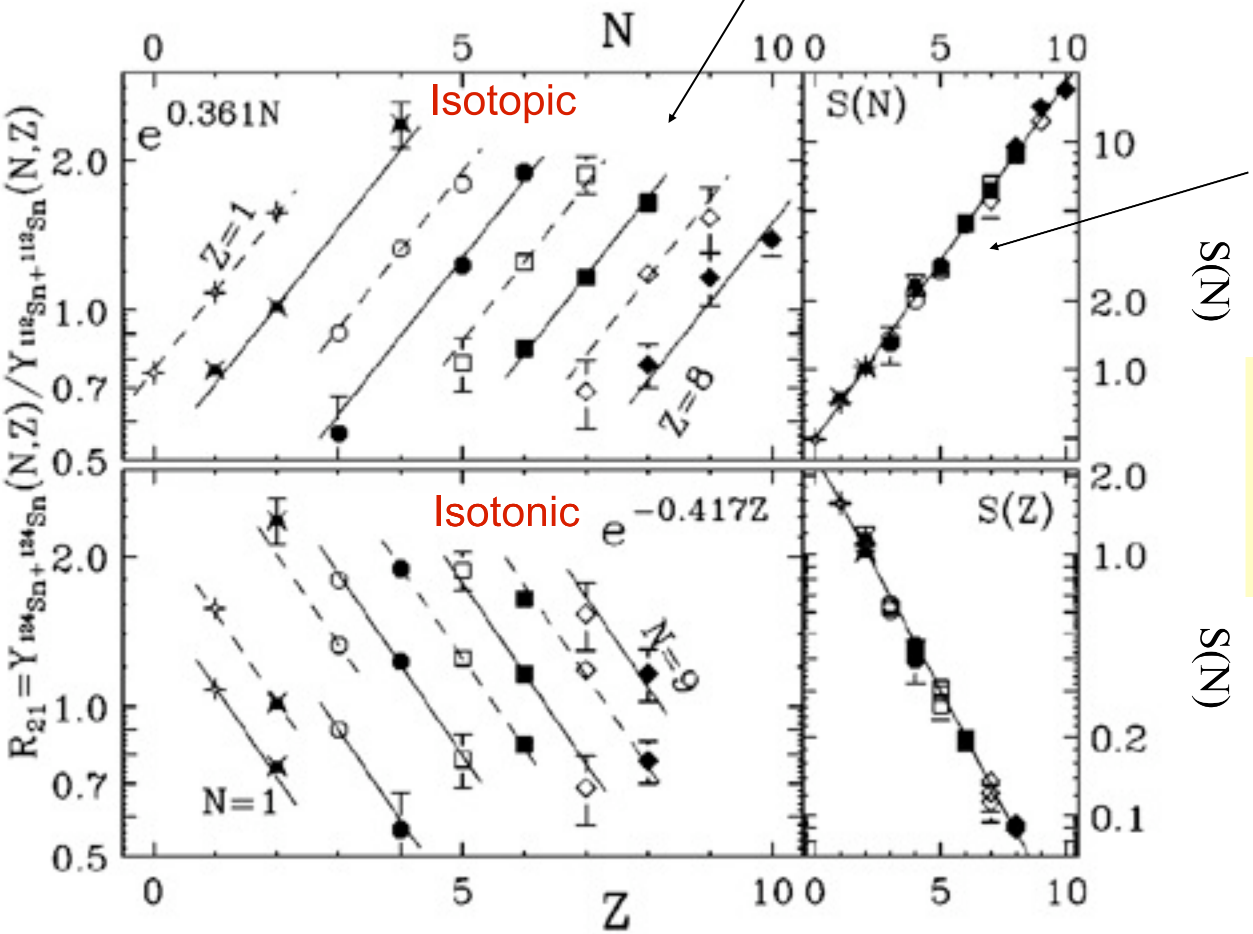
Tsang et al, PRL86, 5023 (2001)

Isotope ratios and isoscaling

$^{112}Sn+^{112}Sn$ vs $^{124}Sn+^{124}Sn$ E/A=50 MeV

Amplify isotopic effects with ratios

$$R_{12}(N,Z) = \frac{Y^{124+124}(N/Z)}{Y^{112+112}(N/Z)} \propto \exp(\alpha \cdot N + \beta \cdot Z)$$



Compact representation

$$S(N) = R_{21} \cdot exp(-\beta Z)$$

Universal scaling law

"Isoscaling"

Tsang et al, PRL86, 5023 (2001)

Accessing the N/Z of the gas

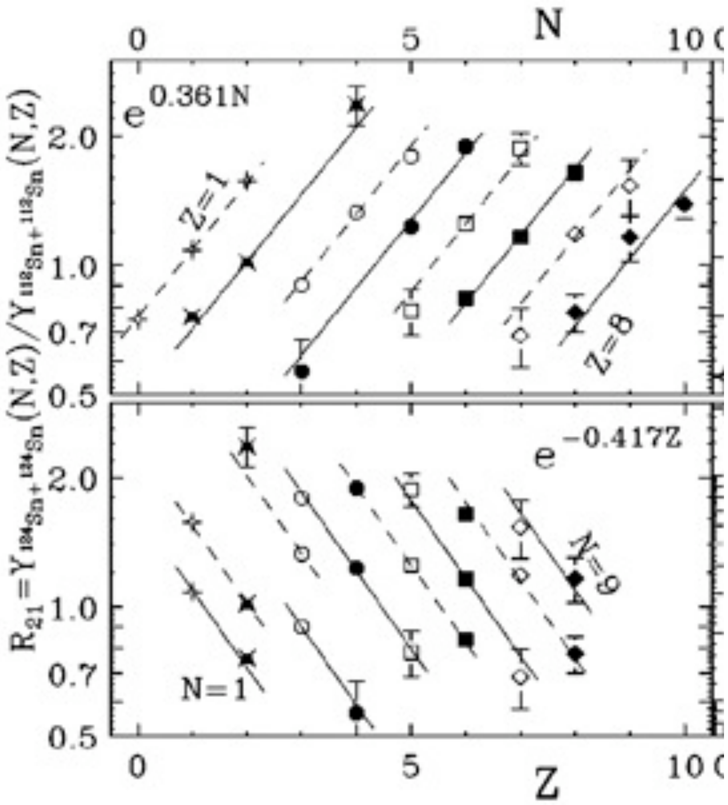
$$R_{12}(N,Z)=\frac{Y^{124+124}(N,Z)}{Y^{112+112}(N,Z)}=\exp(\alpha\cdot N+\beta\cdot Z)$$

Statistical interpretation (grand-canonical ensemble)

$$R_{12}(N,Z)=(\hat{\rho}_n)^N(\hat{\rho}_p)^Z$$

α β

$$\langle\hat{\rho}_n\rangle=\left[\frac{\rho_{free,n}^{124+124}}{\rho_{free,n}^{112+112}}\right]\qquad\langle\hat{\rho}_p\rangle=\left[\frac{\rho_{free,p}^{124+124}}{\rho_{free,p}^{112+112}}\right]$$



Free neutron and proton densities: N/Z of the GAS phase

Accessing the N/Z of the gas

$$R_{12}(N,Z) = \frac{Y^{124+124}(N,Z)}{Y^{112+112}(N,Z)} = \exp(\alpha \cdot N + \beta \cdot Z)$$

Statistical interpretation (grand-canonical ensemble)

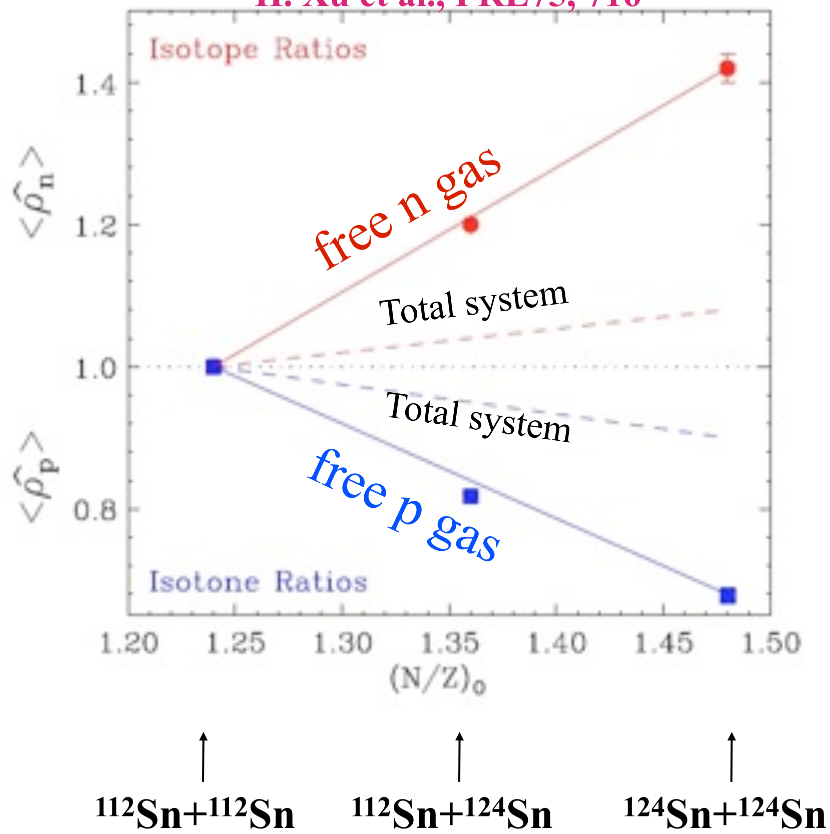
$$R_{12}(N,Z) = \left(\hat{\rho}_n\right)^N \left(\hat{\rho}_p\right)^Z$$

α β

$$\langle \hat{\rho}_n \rangle = \left[\frac{\rho_{free,n}^{124+124}}{\rho_{free,n}^{112+112}} \right] \qquad \langle \hat{\rho}_p \rangle = \left[\frac{\rho_{free,p}^{124+124}}{\rho_{free,p}^{112+112}} \right]$$

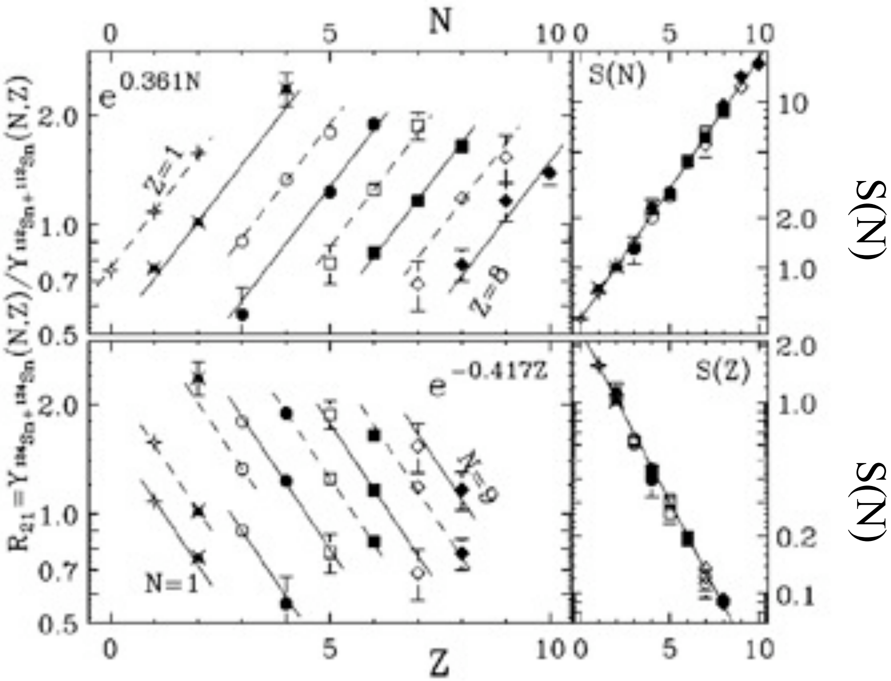
First experimental evidence of isospin fractionation

H. Xu et al., PRL75, 716



Interdisciplinary break

First submission to PRL was rejected…
"Submitted to the wrong journal."



Interdisciplinary break

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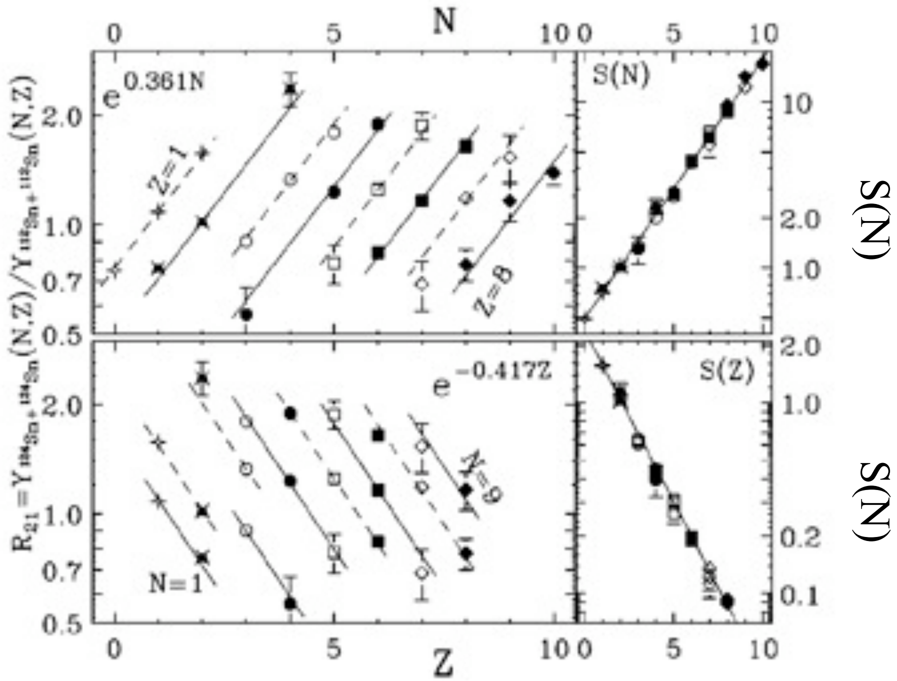


Biological symmetries in fish
(bilateral symmetry?)

Wednesday's night talk "legacy"

("Symmetries in biology")

Can we use Isoscaling and isospin fractionation to probe the symmetry energy $E_{sym}(\rho)$?

"Yes, but…"

Can we use Isoscaling and isospin fractionation to probe the symmetry energy $E_{sym}(\rho)$?

"Yes, but…"

Isoscaling and $E_{sym}(\rho)$: "Yes!"

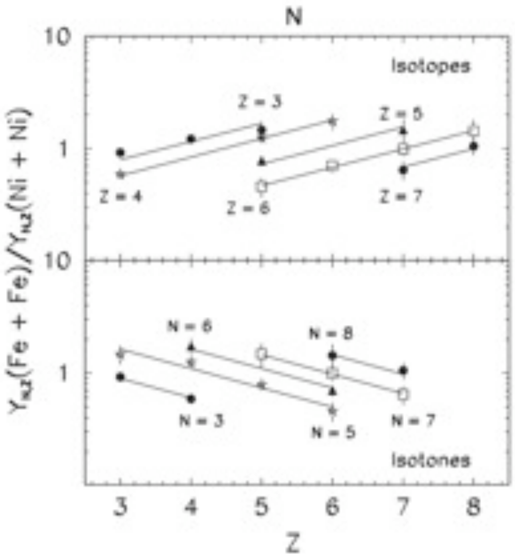
$$\alpha = \frac{4C_{sym}}{T}\cdot\left(\frac{Z_1^2}{A_1^2}-\frac{Z_2^2}{A_2^2}\right)$$

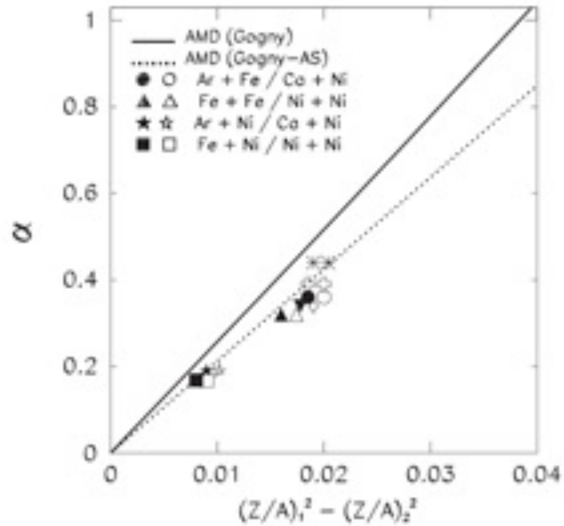
$$E/A = \ldots + C_{sym}(\rho/\rho_0)\times\frac{(N-Z)^2}{A^2}+\ldots$$

Texa A&M Un.

Fe+Fe/Ni+Ni, Ar +Fe/Ca+Ni Ar+Ni/ Ca+Ni Fe+Ni/Ni +Ni

E/A=25-53 MeV





Isoscaling and $E_{sym}(\rho)$: "Yes!"

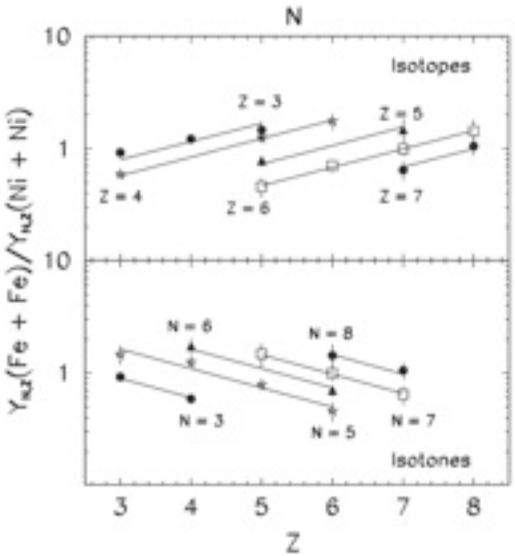
$$\alpha = \frac{4C_{sym}}{T} \cdot \left(\frac{Z_1^2}{A_1^2} - \frac{Z_2^2}{A_2^2} \right)$$

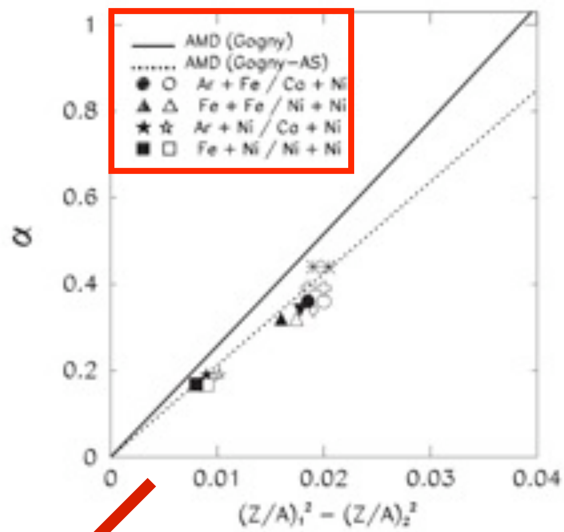
$$E/A = \ldots + C_{sym}(\rho/\rho_0) \times \frac{(N-Z)^2}{A^2} + \ldots$$

Texa A&M Un.

Fe+Fe/Ni+Ni, Ar+Fe/Ca+Ni Ar+Ni/Ca+Ni Fe+Ni/Ni+Ni

E/A=25-53 MeV





Comparisons to dynamical models →Density dependence of the symmetry energy

Isoscaling and $E_{sym}(\rho)$: "Yes, but…"

Isoscaling and $E_{sym}(\rho)$: “Yes, but…”

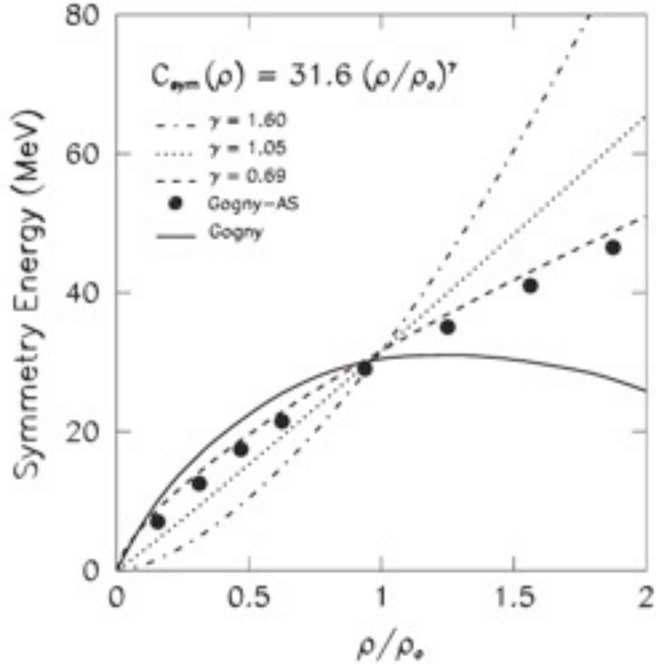
Fe+Fe/Ni+Ni, Ar+Fe+Ca+Ni
Ar+Ni/Ca+Ni Fe+Ni/Ni+Ni

E/A=25-53 MeV

$$C_{sym}(\rho) = 31.6 \cdot \left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Comparison to model simulations $\gamma \approx 0.7$

Faust data @ TAMU



D. Shetty et al. PRC76, 024606

Isoscaling and $E_{sym}(\rho)$: "Yes, but…"

Fe+Fe/Ni+Ni, Ar+Fe+Ca+Ni
Ar+Ni/Ca+Ni Fe+Ni/Ni+Ni

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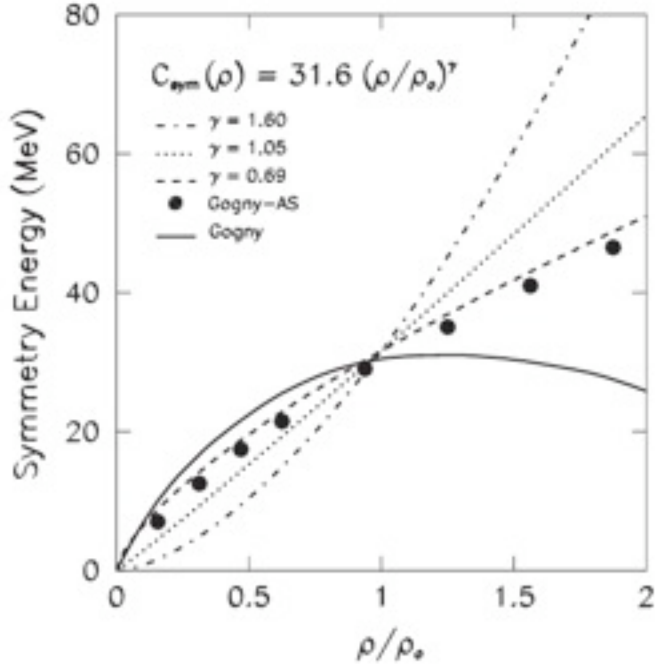
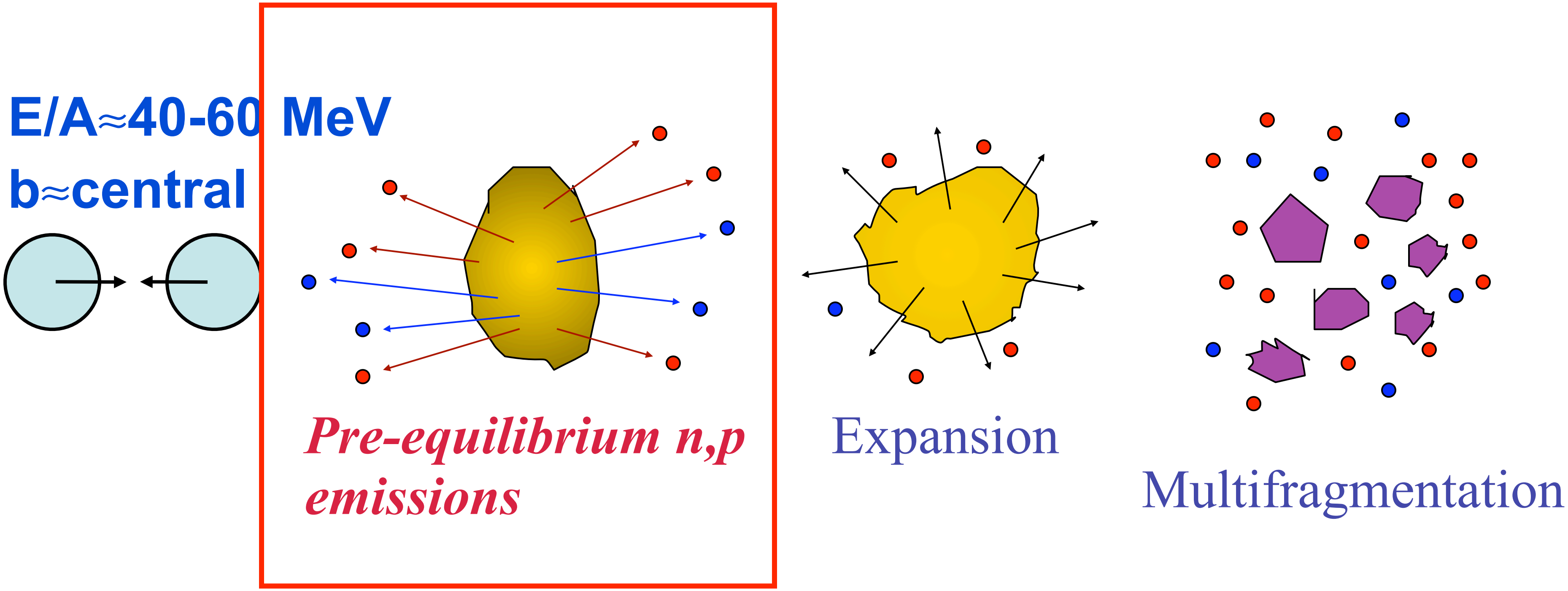


D. Shetty et al. PRC76, 024606

… but with caution:

- Very model-dependent analyses (fragment formation)
- Secondary decays may distort sensitivity to $E_{sym}(\rho)$

$E_{sym}(\rho)$ & pre-equilibrium nucleons



1. **Neutron/proton ratios $R_{n/p}$ and double ratios $DR_{n/p}$**
2. p-p, n-n and n-p correlation functions (HBT)

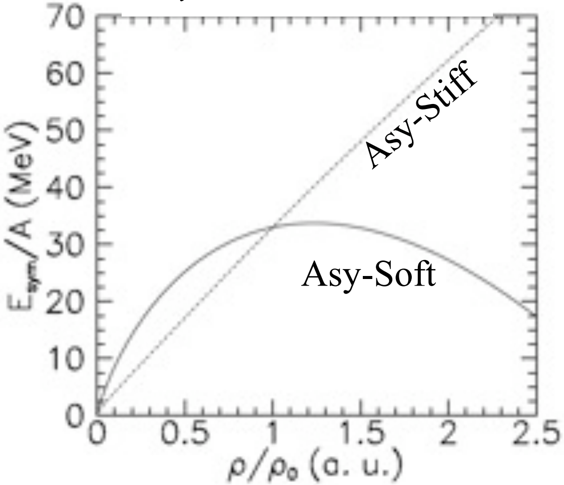
Pre-equilibrium n/p ratios: **R(n/p)**

Simulations:
Quantum molecular dynamics

112Sn+112Sn vs 124Sn+112Sn
E/A=50 MeV

$$E_{sym}(\rho) = \frac{Cs,k}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{Cs,p}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

E_{sym} vs (ρ/ρ_0)



Neutrons and protons evolve under the effect of

1. Nuclear&Coulomb forces
2. NN collisions
3. Density-dependent mean-field

Pre-equilibrium n/p ratios: **R(n/p)**

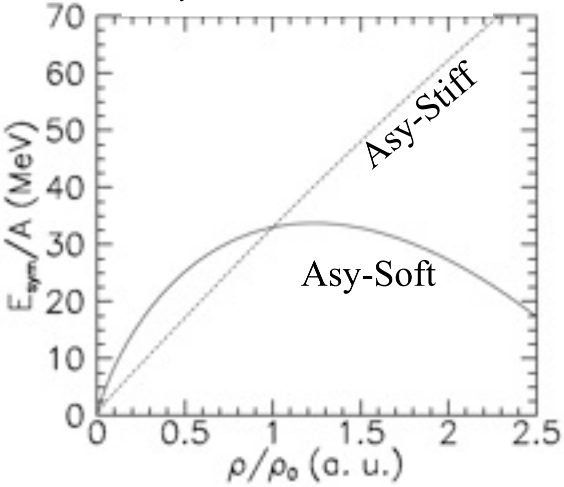
Simulations:
Quantum molecular dynamics

112Sn+112Sn vs 124Sn+112Sn
E/A=50 MeV

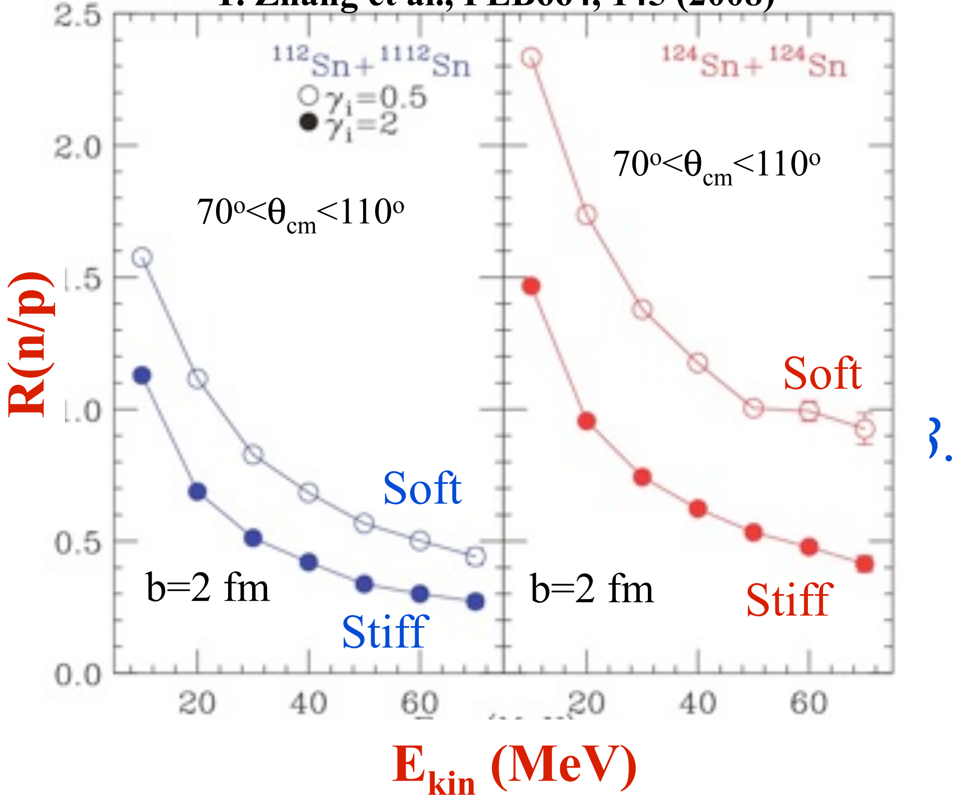
R(n/p) = Y(n)/Y(p) vs E_{kin}

$$E_{sym}(\rho) = \frac{Cs,k}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{Cs,p}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

E_{sym} vs (ρ/ρ_0)



Y. Zhang et al., PLB664, 145 (2008)



In experiments: caution with R(n/p) single ratios…

$$R(n/p) = \left[Y(n)/Y(p)\right]^{124+124}$$

Single n/p ratio may be affected by secondary non-E_{sym} effects (neutron detection efficiency problems, Coulomb repulsion, secondary decays…)

In experiments: caution with R(n/p) single ratios…

$$R(n/p) = \left[Y(n)/Y(p)\right]^{124+124}$$

Single n/p ratio may be affected by secondary non-E_{sym} effects (neutron detection efficiency problems, Coulomb repulsion, secondary decays…)

Double ratios: reduce secondary effects and **enhance effects of symmetry energy**

^{124}Sn+^{124}Sn

^{112}Sn+^{112}Sn

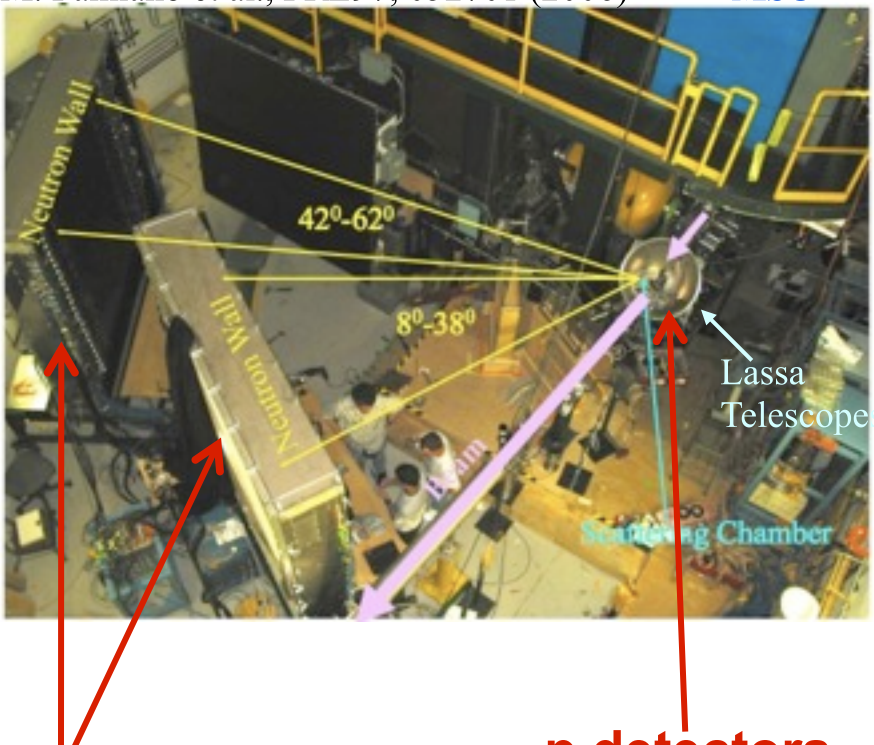
$$DR(n/p) = \frac{\left[Y(n)/Y(p)\right]^{124+124}}{\left[Y(n)/Y(p)\right]^{112+112}}$$

Neutron/proton experiments

$^{112}Sn+^{112}Sn$ & $^{124}Sn+^{124}Sn$ E/A=50 MeV

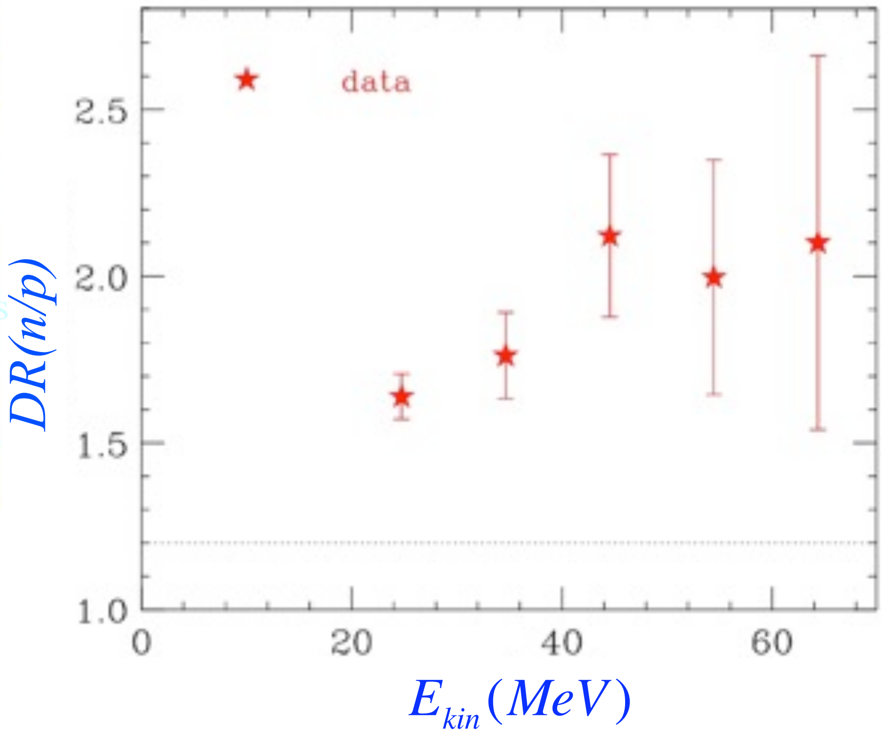
M. Famiano et al., PRL97, 052701 (2006) MSU

$$DR(n/p) = \frac{[Y(n)/Y(p)]^{124+124}}{[Y(n)/Y(p)]^{112+112}}$$



n detectors

p detectors



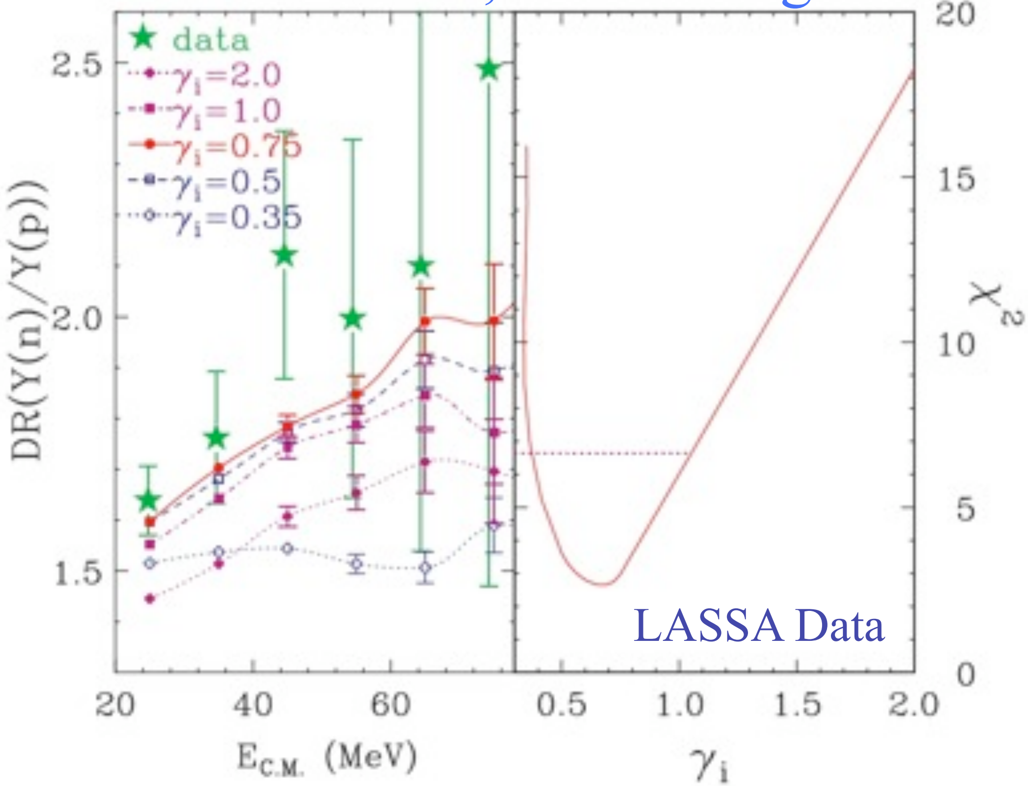
DR(n/p) deviate significantly from trivial value ≈ 1.2

Comparisons to model ==> $E_{sym}(\rho)$

Probing $E_{sym}(\rho)$ with n/p emissions

$^{112}Sn+^{112}Sn$ vs $^{124}Sn+^{124}Sn$ E/A=50 MeV

M. Famiano, M.B. Tsang



M.B. Tsang et al., PRL102, 122701 (2009)

Probing $E_{sym}(\rho)$ with n/p emissions

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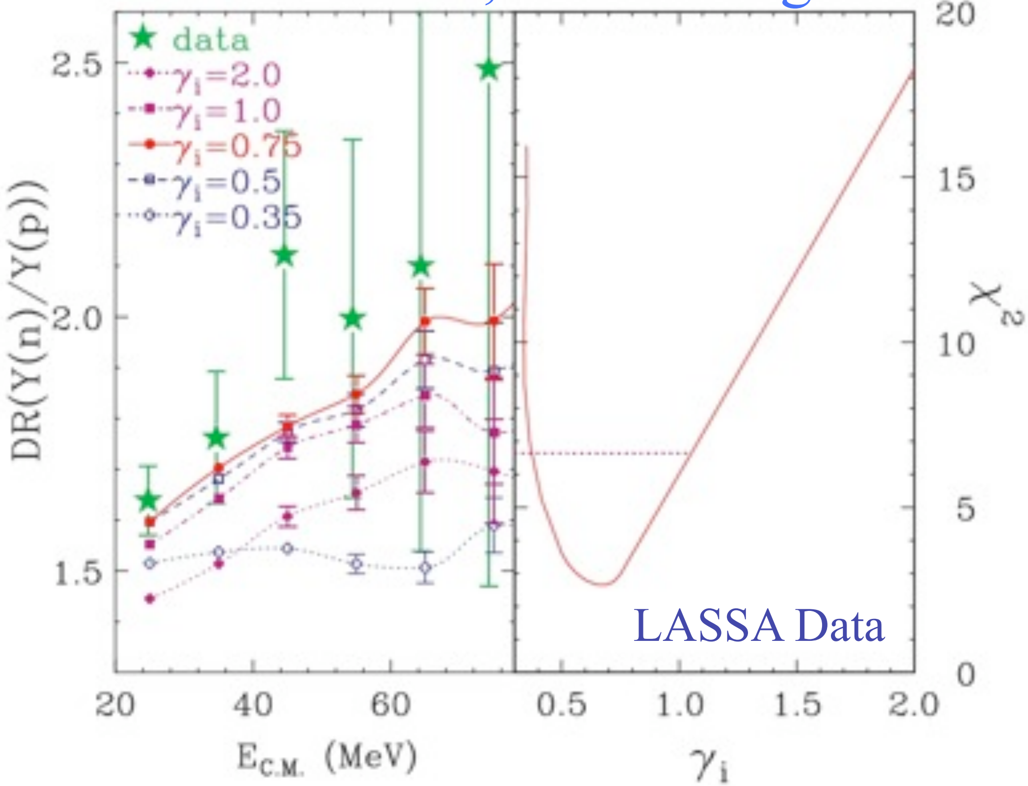


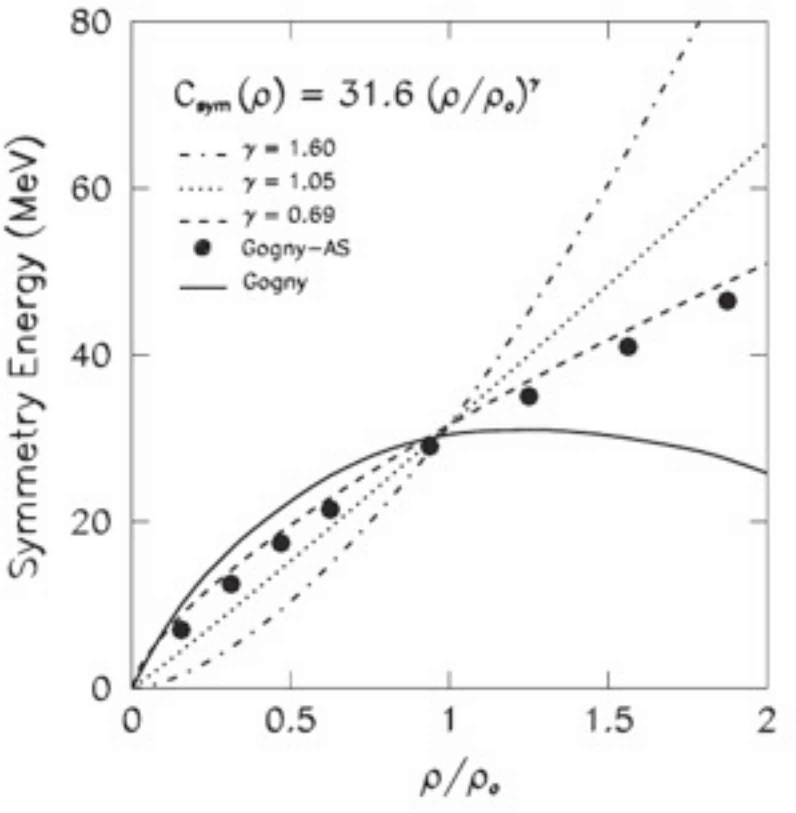
Comparisons to ImQMD

$$E_{sym}(\rho) = \frac{Cs,k}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{Cs,p}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

χ^2 analysis provides $\gamma \approx 0.7$

M.B. Tsang et al., PRL102, 122701 (2009)

$E_{sym}(\rho)$ from central collisions



Multifragmentation and isoscaling (?)

Faust @ TAMU

D. Shettty et al., PRC76, 024606

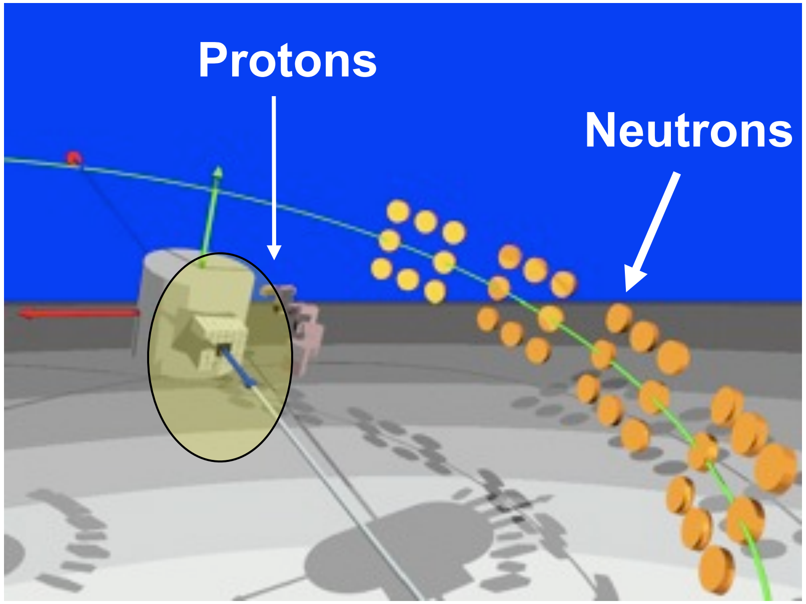
Pre-equilibrium n/p emissions

Lassa & n-Wall @ NSCL-MSU

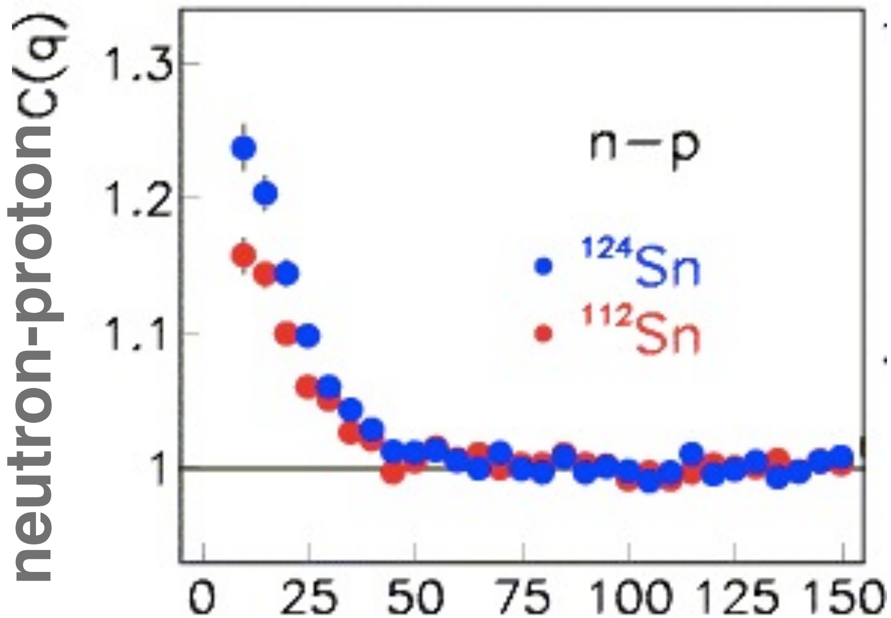
M.B. Tsang et al., PRL102, 122701 (2009)

γ ≈ 0.7 (± 0.2) excluding very soft and very stiff

Neutron-proton correlation functions
Perspective for the future



Ghetti *et al*, PRC 69 (2004) 031605

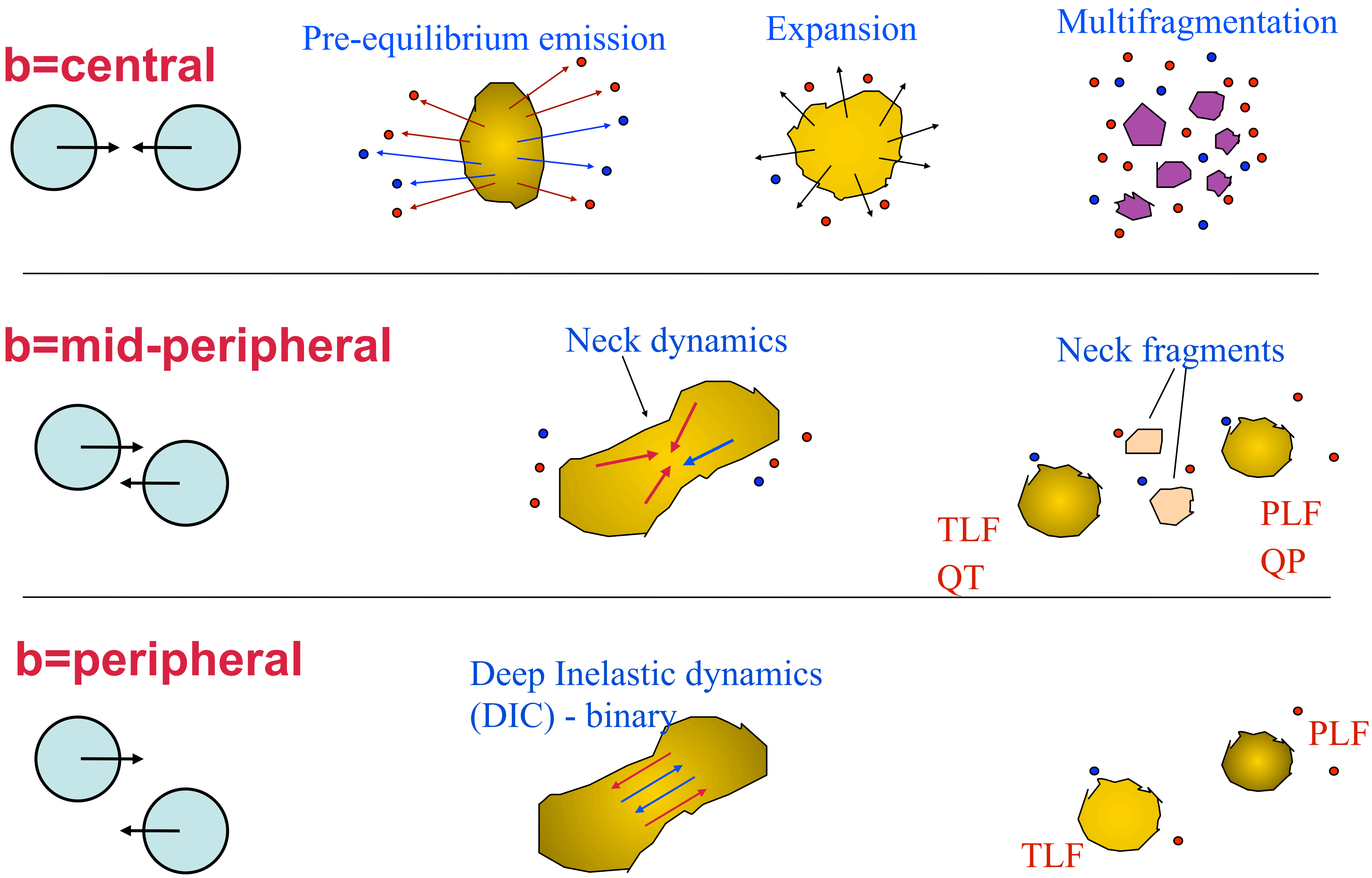


Emission chronology sensitive to Asy-EOS

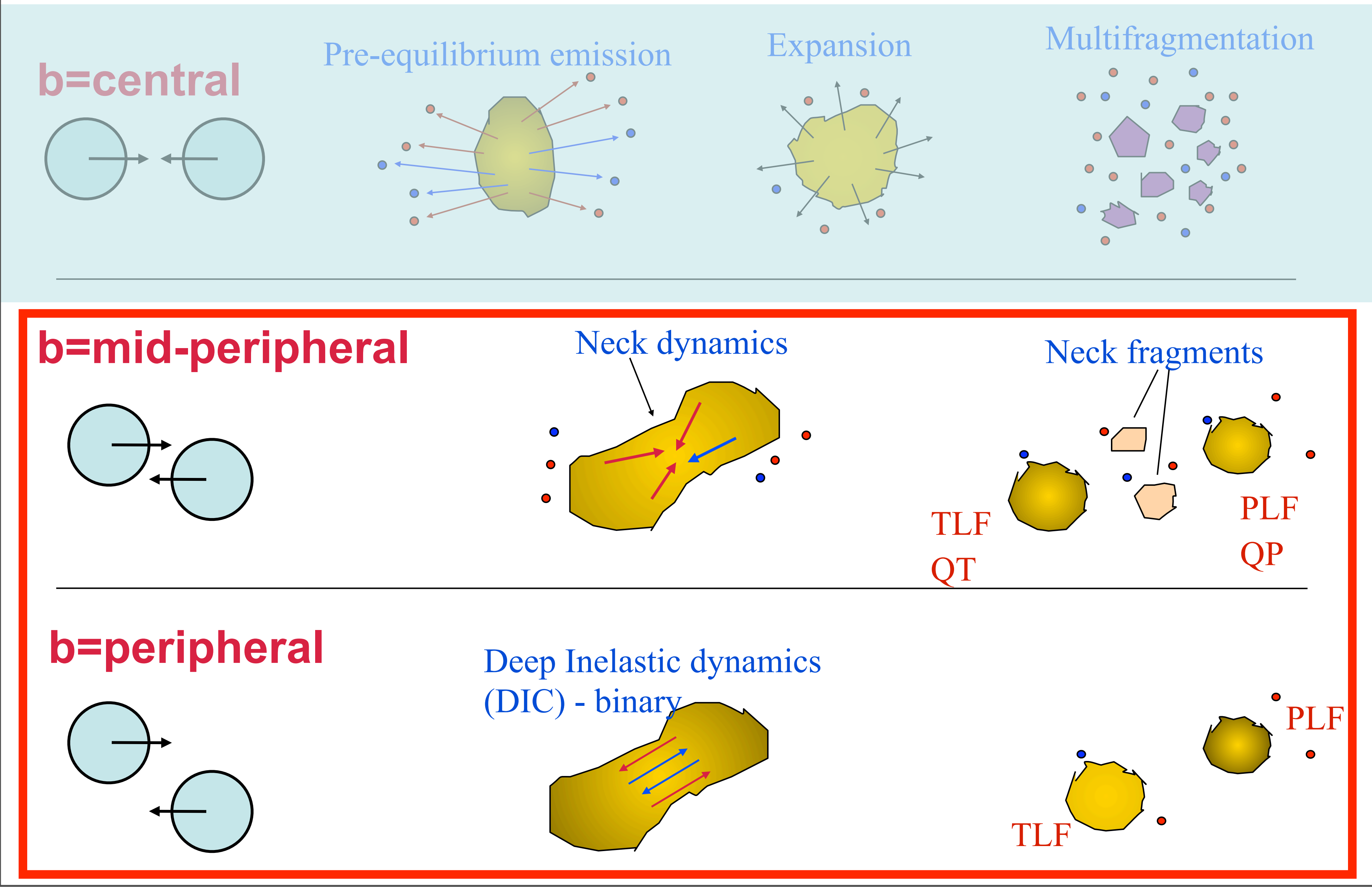
Important task or the future… (LNS, MSU)

Peripheral and Mid-peripheral reactions

HIC at intermediate energies: $E_{sym}(\rho)$ at $\rho<\rho_0$



HIC at intermediate energies: $E_{sym}(\rho)$ at $\rho<\rho_0$

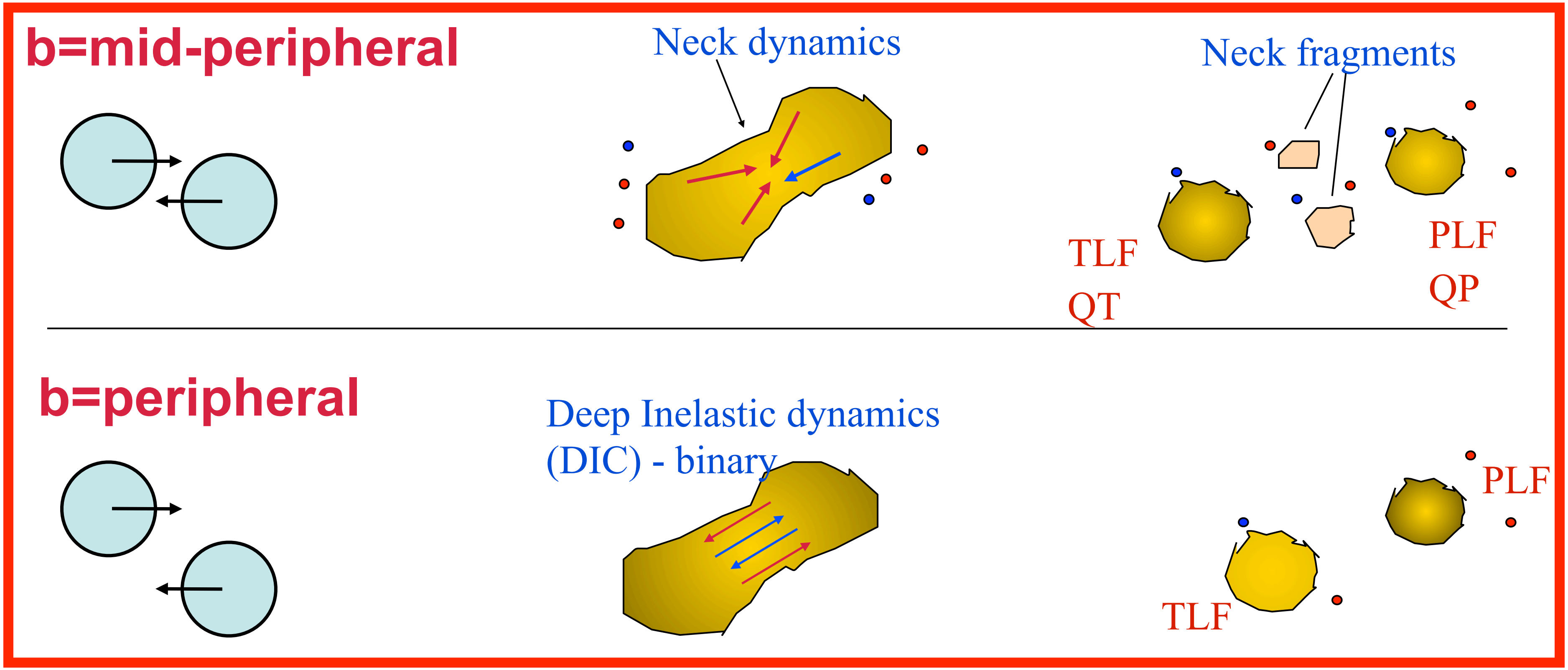


HIC at intermediate energies: $E_{sym}(\rho)$ at $\rho<\rho_0$

Vocabulary

PLF = Projectile-like Fragments QP = Quasi-Projectile

TLF = Target-like Fragments QT = Quasi-Target



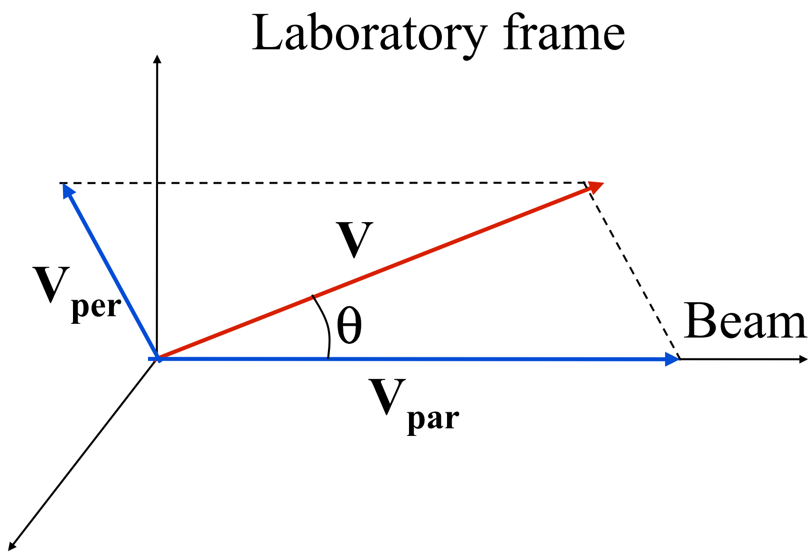
… more vocabulary

Parallel and transverse velocity

$$V_{par} = V_{//} = V_{long} = V \cdot \cos(\theta)$$

$$V_{per} = V_{\perp} = V \cdot \sin(\theta)$$

$$\beta = \frac{V}{c} \qquad \beta_{//} = \beta \cdot \cos(\theta) \qquad \beta_{\perp} = \beta \cdot \cos(\theta)$$



Rapidity, transverse momentum

$$y = \frac{1}{2}\frac{\log\left(1+\beta_{//}\right)}{\log\left(1-\beta_{//}\right)}$$

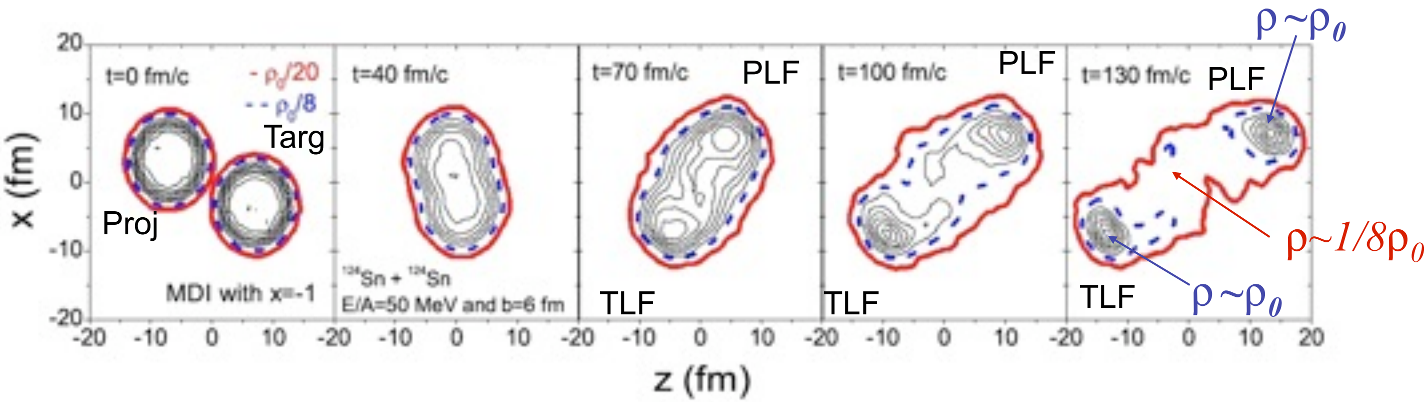
$$P_{\perp} = P \cdot \sin(\theta)$$

Mid-peripheral collisions

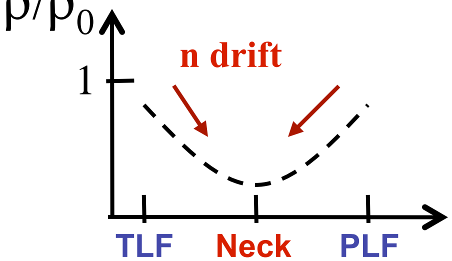
1. Isospin drift
2. Isospin diffusion

Both sensitive to the density dependence of the symmetry energy $E_{sym}(\rho)$

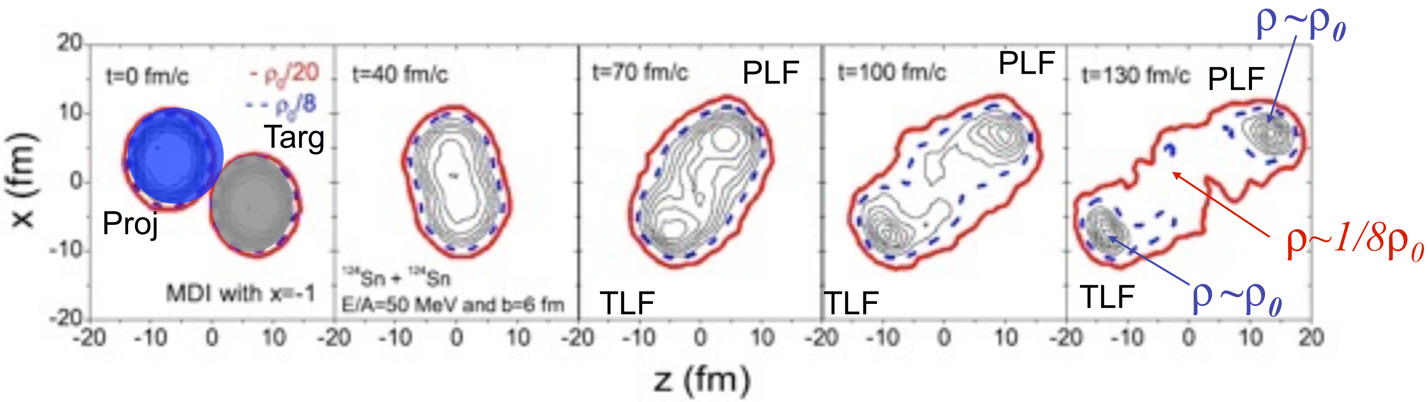
Isospin drift & diffusion



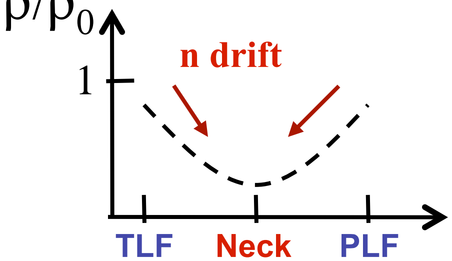
Isospin drift



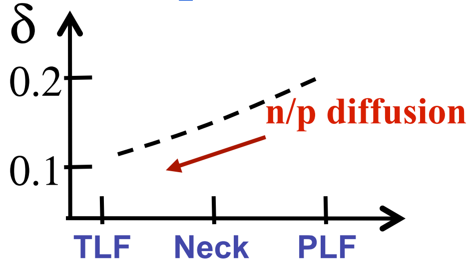
Isospin drift & diffusion



Isospin drift

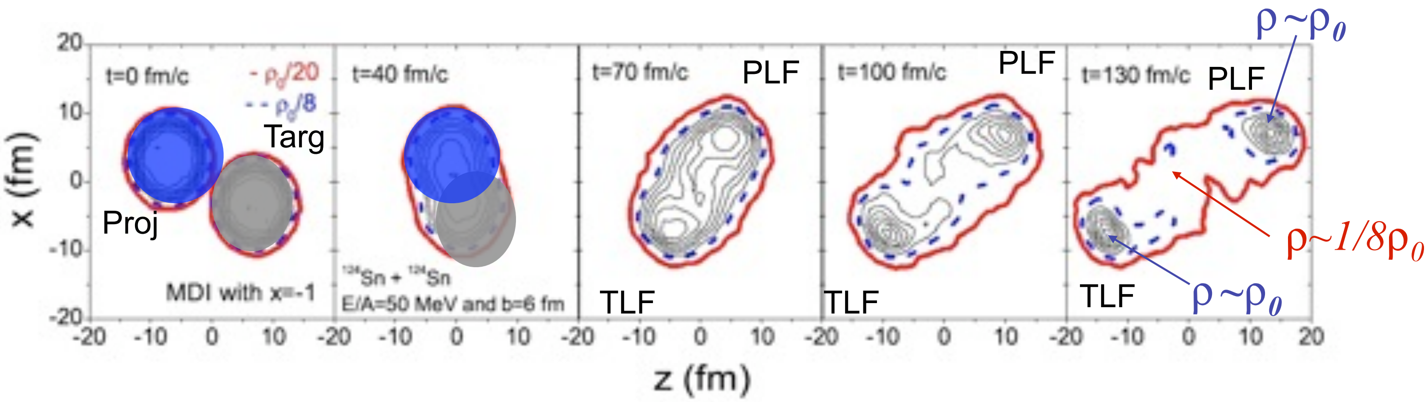


Isospin diffusion

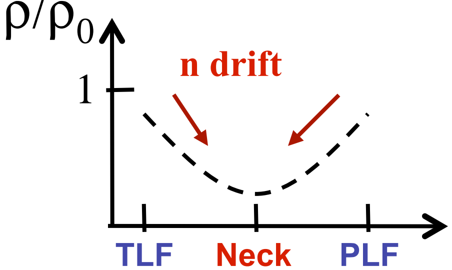


$$\delta = \frac{N - Z}{N + Z}$$

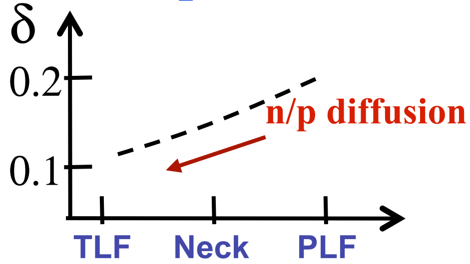
Isospin drift & diffusion



Isospin drift



Isospin diffusion



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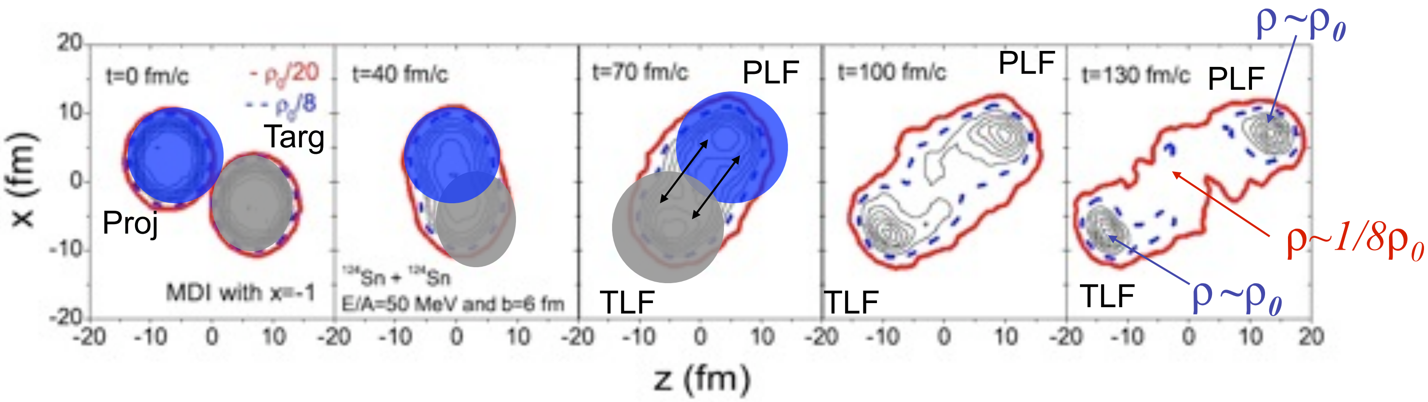
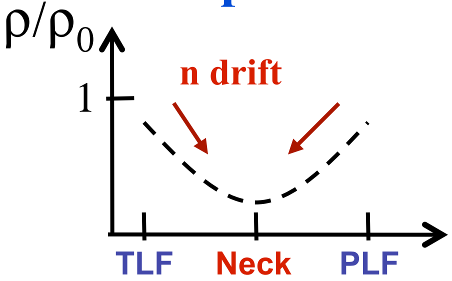
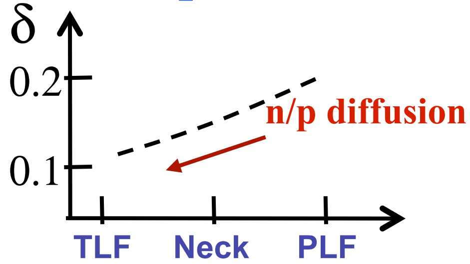
Isospin drift & diffusion



Isospin drift



Isospin diffusion



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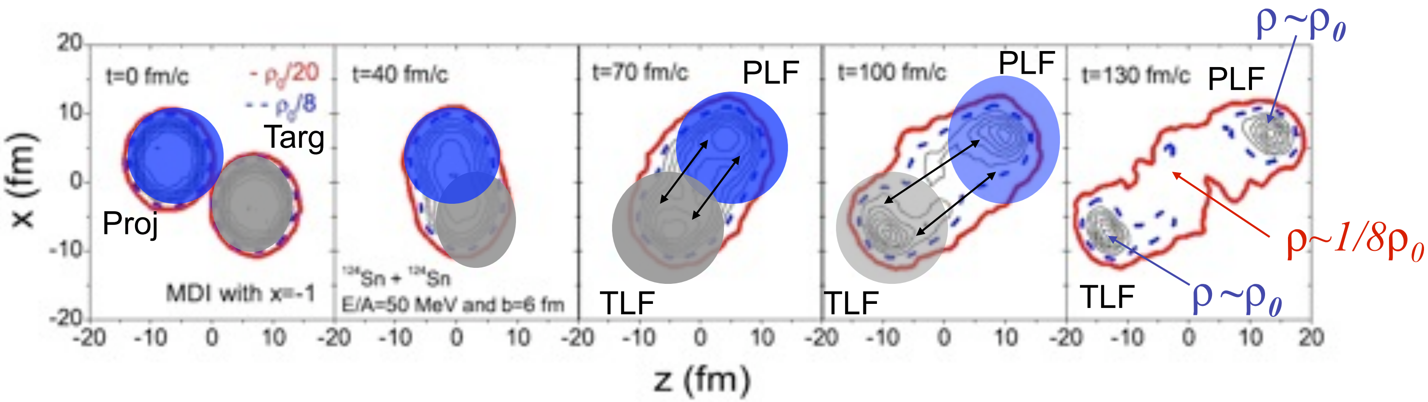
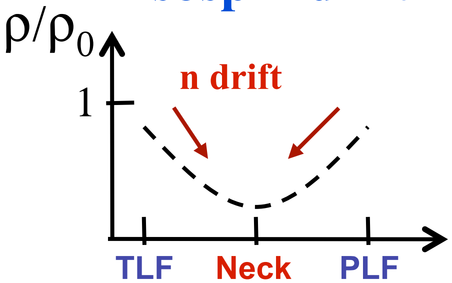
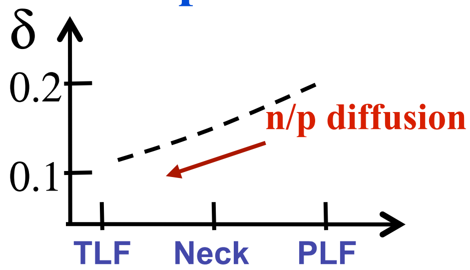
Isospin drift & diffusion



Isospin drift



Isospin diffusion



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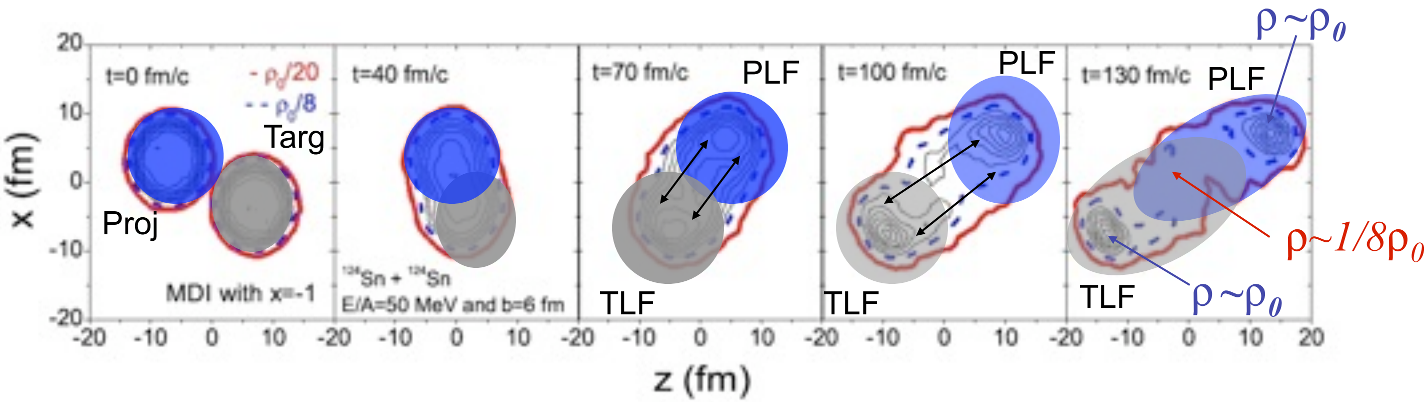
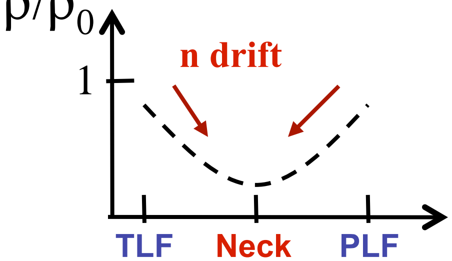
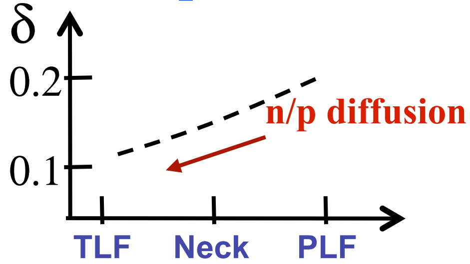
Isospin drift & diffusion



Isospin drift



Isospin diffusion



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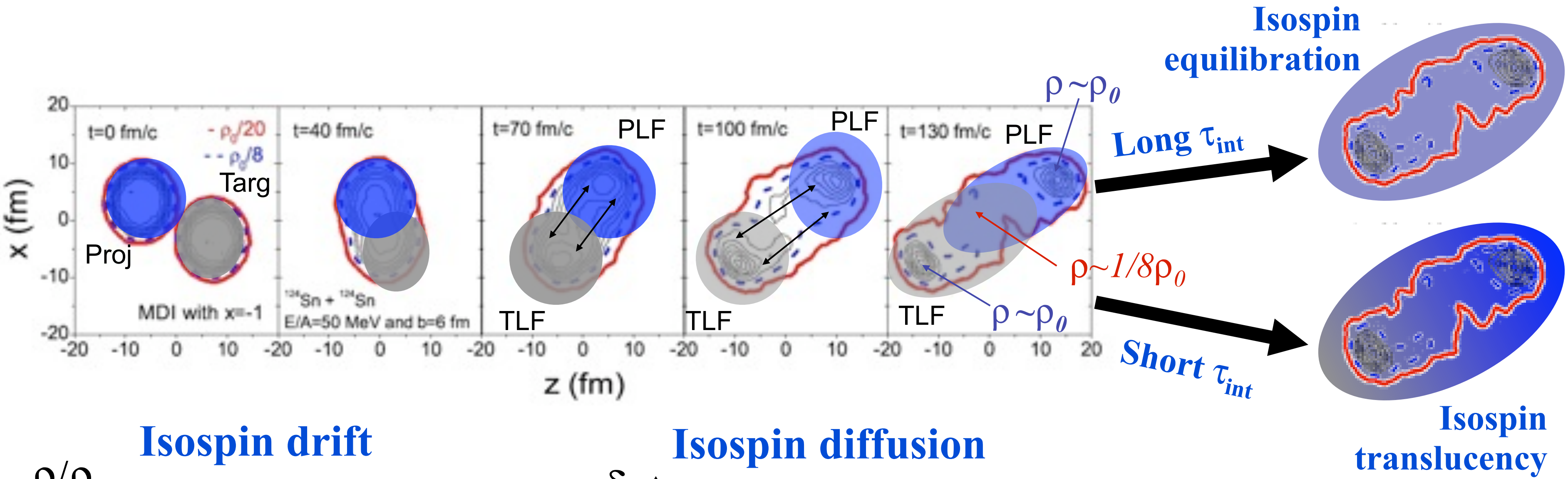
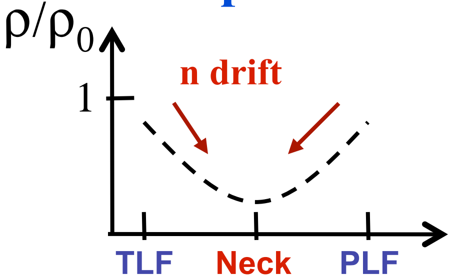
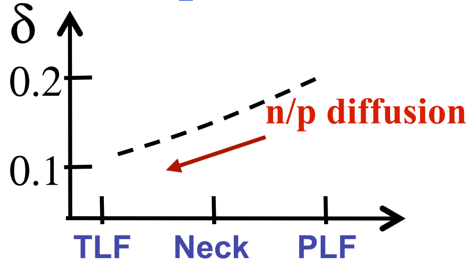
Isospin drift & diffusion



Isospin drift

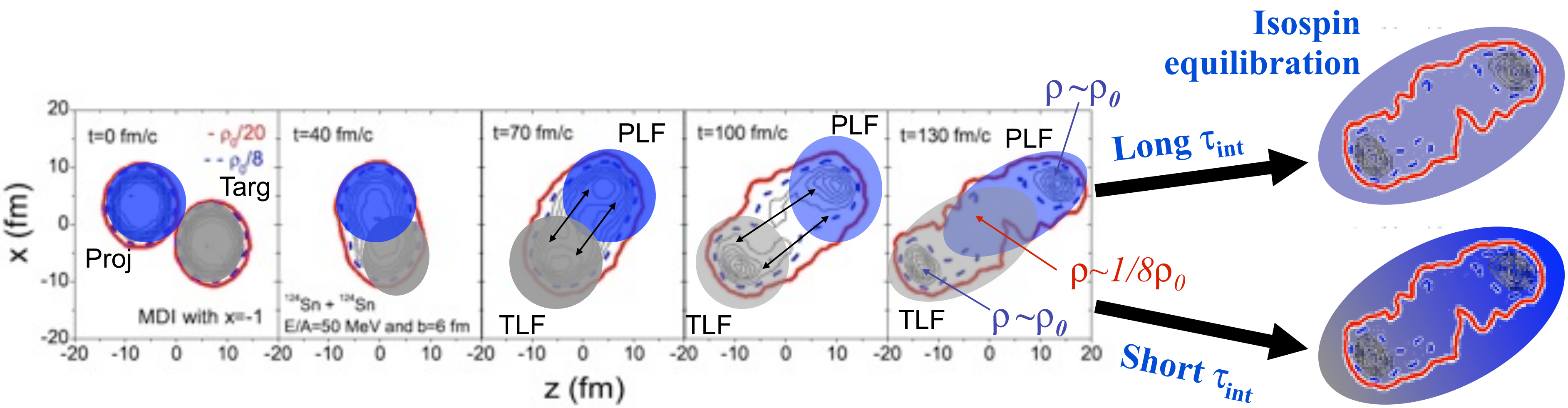


Isospin diffusion

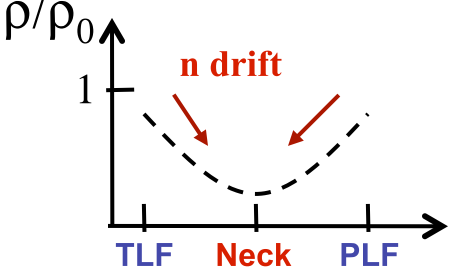


$$\delta = \frac{N - Z}{N + Z}$$

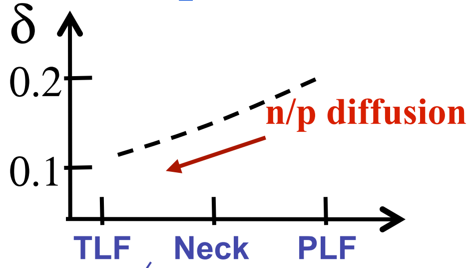
Isospin drift & diffusion



Isospin drift



Isospin diffusion

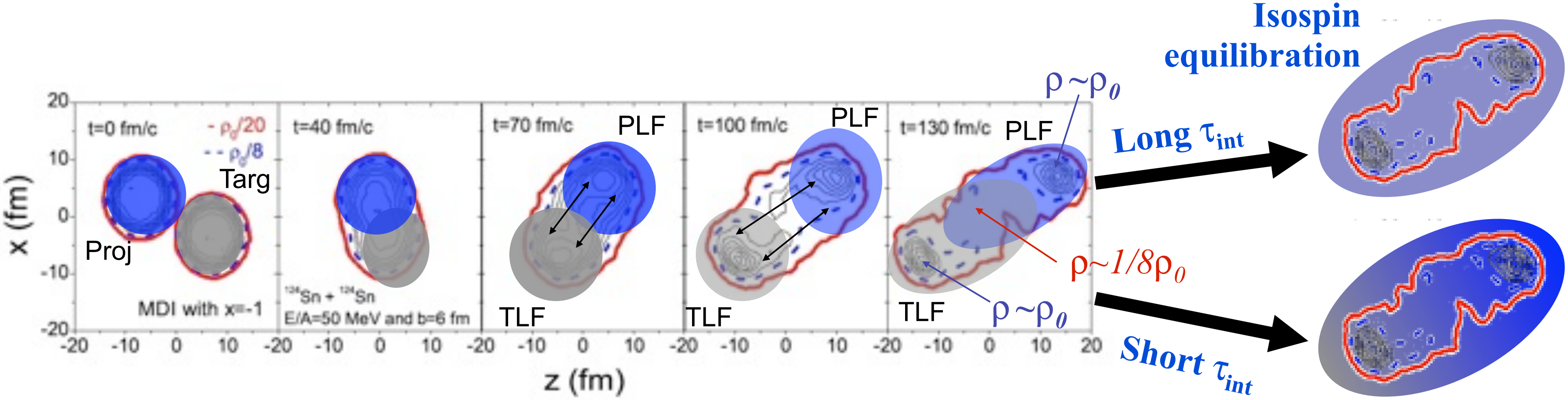


$$\delta = \frac{N-Z}{N+Z}$$

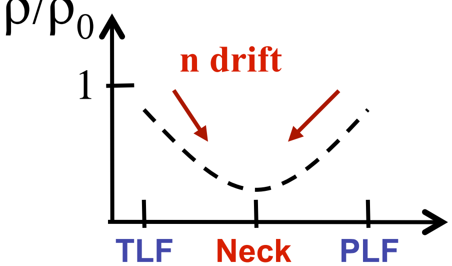
$$\boldsymbol{j}_n - \boldsymbol{j}_p = \left(D_n^{\rho} - D_p^{\rho}\right)\nabla\rho - \left(D_n^{\delta} - D_p^{\delta}\right)\nabla\delta$$

$$\propto \frac{\partial E_{sym}}{\partial\rho} \qquad \propto E_{sym}$$

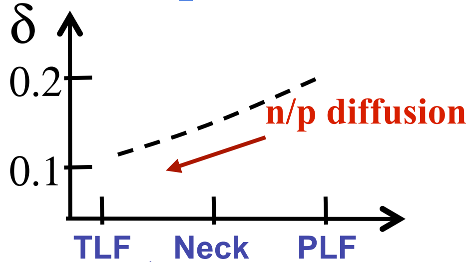
Isospin drift & diffusion



Isospin drift



Isospin diffusion



$$\delta = \frac{N-Z}{N+Z}$$

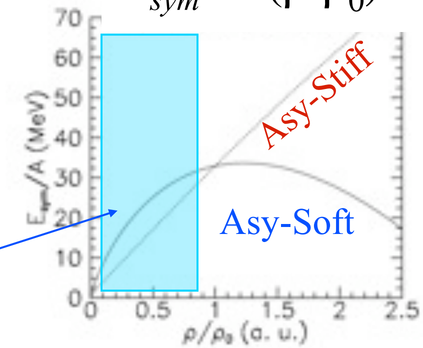
$$\boldsymbol{j}_n - \boldsymbol{j}_p = \left(D_n^{\rho} - D_p^{\rho}\right)\nabla\rho - \left(D_n^{\delta} - D_p^{\delta}\right)\nabla\delta$$

$$\propto \frac{\partial \boldsymbol{E}_{sym}}{\partial \rho} \qquad \propto \boldsymbol{E}_{sym}$$

Stiff > Soft   Soft > Stiff

Low ρ/ρ_0

E_{sym} vs (ρ/ρ_0)



Refs about isospin drift and neck emission

Piantelli P et al., 2006 *Phys. Rev.* **C74** 034609.

Piantelli P et al., 2007 *Phys. Rev.* **C76** 061601.

Milazzo P et al., 2005 *Nucl. Phys.* **A756** 39.

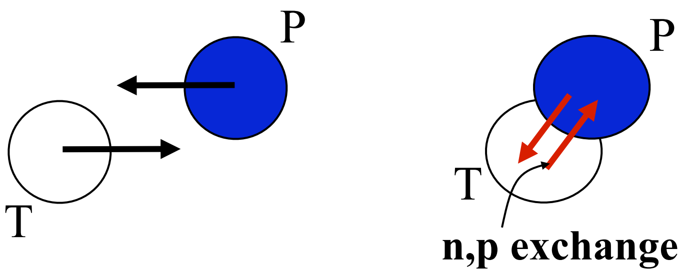
Theriault D et al., 2006 *Phys. Rev.* **C74** 051602.

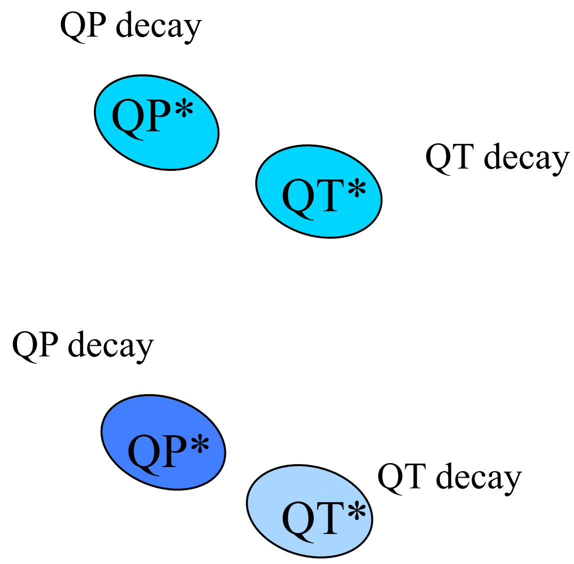
Hudan S et al., 2005 *Phys. Rev.* **C71** 054604.

De Filippo E et al.(Chimera Collab.), 2005 *Phys. Rev.* **C71** 044602
2005 *Phys. Rev.* **C71** 064604

How to probe the N/Z of the QP and QT?

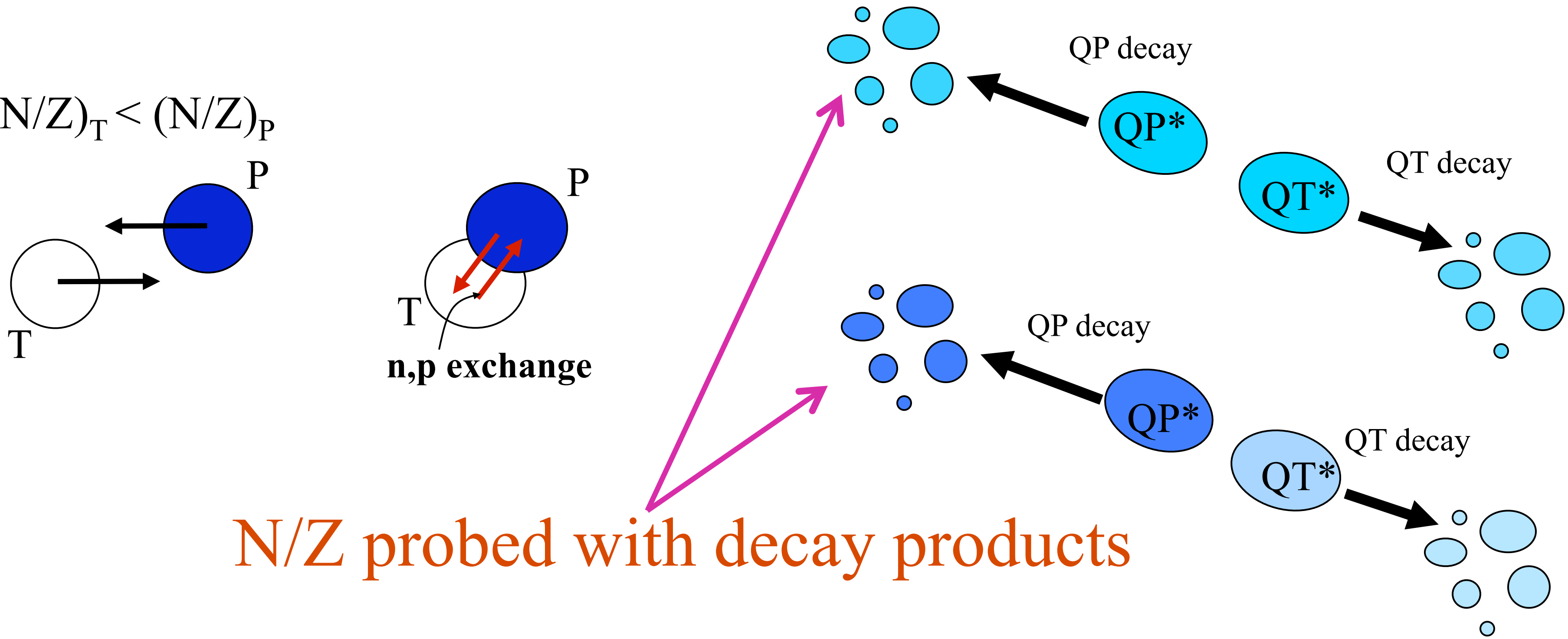
$(N/Z)_T < (N/Z)_P$





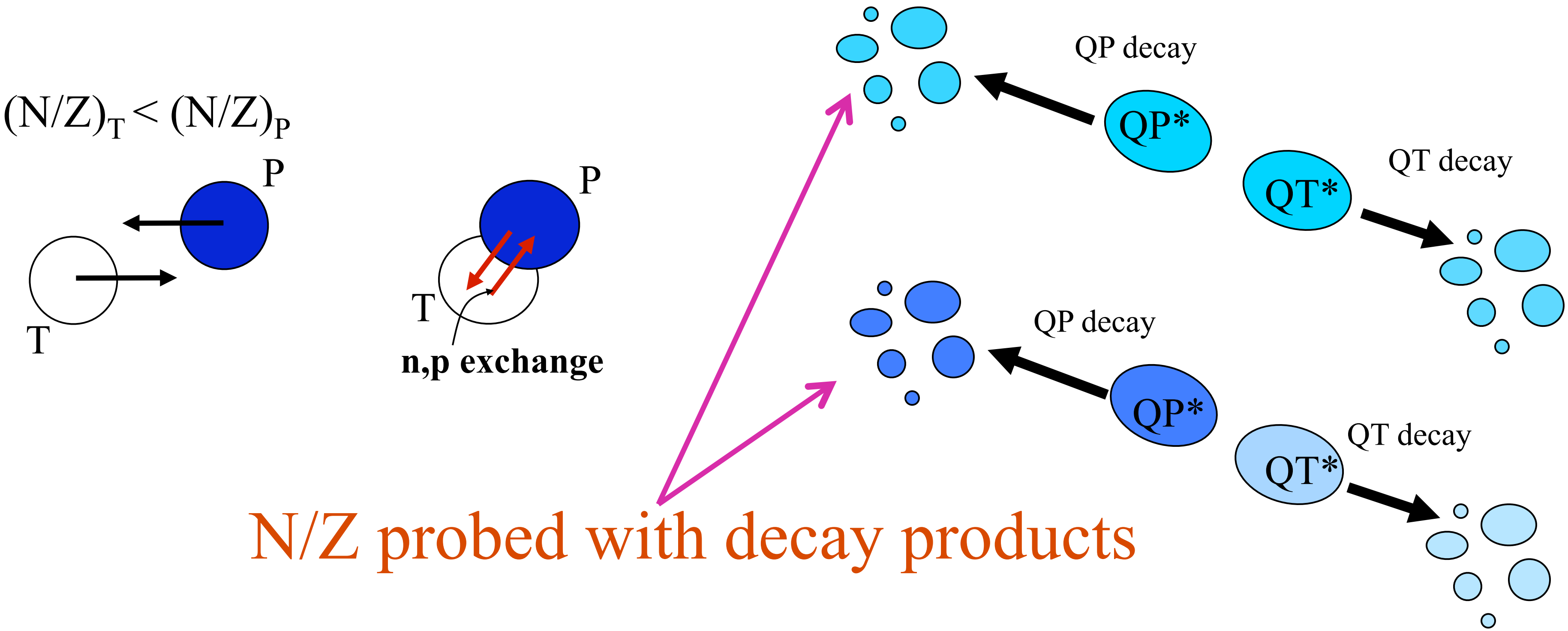
How to probe the N/Z of the QP and QT?

$(N/Z)_T < (N/Z)_P$



N/Z probed with decay products

How to probe the N/Z of the QP and QT?



Need to measure observables $X \propto \delta^* = (N-Z)/(N+Z)$

- **X = Reconstructed $(N/Z)_{QP^*}$** Indra @ GANIL
- **$X = Y(^7Li)/Y(^7Be)$** Lassa @ MSU

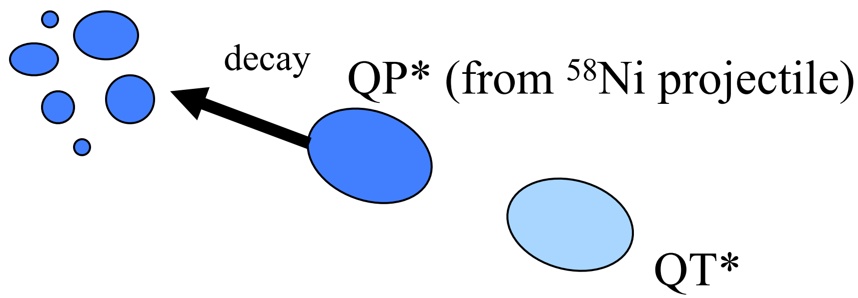
Isospin diffusion in Indra data

^{58}Ni+^{58}Ni	$(N/Z)_{Proj}=(N/Z)_{Targ} \approx 1.07$	No Diffusion
^{58}Ni+^{197}Au	$(N/Z)_{Proj}<(N/Z)_{Targ} \approx 1.5$	Diffusion Target→Proj
	E/A=52, 74 MeV	

Collect all fragments from QP* decay

$$\left(\frac{N}{Z}\right)_{QP*} \qquad V_{QP*}^{REC}$$

$$V_{rel} = V_{QP}^{rec} \times \frac{A_{tot}}{A_{target}}$$



Dissipated energy

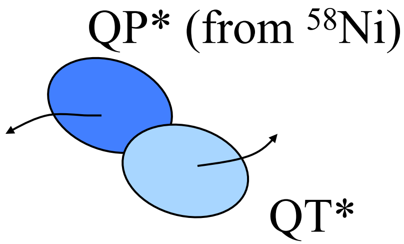
$$E_{\rm diss} = E_{\rm c.m.} - \frac{1}{2}\mu V_{\rm rel}^2$$

E. Galichet et al., PRC79, 064614 (2009)

Dissipated energy and contact time

E. Galichet et al., PRC79, 064614 (2009)

Large impact parameter b

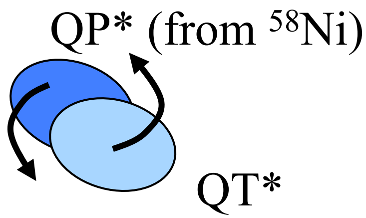


Weak friction

Short contact time τ_{int}

Small dissipation, E_{diss}

Small impact parameter b



Strong friction

Long contact time τ_{int}

Large dissipation, E_{diss}

E_{diss} ⟶

τ_{int} ⟶

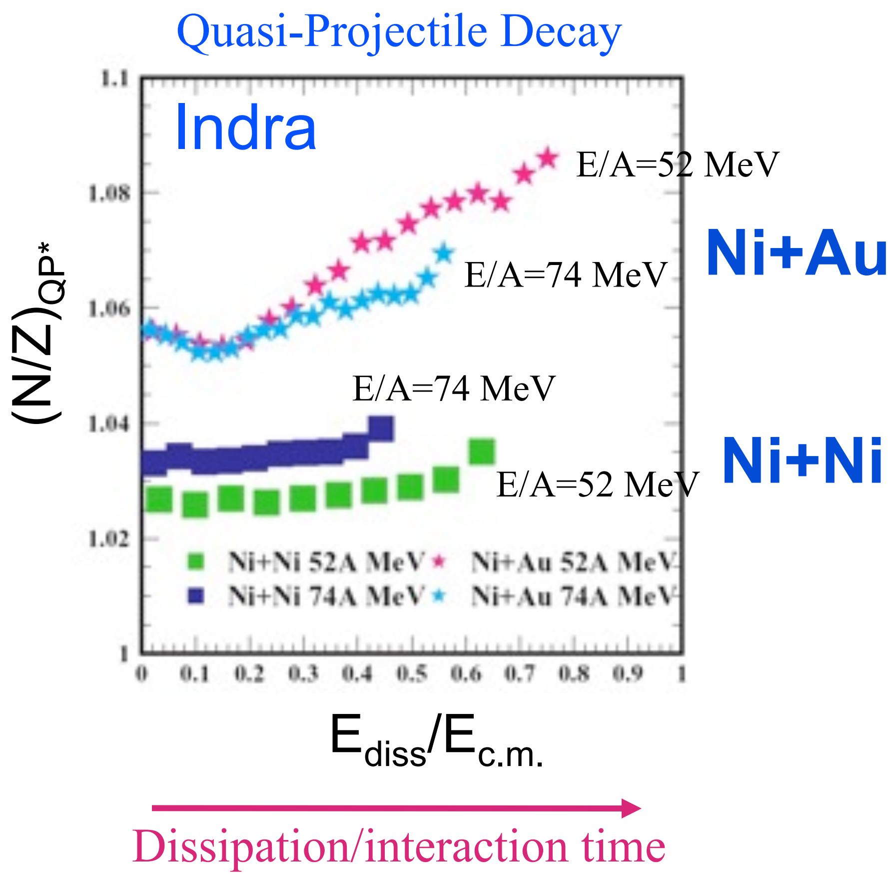
b ⟵

Isospin diffusion/equilibration

E. Galichet et al., PRC79, 064614 (2009)

$^{58}Ni+^{58}Ni$ E/A=52, 74 MeV

$^{58}Ni+^{197}Au$ E/A=52, 74 MeV



Isospin diffusion/equilibration

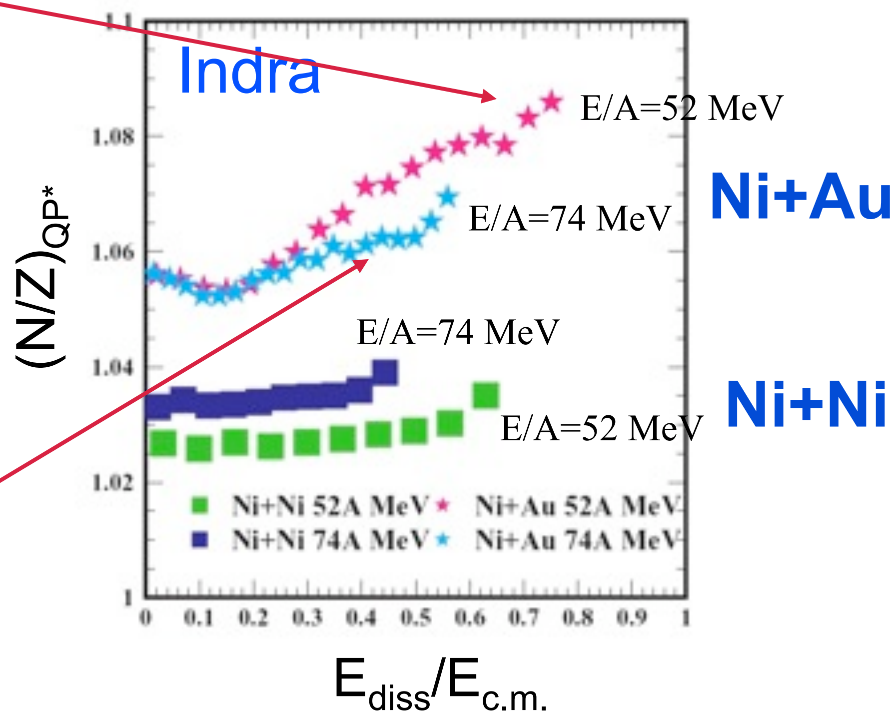
^{58}Ni+^{58}Ni E/A=52, 74 MeV

^{58}Ni+^{197}Au E/A=52, 74 MeV

E. Galichet et al., PRC79, 064614 (2009)

Long τ_{int}: Large E_{diss}
N/Z equilibration

Quasi-Projectile Decay

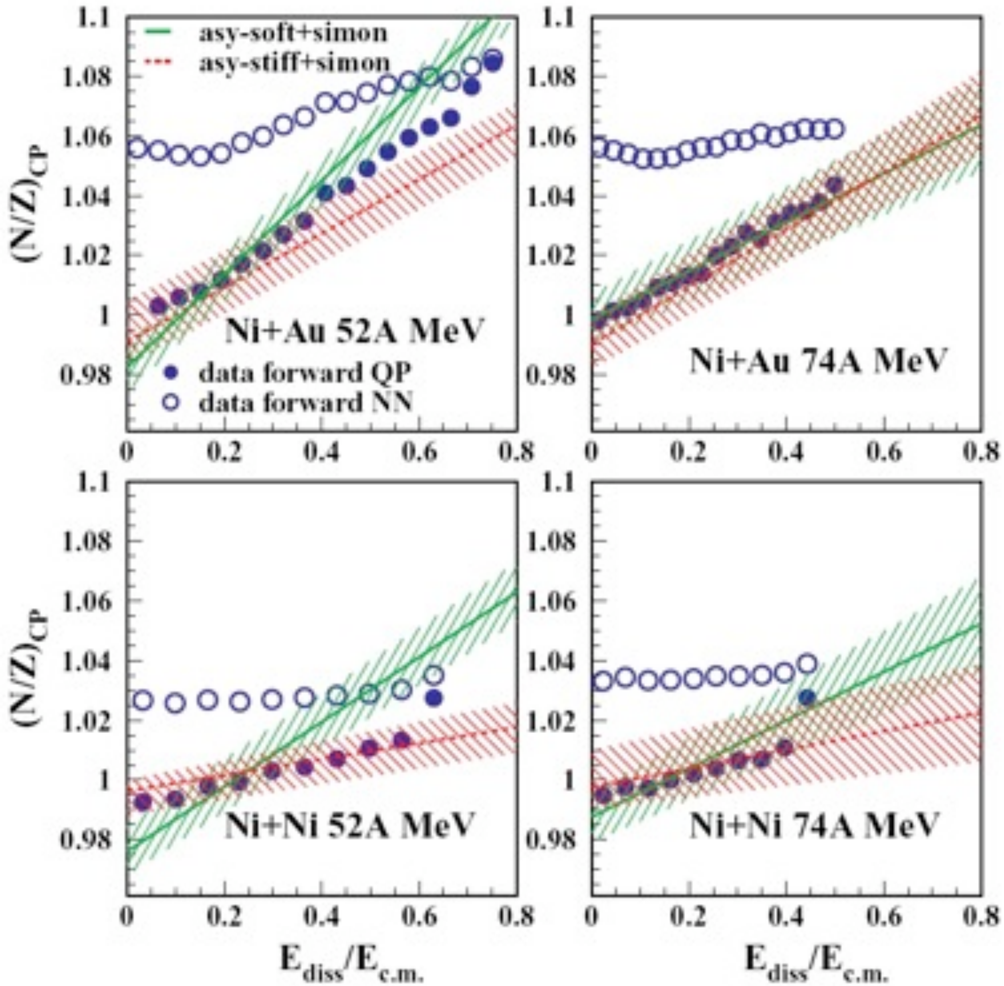


Short τ_{int}: Small E_{diss}
N/Z translucency

Dissipation/interaction time

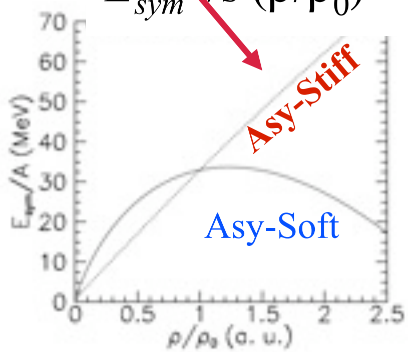
Probing $E_{sym}(\rho)$

Comparisons to SMF



Data closer to Asy-Stiff parameterization

E_{sym} vs (ρ/ρ_0)



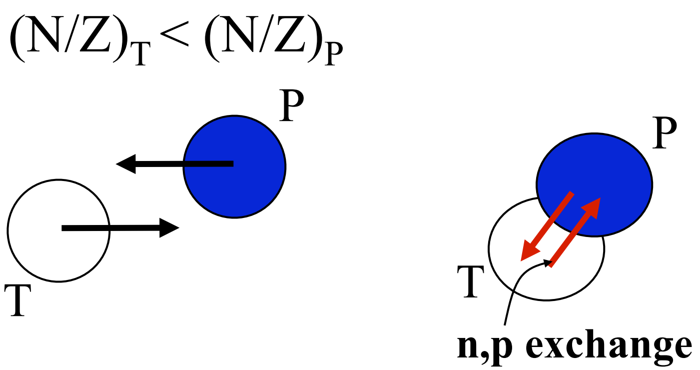
$$E_{sym}(\rho) \propto \left(\frac{\rho}{\rho_0}\right)^{\gamma} \qquad \gamma \approx 1$$

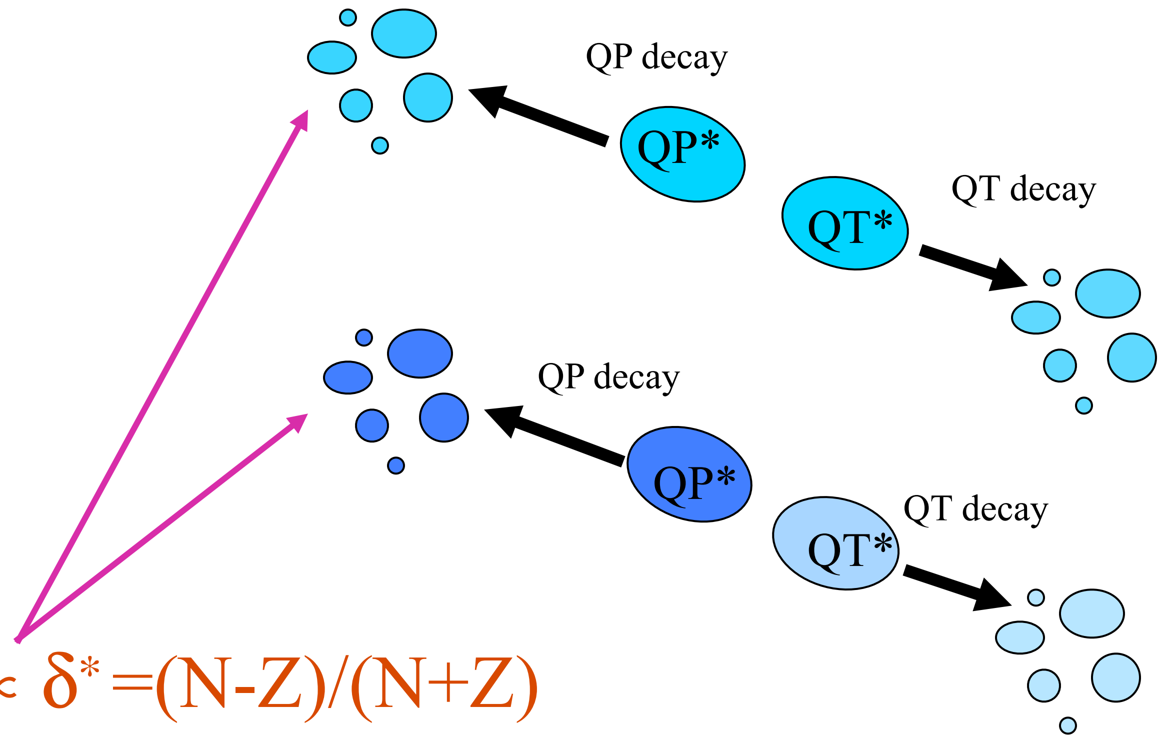
E. Galichet et al., PRC79, 064615 (2009)

Isospin diffusion in Sn+Sn collisions

$^{112}Sn+^{112}Sn$, $^{112}Sn+^{124}Sn$, $^{124}Sn+^{112}Sn$, $^{124}Sn+^{124}Sn$ E/A=50 MeV

N/Z=1.24 N/Z=1.36 N/Z=1.36 N/Z=1.48



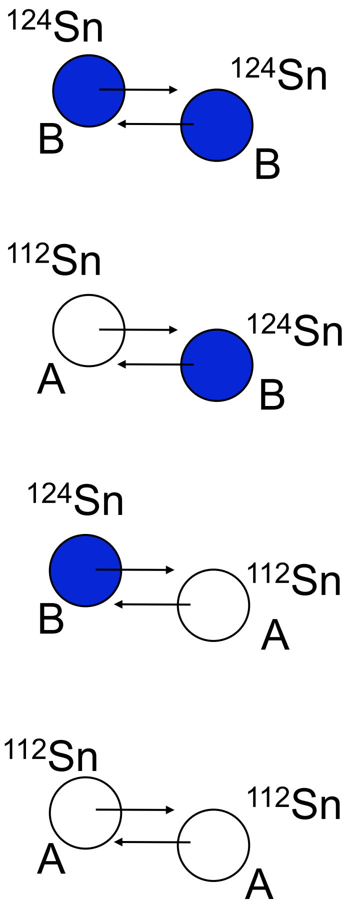


Observables $X \propto \delta^* = (N-Z)/(N+Z)$

$X = Y(^7Li)/Y(^7Be)$ Lassa @ MSU

Isospin imbalance ratios

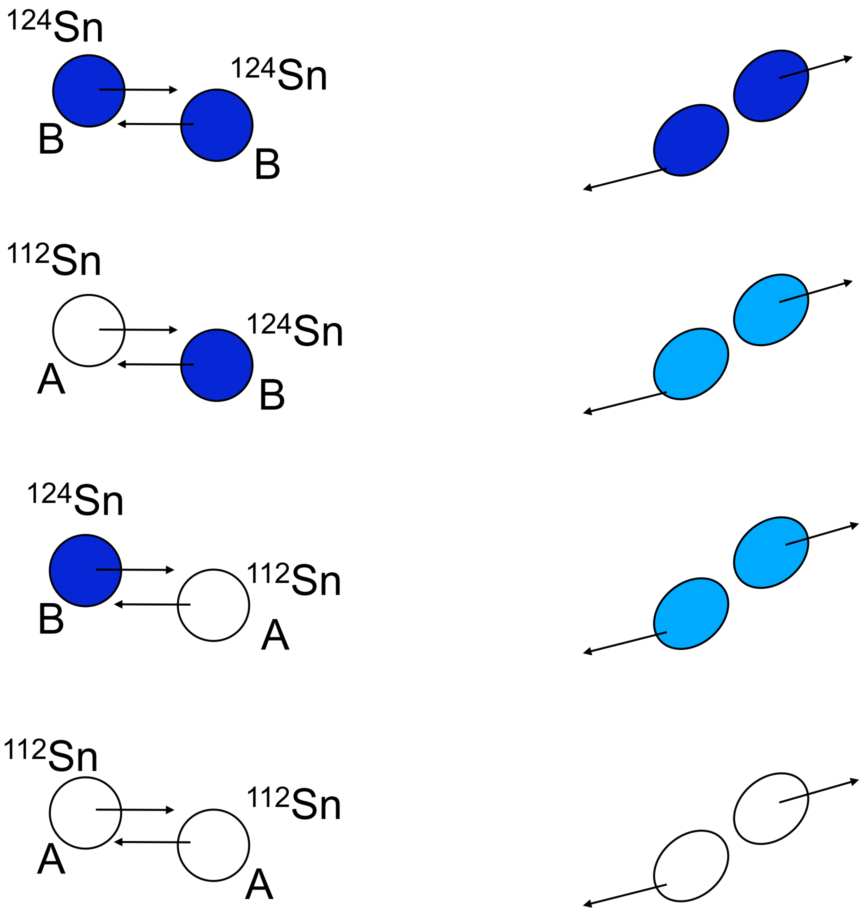
Before collision



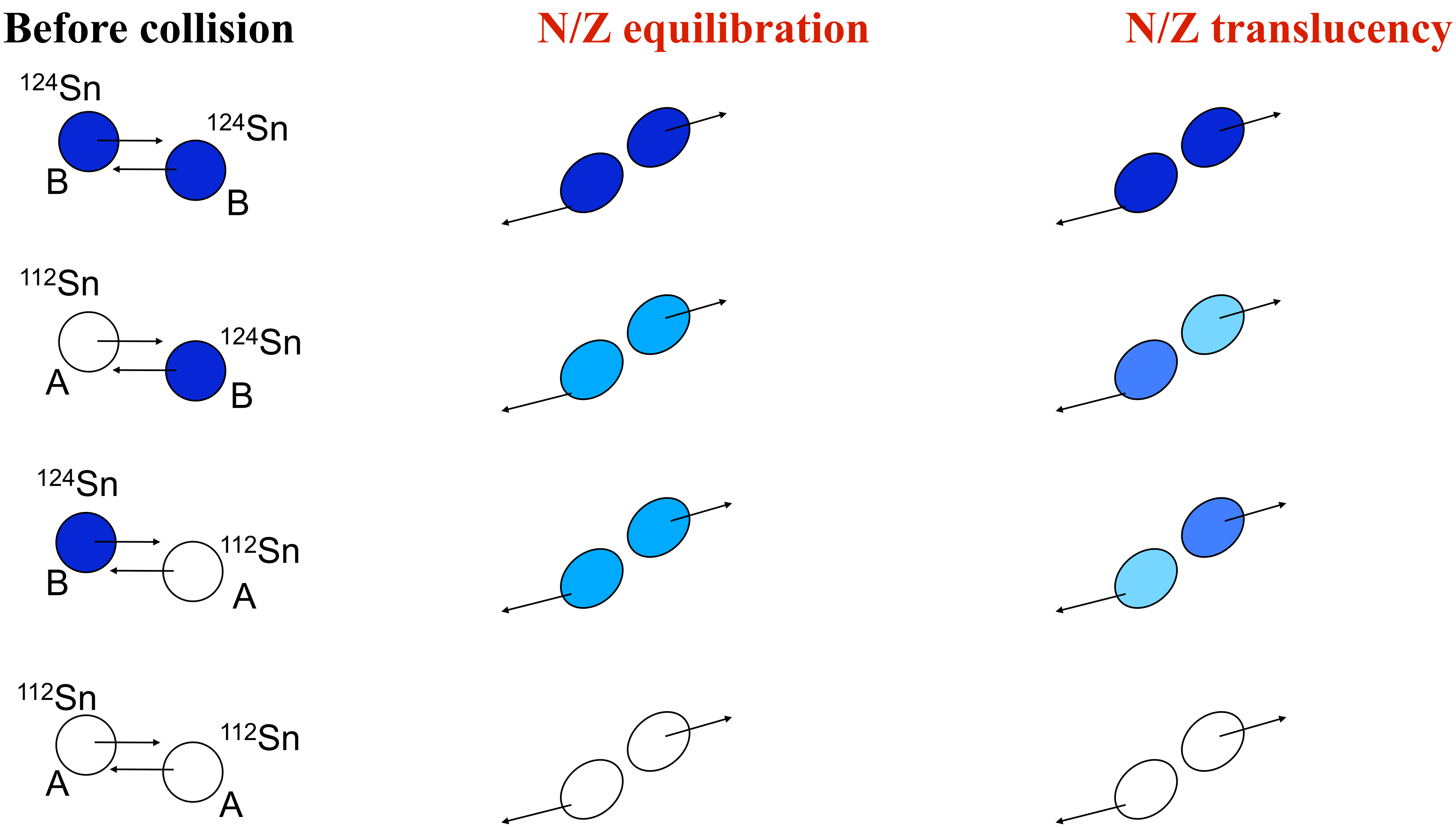
Isospin imbalance ratios

Before collision

N/Z equilibration



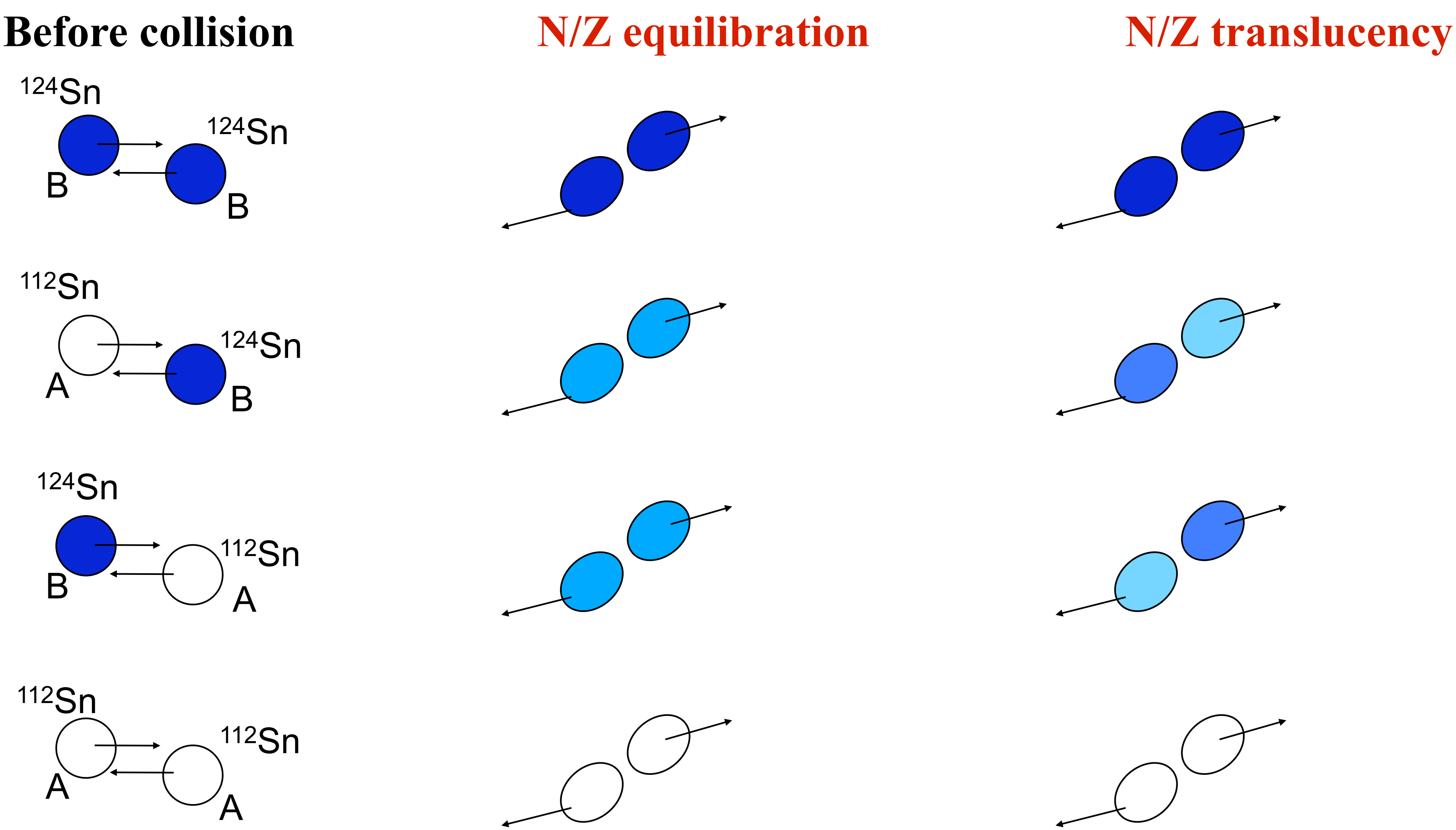
Isospin imbalance ratios



Isospin imbalance ratios

Measured observable in final state

Ex.: $x = Y(^7Li)/Y(^7Be)$



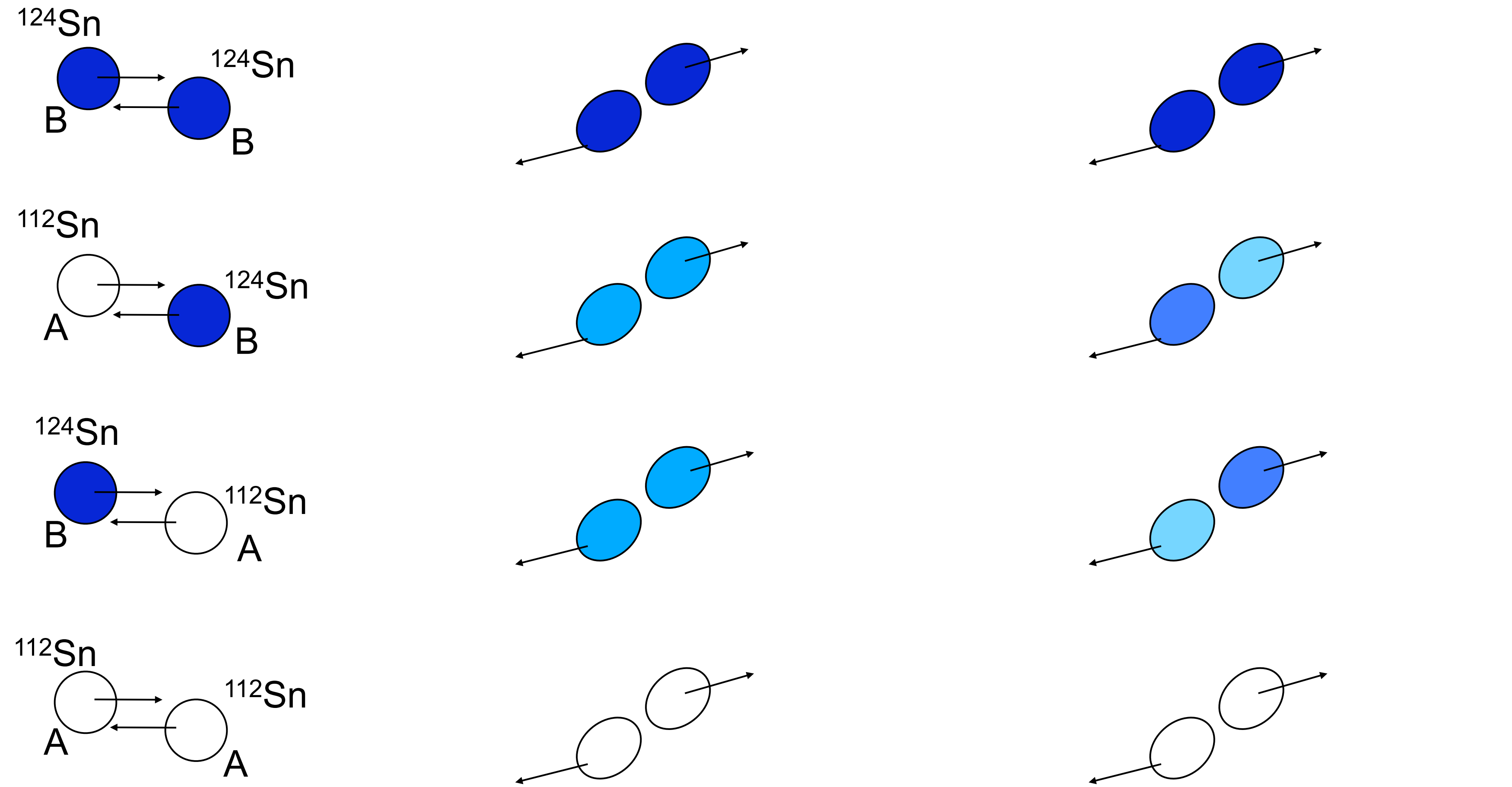
Isospin imbalance ratios

Measured observable in final state

Ex.: x=Y(⁷Li)/Y(⁷Be)

$$R_i(x) = \frac{2x - x_{B+B} - x_{A+A}}{x_{B+B} - x_{A+A}}$$

Before collision **N/Z equilibration** **N/Z translucency**



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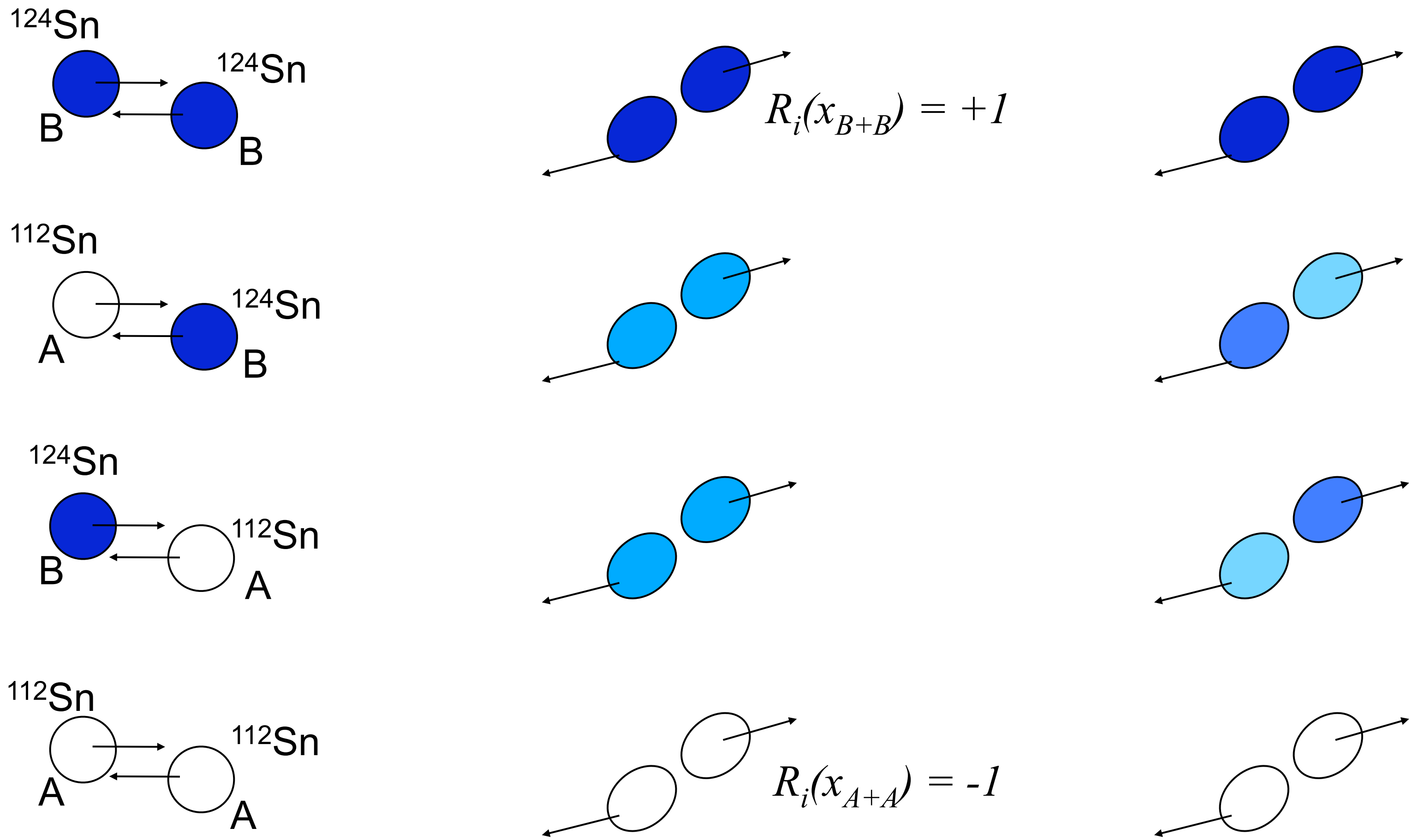
Before collision | **N/Z equilibration** | **N/Z translucency**



Isospin imbalance ratios

Measured observable in final state

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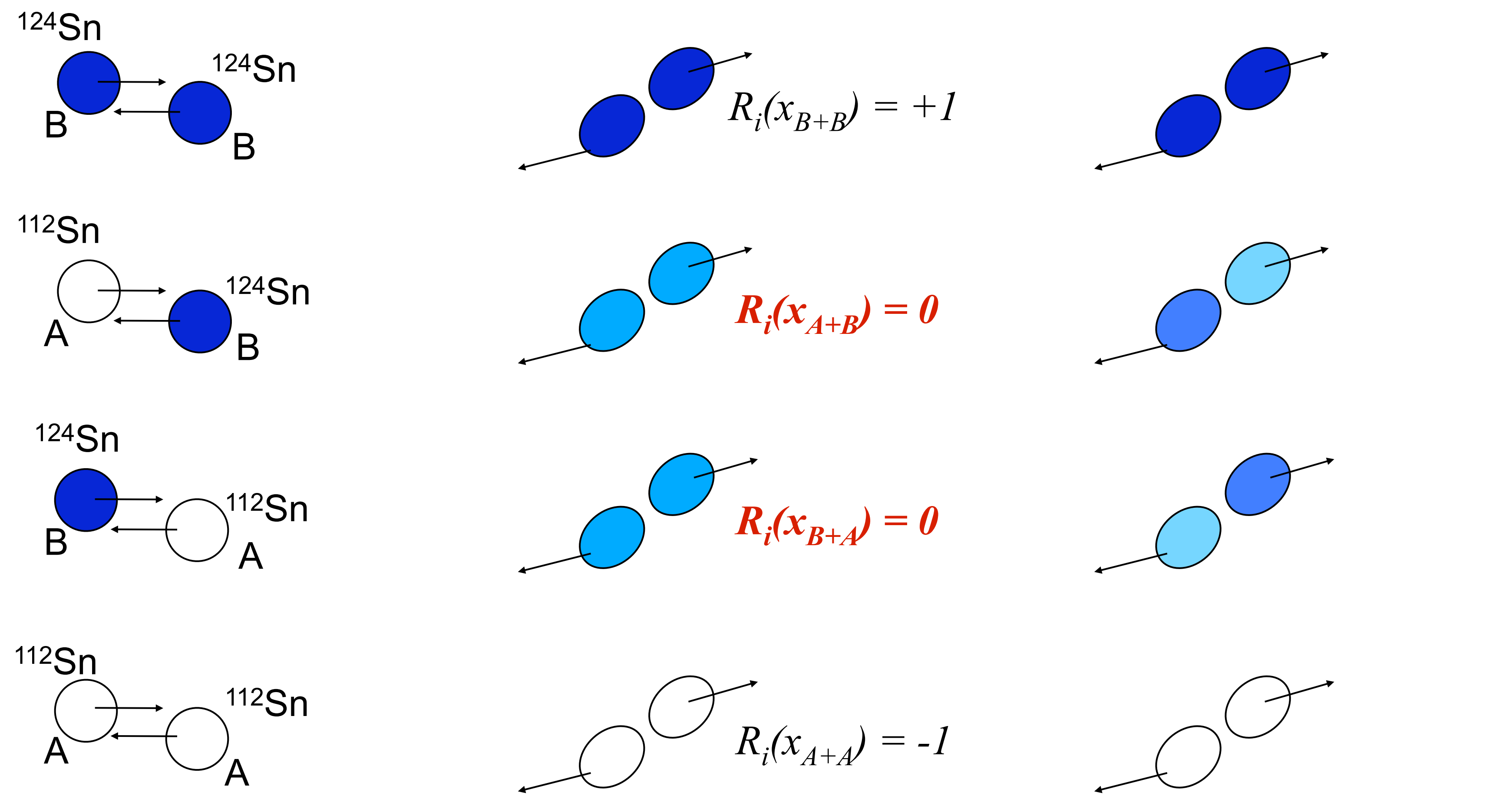

Isospin imbalance ratios

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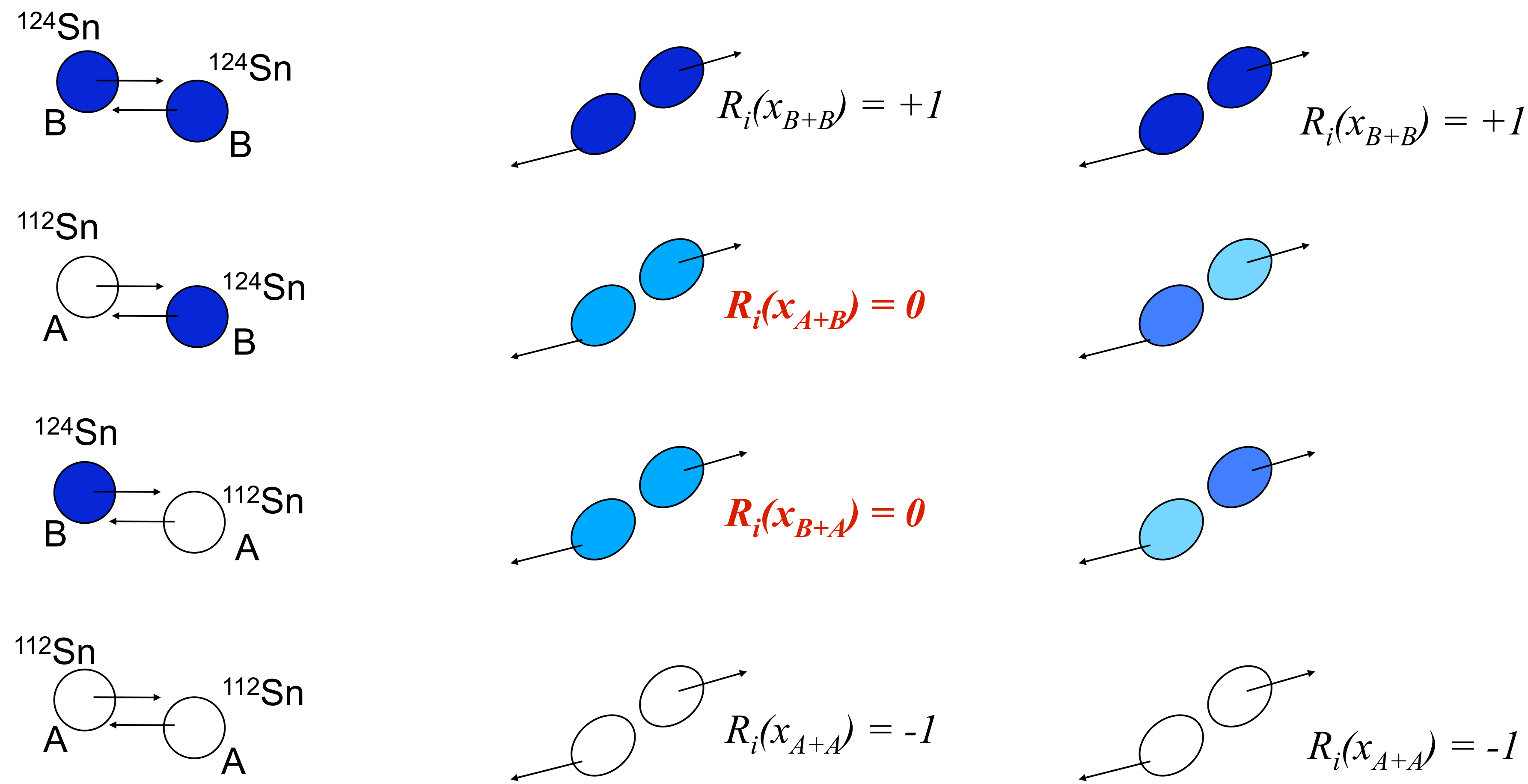
Before collision | **N/Z equilibration** | **N/Z translucency**



Isospin imbalance ratios

Measured observable in final state

$$R_i(x) = \frac{2x - x_{B+B} - x_{A+A}}{x_{B+B} - x_{A+A}}$$

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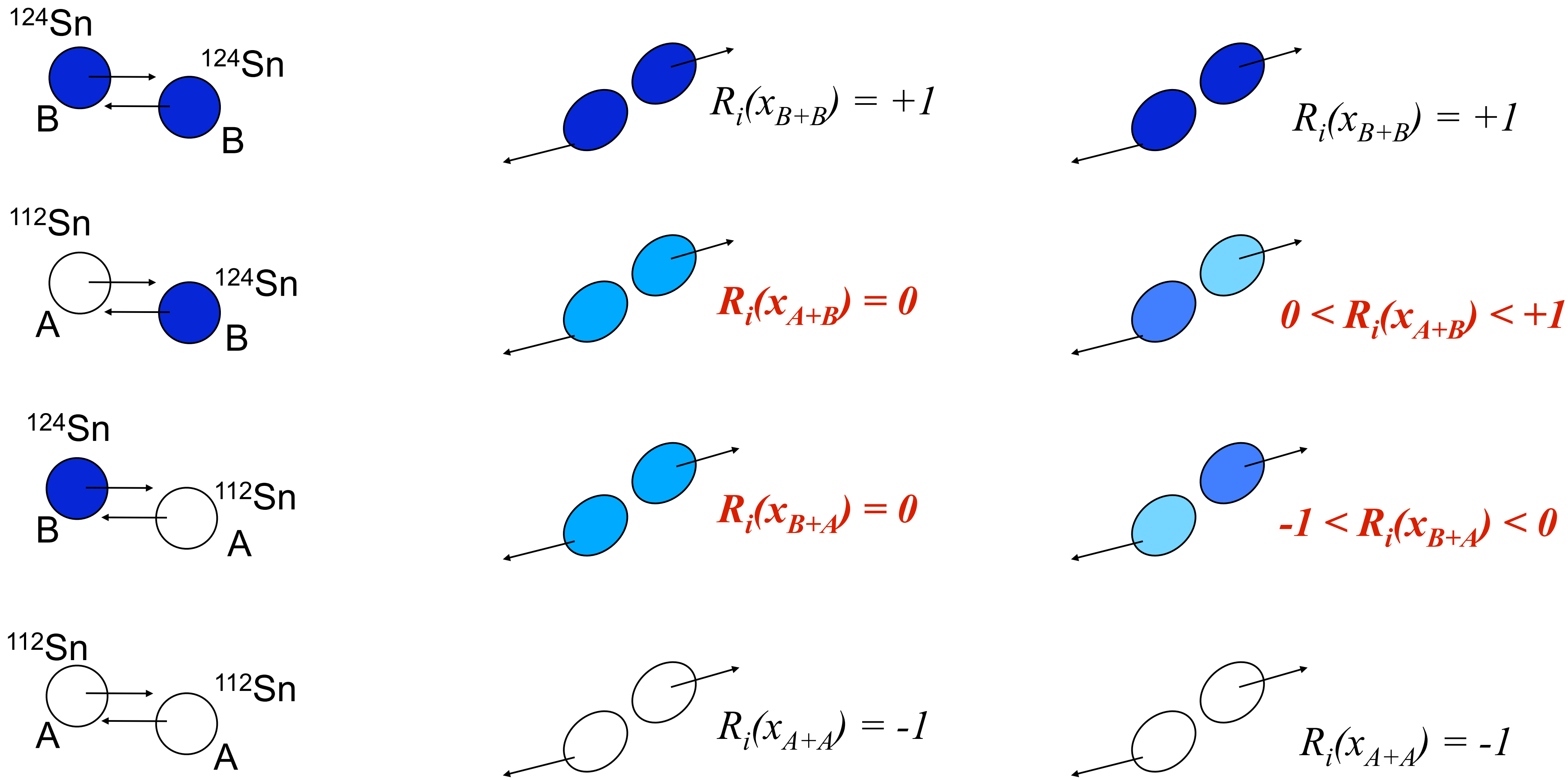
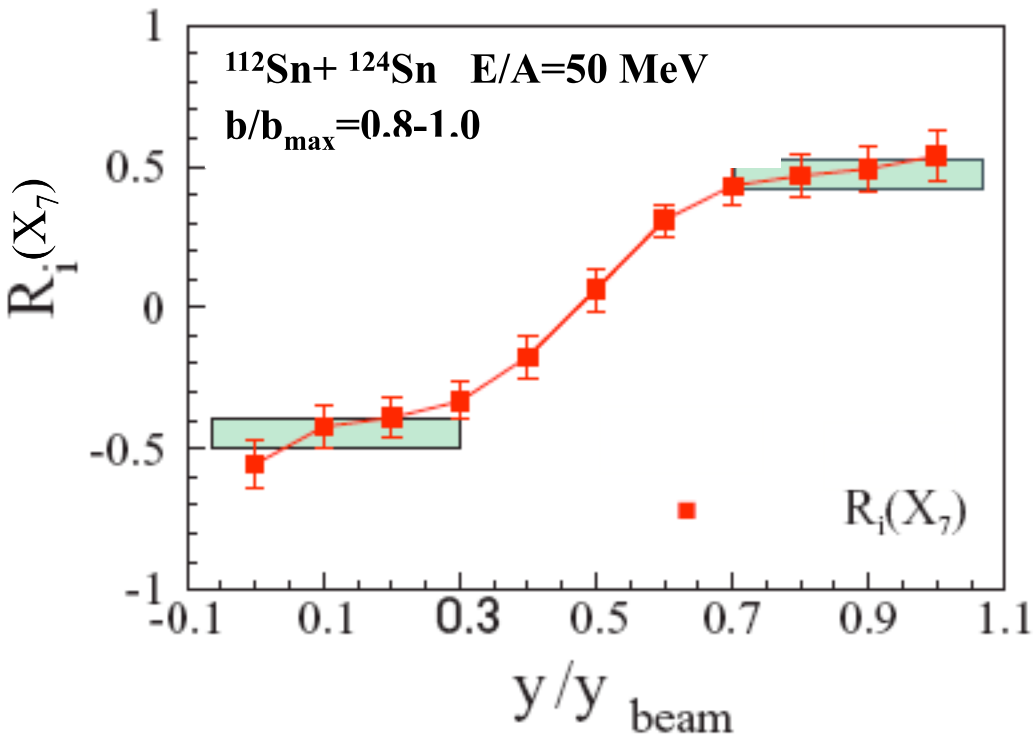
Before collision | **N/Z equilibration** | **N/Z translucency**



Imbalance ratios vs rapidity $^{112,124}Sn + ^{112,124}Sn$ E/A=50 MeV

$$y = \frac{1}{2}\frac{\log(1+\beta_{//})}{\log(1-\beta_{//})}$$

$X_7 = Y(^7Li)/Y(^7B$



T.X. Liu et al., PRC78, 034603 (2007)

Imbalance ratios vs rapidity $^{112,124}Sn + ^{112,124}Sn$ E/A=50 MeV

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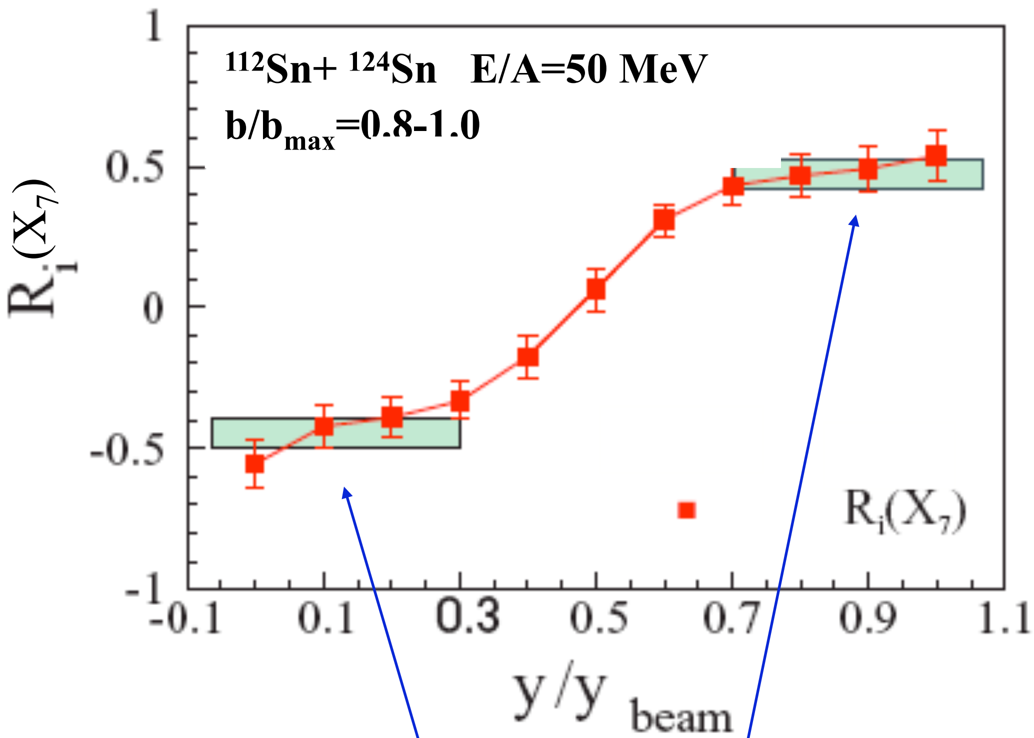
$X_7 = Y(^7Li)/Y(^7B$



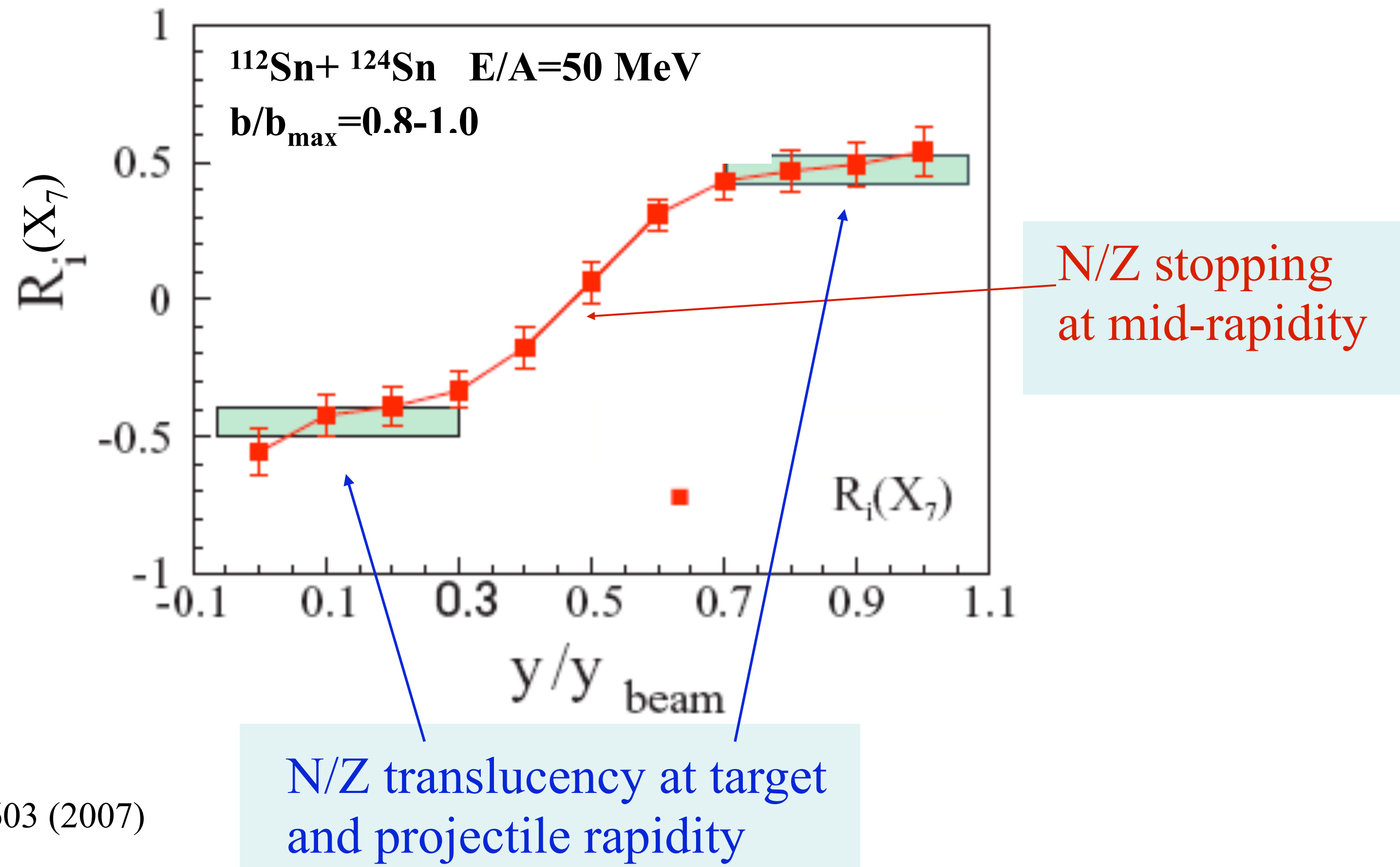
N/Z translucency at target and projectile rapidity

T.X. Liu et al., PRC78, 034603 (2007)

Imbalance ratios vs rapidity $^{112,124}Sn + ^{112,124}Sn$ E/A=50 MeV

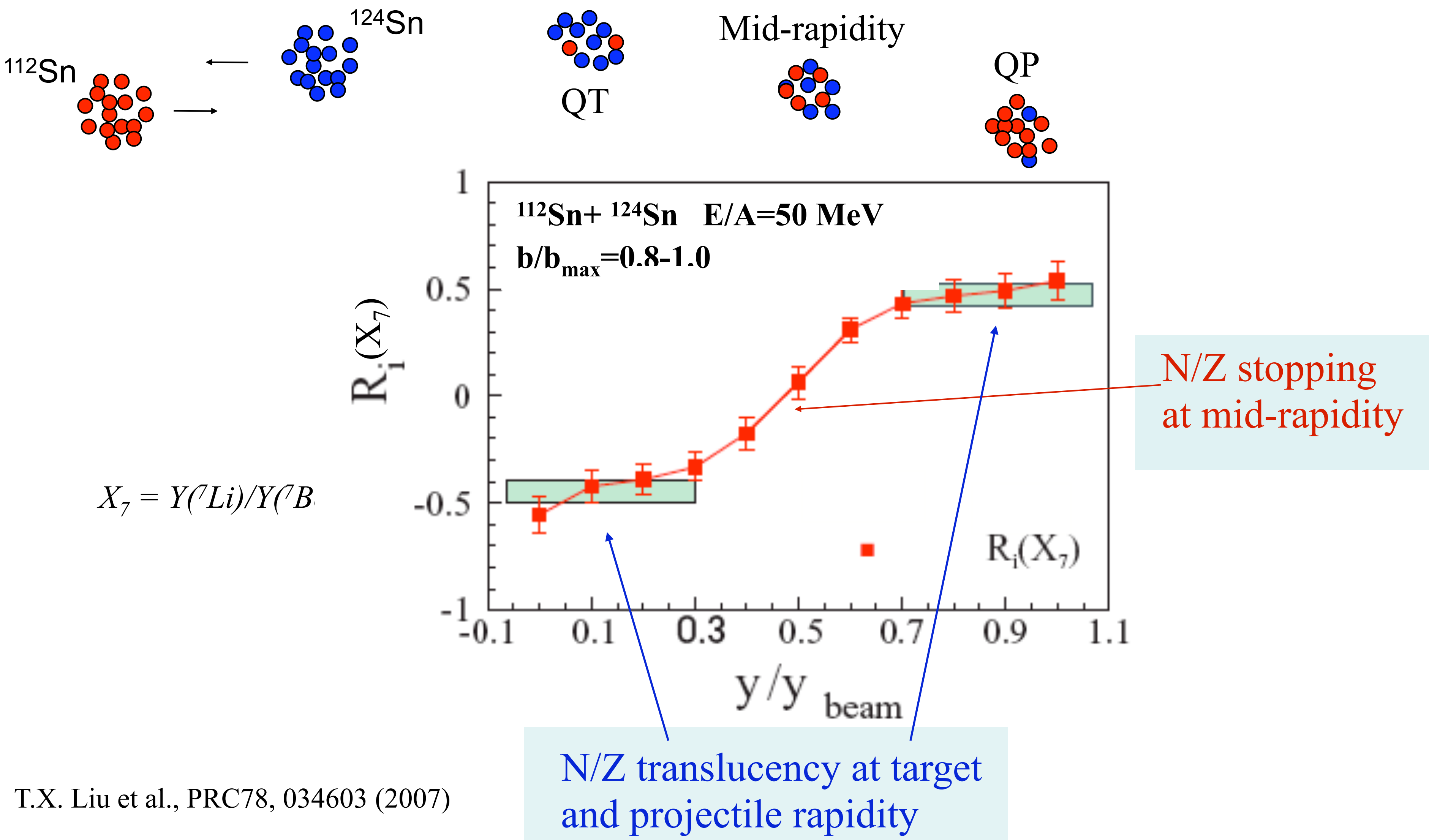
$$y = \frac{1}{2}\frac{\log(1+\beta_{//})}{\log(1-\beta_{//})}$$

$X_7 = Y(^7Li)/Y(^7Be)$



T.X. Liu et al., PRC78, 034603 (2007)

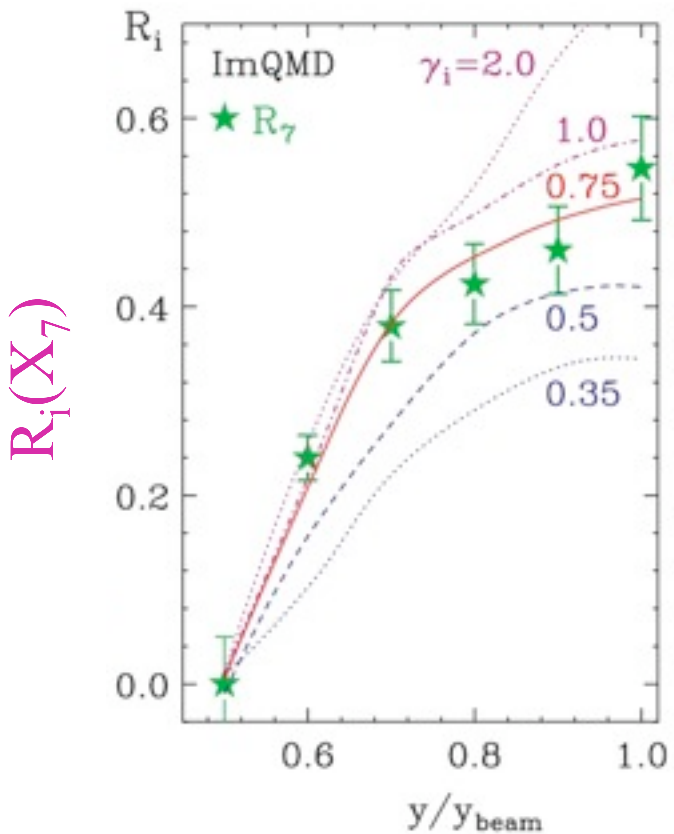
Imbalance ratios vs rapidity $^{112,124}Sn + ^{112,124}Sn$ E/A=50 MeV



T.X. Liu et al., PRC78, 034603 (2007)

Extracting the $E_{sym}(\rho)$ from isospin diffusion

$^{112,124}Sn+^{112,124}Sn$ E/A=50 MeV



Comparison to ImQMD model simulations to probe $E_{sym}(\rho)$

$$E_{sym}(\rho) = \frac{Cs,k}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{Cs,p}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

b=6 fm $\gamma \approx$ 0.45-1.0

b=7 fm $\gamma \approx$ 0.35-0.8

M.B. Tsang et al., PRL102, 122701 (2009)

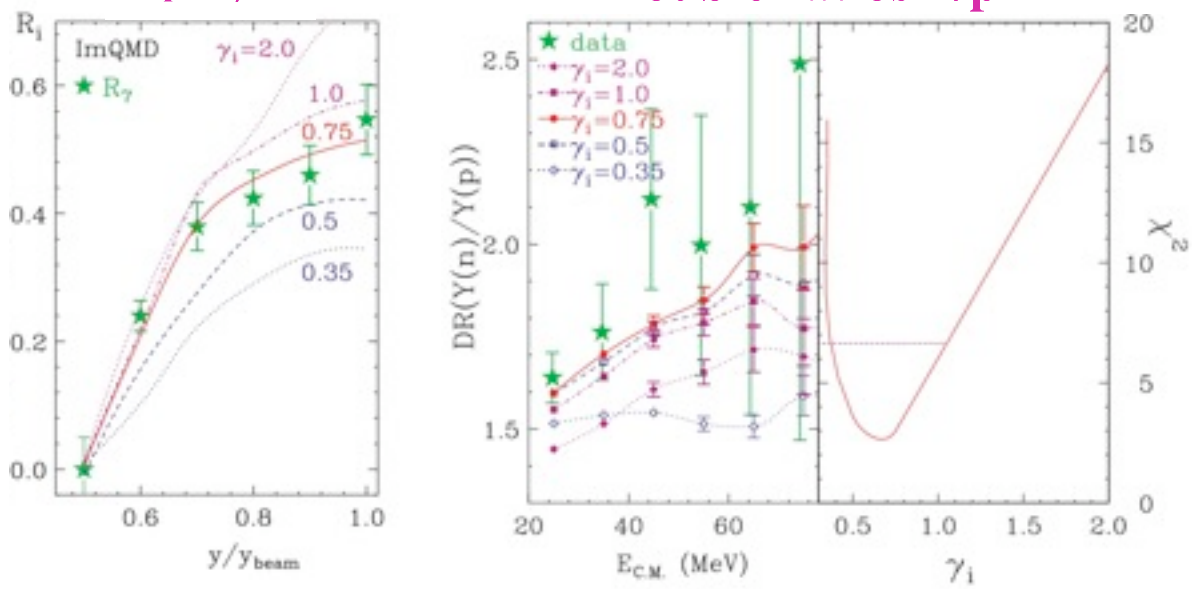
Towards a consistent picture

Same $E_{sym}(\rho)$ parameterization for multiple probes

$$E_{sym}(\rho) = 12.5 \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3} + 17.5 \cdot \left(\frac{\rho}{\rho_0}\right)^{\gamma} \qquad 0.4 < \gamma < 1$$

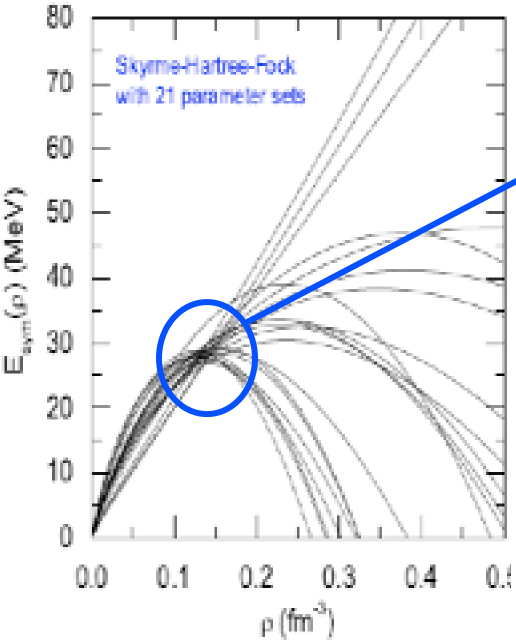
Isospin Diffusion → **$R_i(X_7)$**

N/p pre-equilibrium emission → **Double ratios n/p**



M.B. Tsang et al., PRL102, 122701 (2009)

E_{sym} slope and curvature



$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \ldots$$

$$S_0 = E_{sym}(\rho_0)$$

strength

$$L = 3\rho_0 \left|\frac{\partial E_{sym}(\rho)}{\partial \rho}\right|_{\rho_0} = \left(\frac{3}{\rho_0}\right) p_0$$

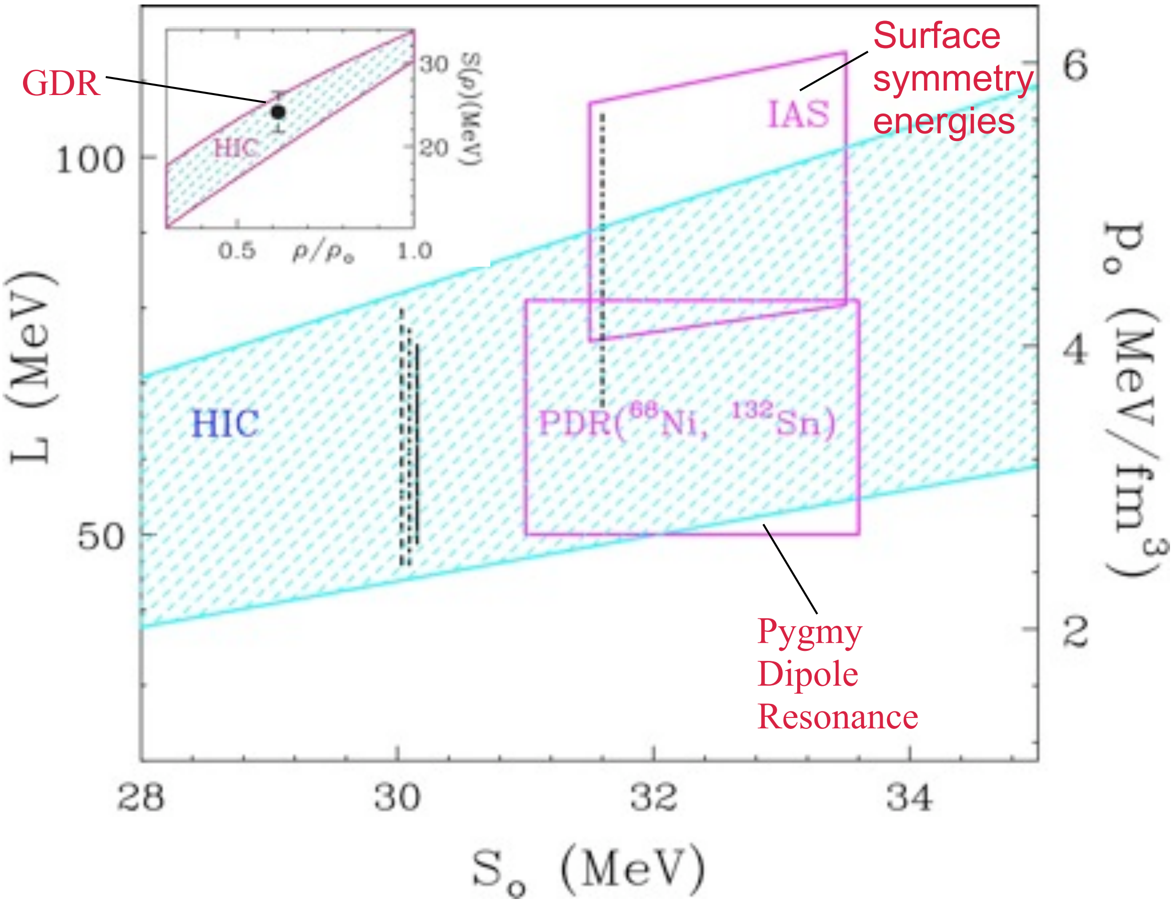
slope

$$K_{sym} = 9\rho_0^2 \left|\frac{\partial^2 E_{sym}(\rho)}{\partial \rho^2}\right|_{\rho_0}$$

curvature

S_0, L and K_{sym} relevant to neutron stars, neutron skins, nuclear collective motion (GMR, GDR, PDR), …

Consensus from different communities



$$S_0 = E_{sym}(\rho_0)$$

Strength at $\rho=\rho_0$

$$L = 3\rho_0 \left|\frac{dE_{sym}(\rho)}{d\rho}\right|_{\rho_0} = \left(\frac{3}{\rho_0}\right) p_0$$

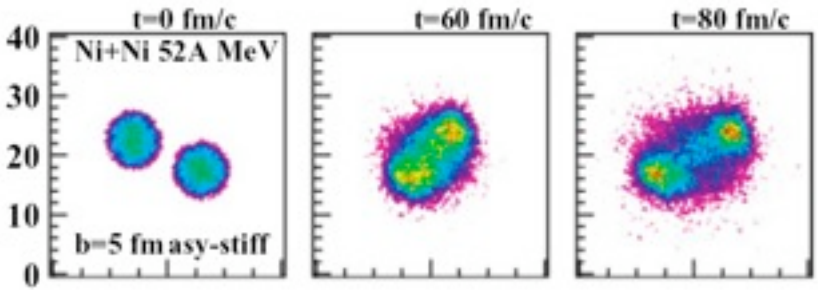
Slope

$$K_{sym} = 9\rho_0^2 \left|\frac{\partial^2 E_{sym}(\rho)}{\partial \rho^2}\right|_{\rho_0}$$

Curvature

Heavy-ion collisions and $E_{sym}(\rho)$

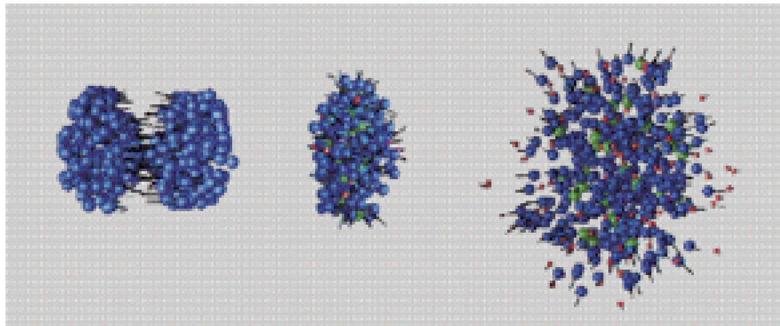
Intermediate energies: E/A=20-100 MeV



SMF - Baran, Colonna, Di Toro, Greco

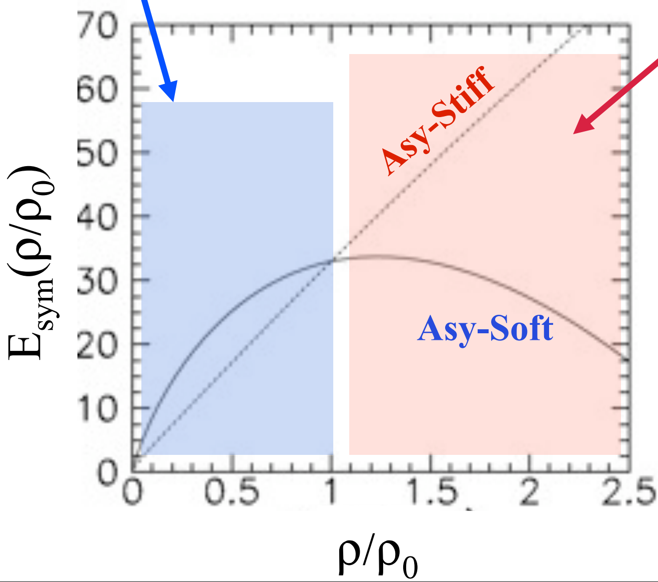
Ganil, Eurisol, Frib, Lns, Nscl, Spiral2, Tamu, …

High energies: E/A>200 MeV



CSR, GSI/Fair, FRIB, Riken, …

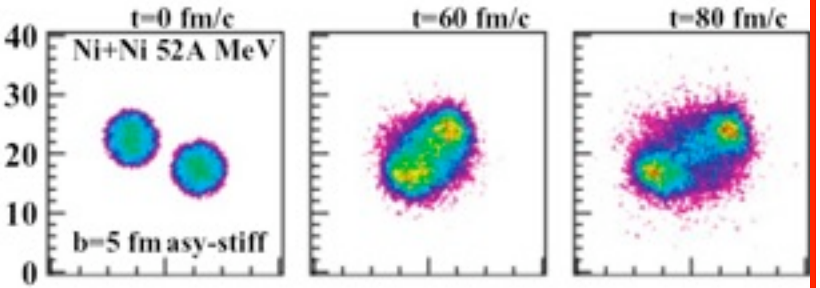
Low density



High density

Heavy-ion collisions and $E_{sym}(\rho)$

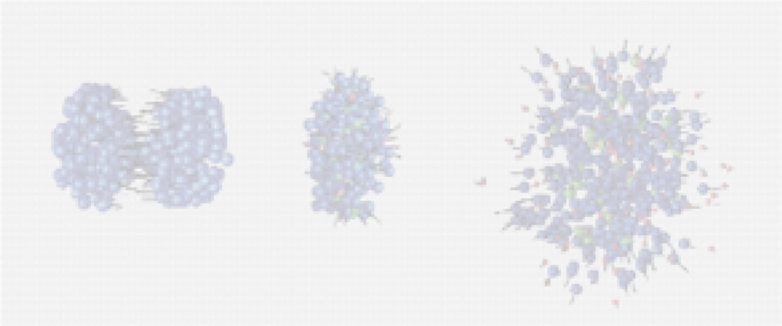
Intermediate energies: E/A=20-100 MeV



SMF - Baran, Colonna, Di Toro, Greco

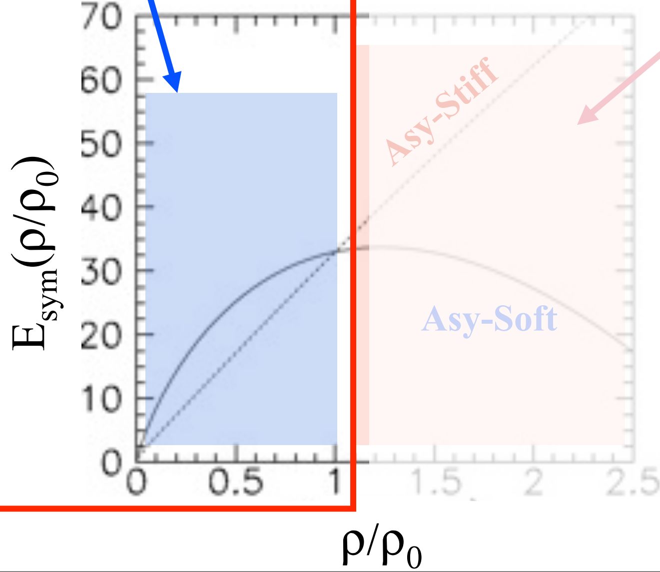
Ganil, Eurisol, Frib, Lns, Nscl, Spiral2, Tamu, …

High energies: E/A>200 MeV



CSR, GSI/Fair, FRIB, Riken, …

Low density

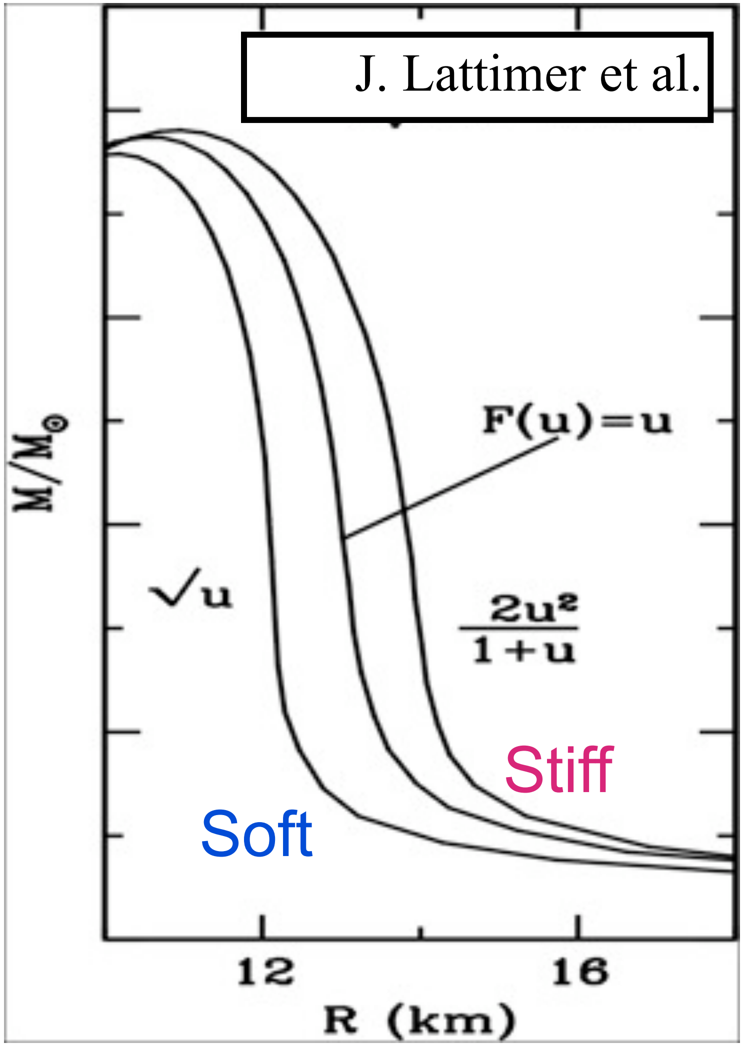


High density

Conclusions: Symmetry energy at

- Heavy-ion collisions at intermediate energies
  - Central collisions: pre-equilibrium n/p emission, isoscaling, isospin fractionation
  - Mid-peripheral collisions: isospin drift and isospin diffusion
- Consistent picture γ=0.4-1.0
- Agreement on L, K_{sym}, S_0 with other communities

Neutron star radii vs $E_{sym}(\rho)$

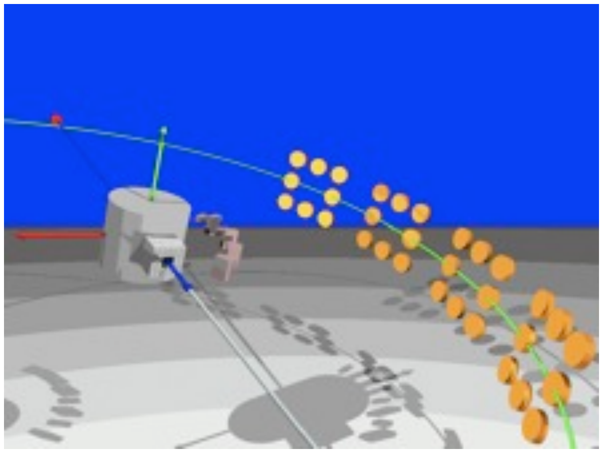


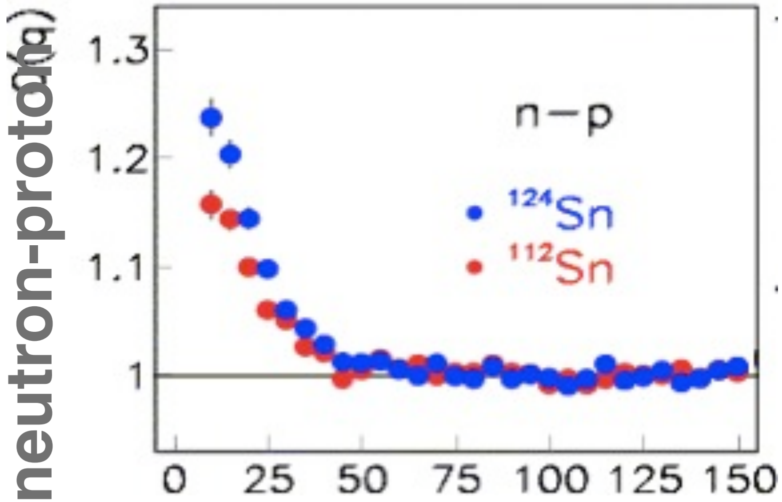
The relevance of the $E_{sym}(\rho)$ on the radius

Need to reduce error bars

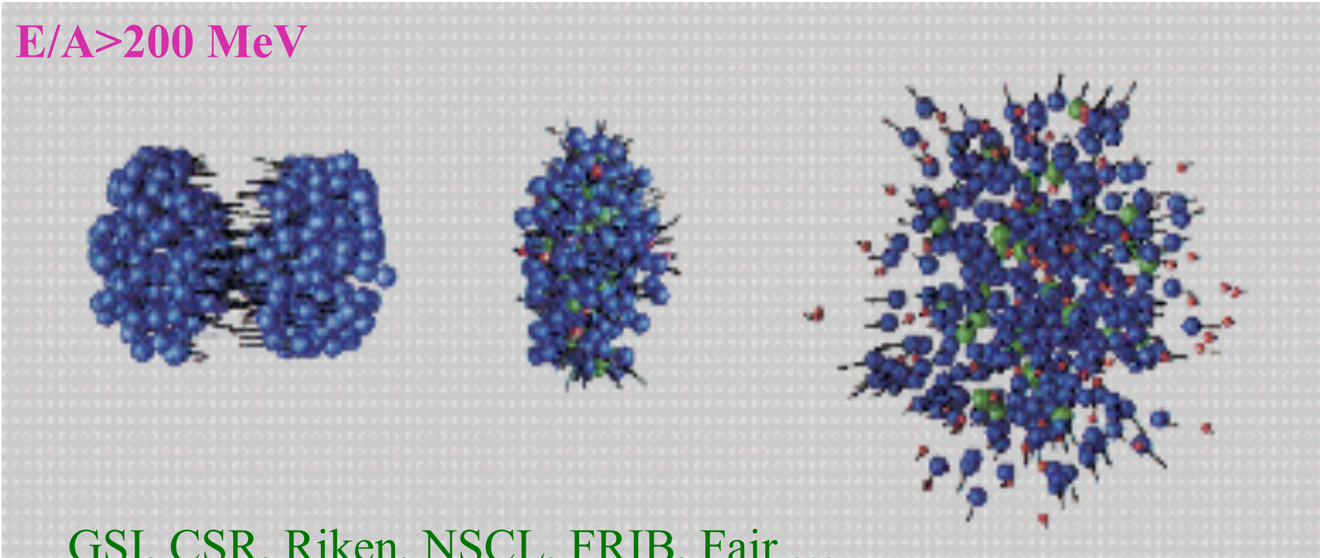
Outlook

- Reduce uncertainties (reduce model dependencies, improve measurements, more sensitive observables)
- Increase sensitivity to $E_{sym}(\rho)$ with radioactive beams (larger δ-asymmetries)
- pp, nn and np correlation functions



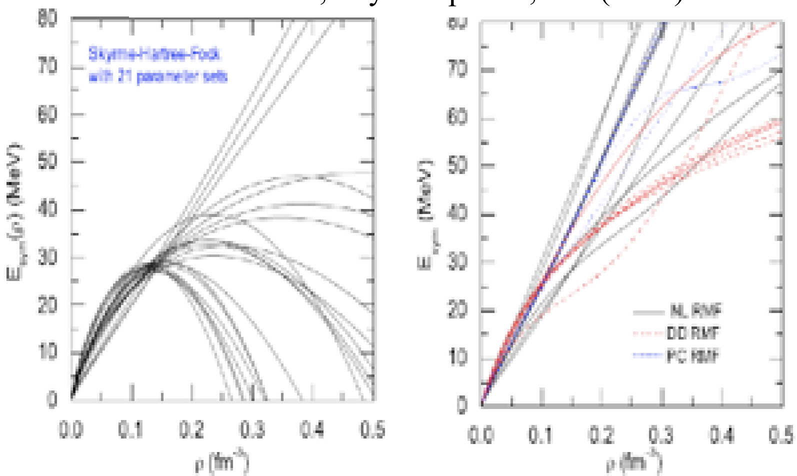


Studying $E_{sym}(\rho)$ at supra-saturation densities



GSI, CSR, Riken, NSCL, FRIB, Fair …

B.A. Li et al., Phys. Rep. 464, 113 (2008)



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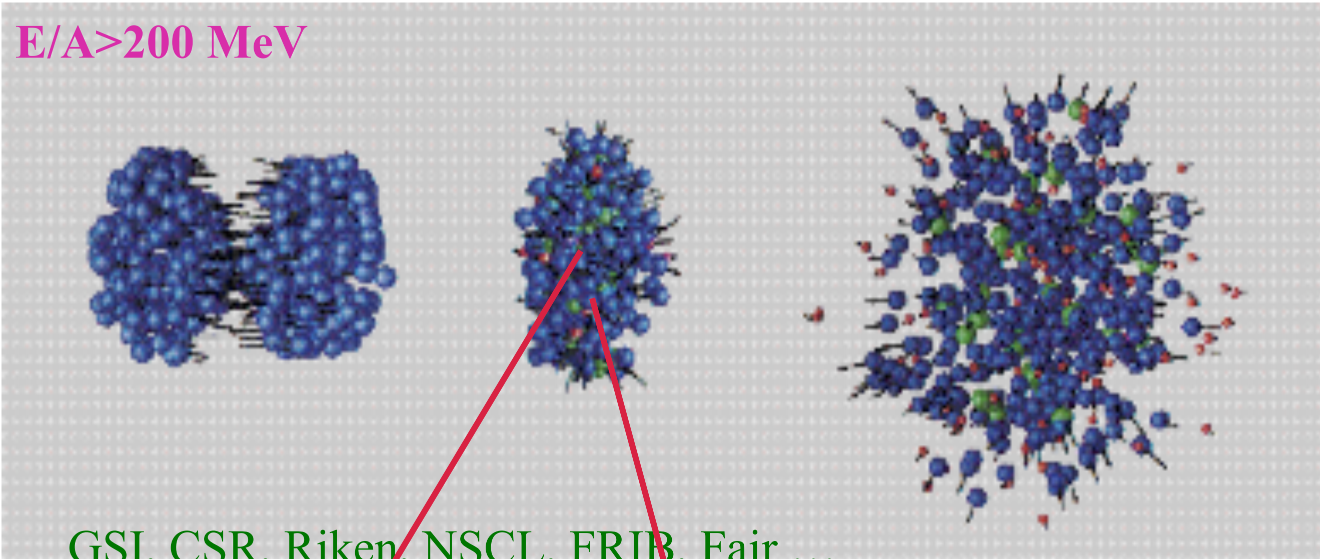

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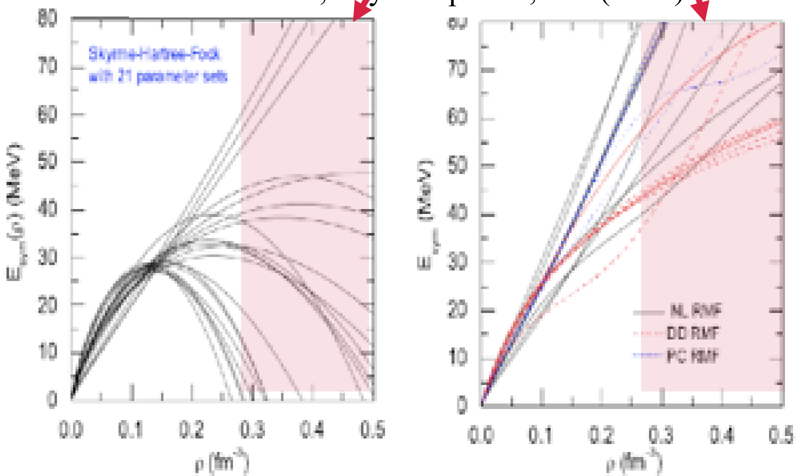
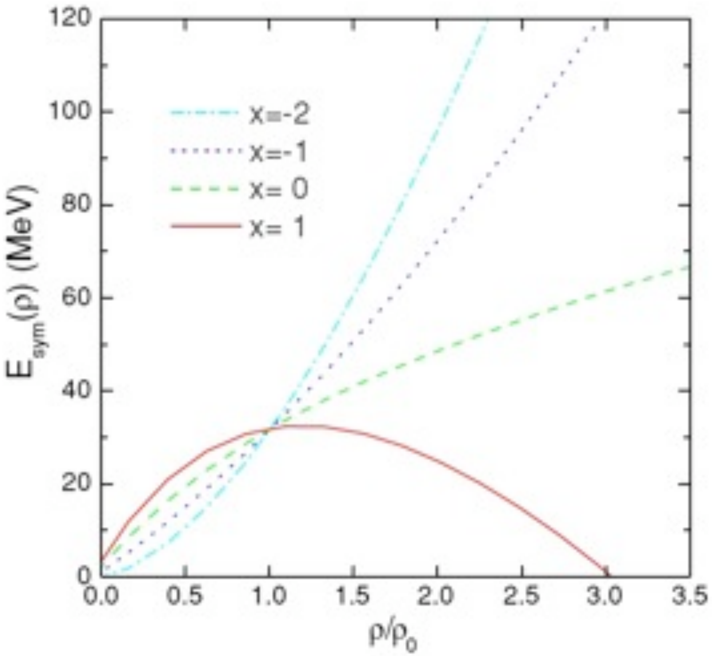
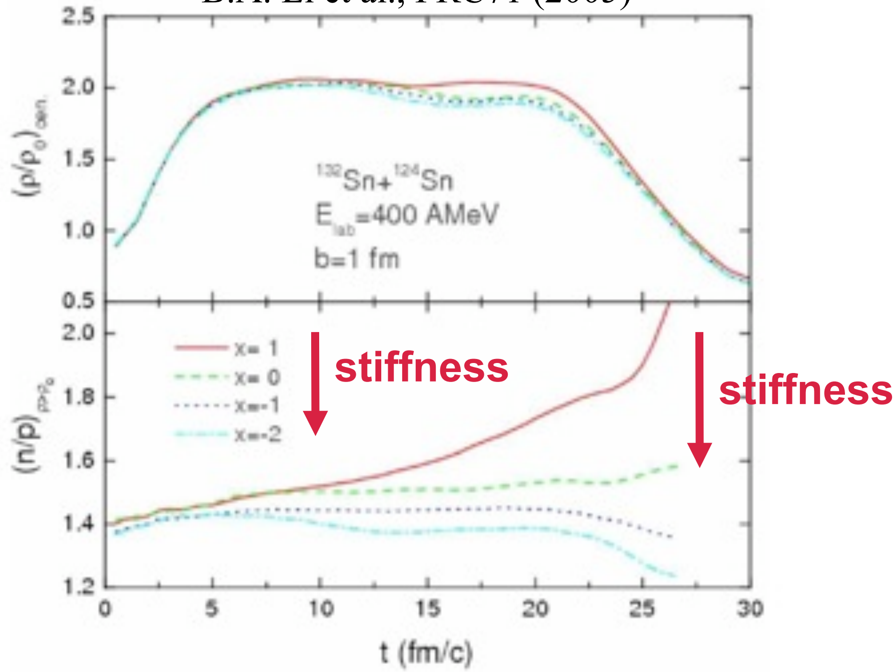


Effects of the E_{sym} at high density



B.A. Li et al., PRC71 (2005)



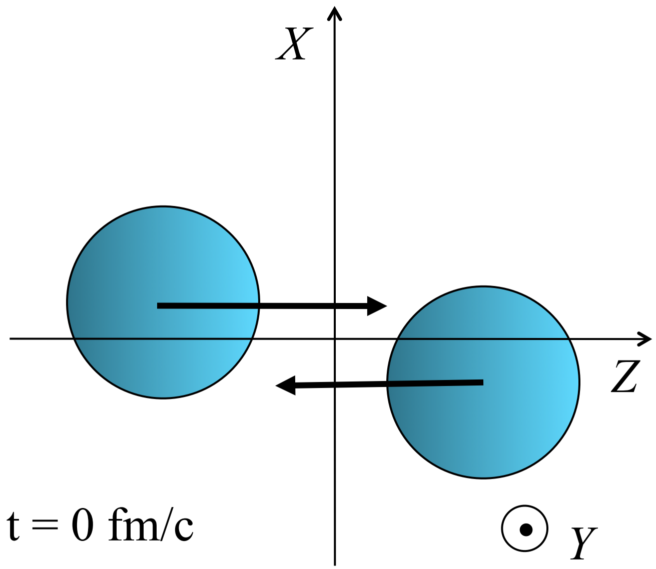
- **N/Z of high density regions sensitive to $E_{sym}(\rho)$**
- High ρ/ρ_0 : asy-stiff more repulsive on neutrons - opposite of sub-saturation trend

Probes at supra-saturation

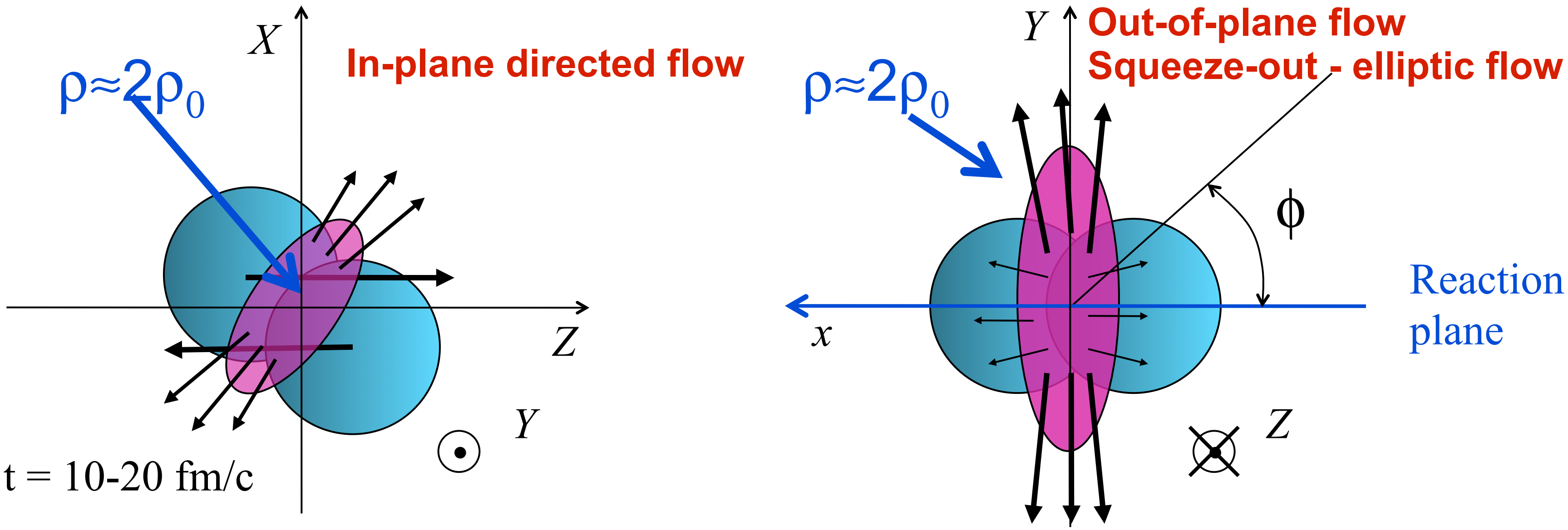
1. n/p directed and elliptic flow
2. Particle production in high density regions: π^-/π^+ and K^0/K^+
3. n/p and t/^{3}He spectra squeezed-out of participant region (ρ~2-3ρ0)

Caution with momentum dependent interaction

Directed and Elliptic flow

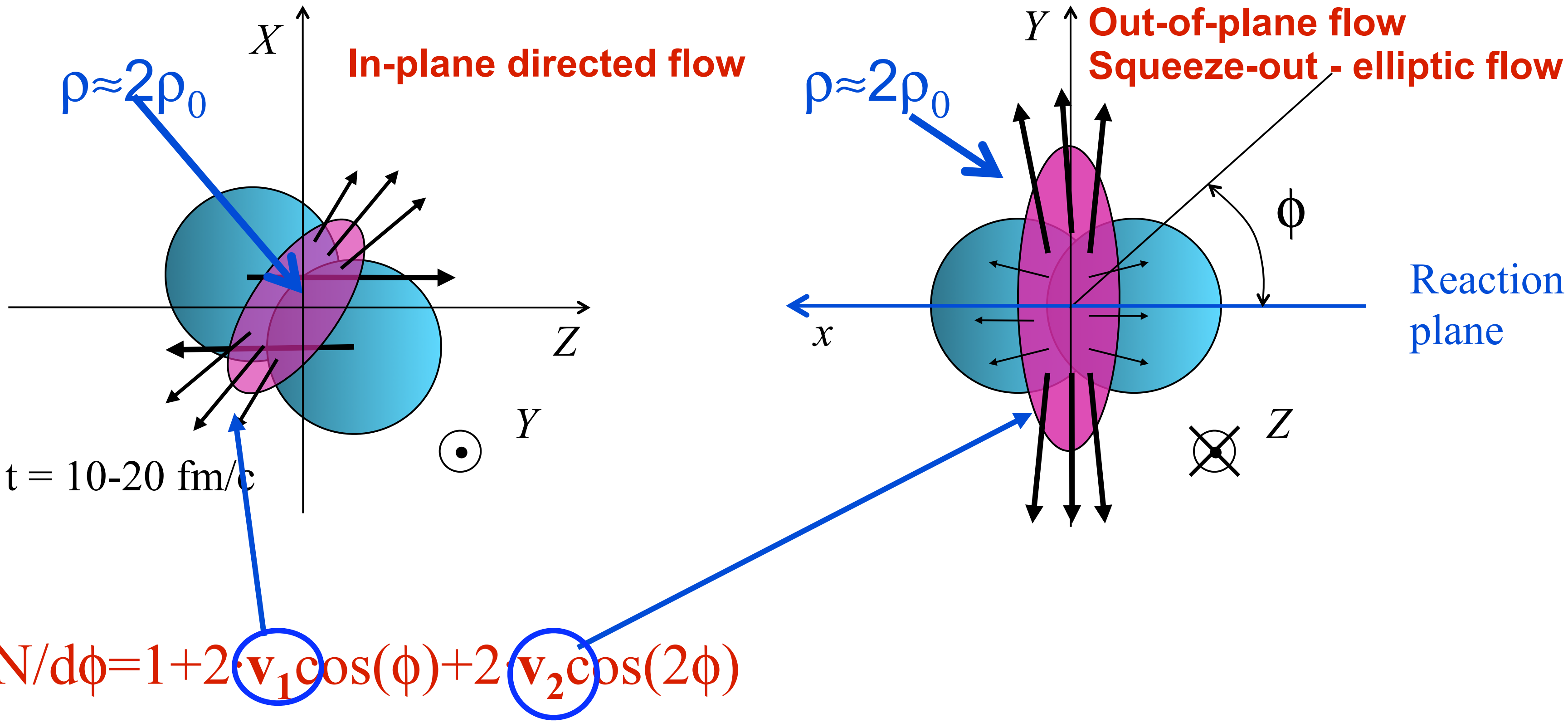


Directed and Elliptic flow



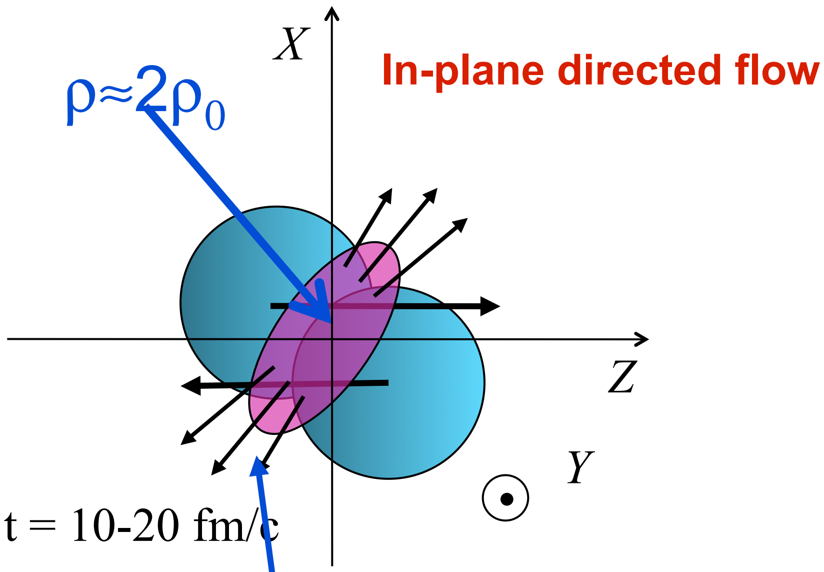
$dN/d\phi = 1 + 2\cdot\mathbf{v_1}\cos(\phi) + 2\cdot\mathbf{v_2}\cos(2\phi)$

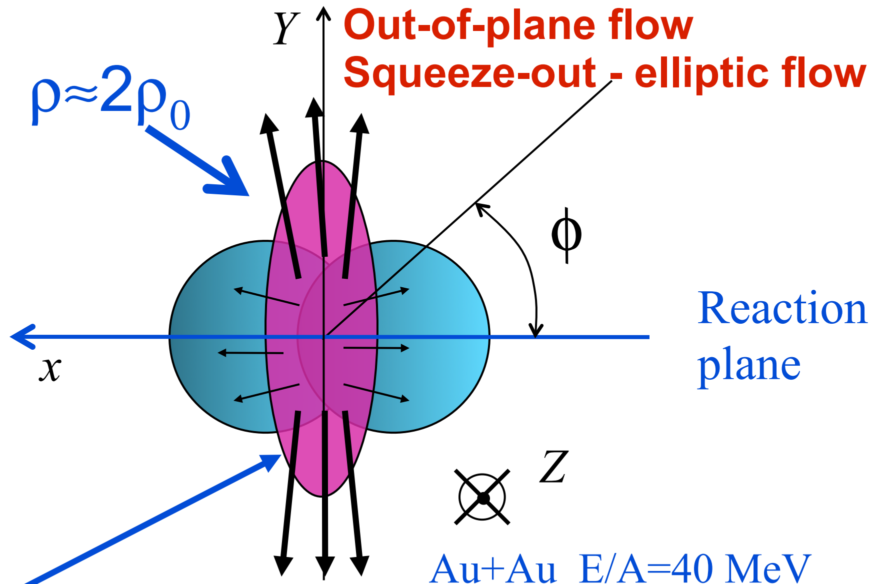
Directed and Elliptic flow



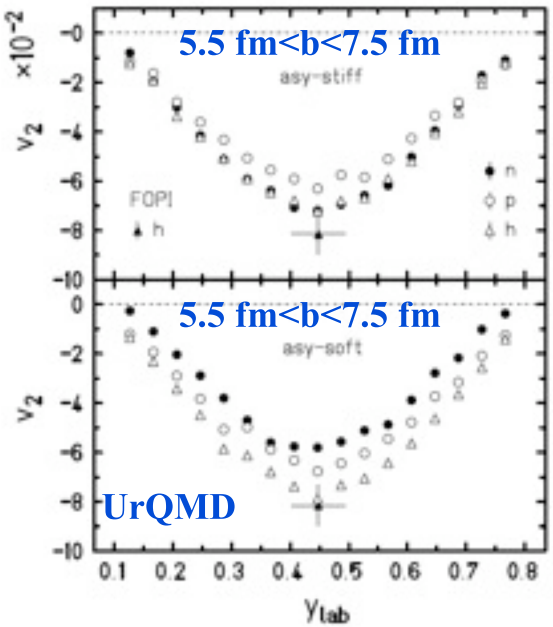
$$dN/d\phi = 1 + 2\cdot\mathbf{v_1}\cos(\phi) + 2\cdot\mathbf{v_2}\cos(2\phi)$$

Directed and Elliptic flow





Au+Au E/A=40 MeV



$dN/d\phi = 1 + 2\cdot \mathbf{v_1}\cos(\phi) + 2\cdot \mathbf{v_2}\cos(2\phi)$

Squeeze-out of neutrons sensitive to $E_{sym}(\rho)$

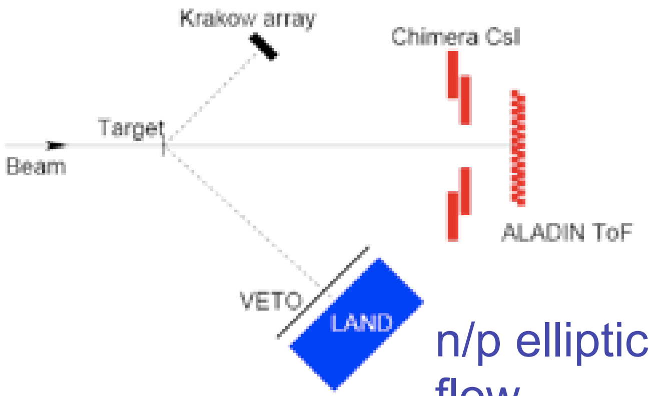
Chimera+Land @ GSI (2010-2011)

Au+Au @ E/A=400 MeV

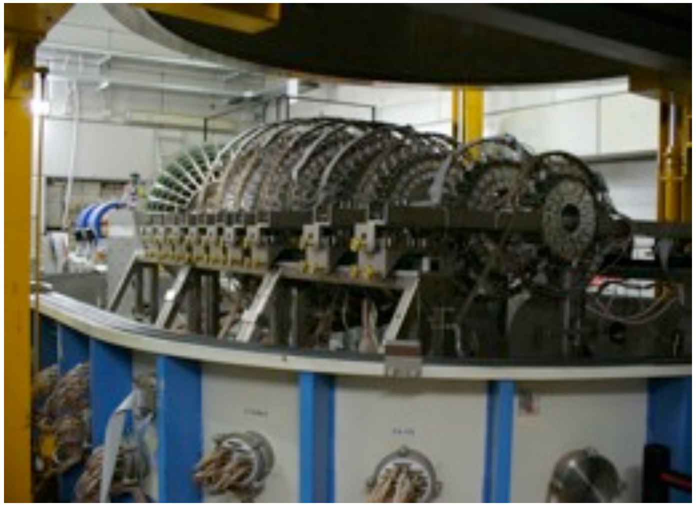
Measure n and p elliptic flow

Nov 2010 at GSI: test of LAND/Chimera coupling with Au beam

Au+Au E/A=400 MeV



n/p elliptic flow



Conclusions

- Intermediate energies: $\rho<\rho_0$
  - Consensus over γ = 0.4-1.0 for the density dependence of the symmetry energy
  - Need to reduce uncertainties (better experiments)
  - Increase sensitivity to E_{sym}: larger N/Z asymmetries (radioactive beams…)

- Relativistic energies: $\rho<\rho_0$
  - Work need to be done: n/p elliptic flow, meson production, …