IBA

Assume valence fermions couple in pairs to bosons of spins 0^+ and 2^+

0^+	*s*-boson
2^+	*d*-boson

- Valence nucleons only
- *s*, *d* bosons – creation and destruction operators

$H = H_s + H_d + H_{\text{interactions}}$

Number of bosons fixed: $N = n_s + n_d$

= ½ # of val. protons + ½ # val. neutrons

IBA Models

IBA – 1 No distinction of p, n

IBA – 2 Explicitly write p, n parts

IBA – 3, 4 Take isospin into account

p-n pairs

IBFM Int. Bos. Fermion Model

Odd A nuclei

$H = H_{e-e} + H_{s.p.} + H_{int}$

core

IBFFM Odd – odd nuclei

[(f, p) bosons for π = - states spdf IBA]

Parameters

Different models have different numbers of parameters. Be careful in evaluating/comparing different models. Be alert for hidden parameters. Lots of parameters are not necessarily bad – they may be mandated by the data, but look at them with your eyes open.

Background, References

- F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, England, 1987).

- F. Iachello and P. Van Isacker, *The Interacting Boson-Fermion Model* (Cambridge University Press, Cambridge, England, 2005)

- R.F. Casten and D.D. Warner, Rev. Mod. Phys. **60** (1988) 389.

- R.F. Casten, *Nuclear Structure from a Simple Perspective*, 2nd Edition (Oxford Univ. Press, Oxford, UK, 2000), Chapter 6 (the basis for most of these lectures).

- D. Bonatsos, *Interacting boson models of nuclear structure*, (Clarendon Press, Oxford, England, 1989)

- Many articles in the literature

IBA has a deep relation to Group theory

That relation is based on the operators that create, destroy *s* and *d* bosons

$s^\dagger$, s, $d^\dagger$, d operators

Ang. Mom. 2

$d^\dagger_\mu$, d_μ $\mu = 2, 1, 0, -1, -2$

Hamiltonian is written in terms of *s, d* operators

Since boson number is conserved for a given nucleus, *H* can only contain "bilinear" terms: 36 of them.

$s^\dagger s$, $s^\dagger d$, $d^\dagger s$, $d^\dagger d$

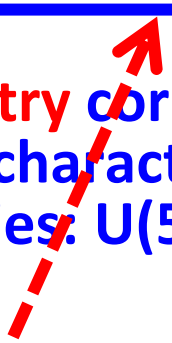
Gr. Theor. classification of Hamiltonian

Group is called

U(6)

OK, here's what you need to remember from the Group Theory

- **Group Chain:** $U(6) \supset U(5) \supset O(5) \supset O(3)$



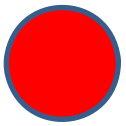
- **A dynamical symmetry corresponds to a certain structure/shape of a nucleus and its characteristic excitations. The IBA has three dynamical symmetries: U(5), SU(3), and O(6).**
- **Each term in a group chain representing a dynamical symmetry gives the next level of degeneracy breaking.**
- **Each term introduces a new quantum number that describes what is different about the levels.**
- **These quantum numbers then appear in the expression for the energies, in selection rules for transitions, and in the magnitudes of transition rates.**

OK, here's the key point :

Concept of a Dynamical Symmetry

Spectrum generating algebra !!

Group theory of the IBA



U(6) 36 generators conserve N

U(5) 25 generators conserve n_d

Suppose:

$$H = \alpha_1 C_{U(6)} + \alpha_2 C_{U(5)} \qquad (1)$$

All states of a given nucleus have same N. So, if $\alpha_2 = 0$, *i.e.*, $H = \alpha_1 C_{U(6)}$ only, then all states would be degenerate.

But these states have different n_d. Thus, if we consider the full eq. 1, then the degeneracy is broken because $C_{U(5)}$ gives $E = f(n_d)$. In group notation

$$U(6) \supset U(5) \supset \ldots$$

Dyn. Symm.

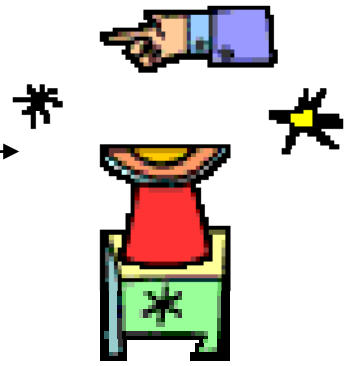
Recall: $O(3) \supset O(2)$

Group Structure of the IBA

s boson : 1

d boson : 5

U(6)

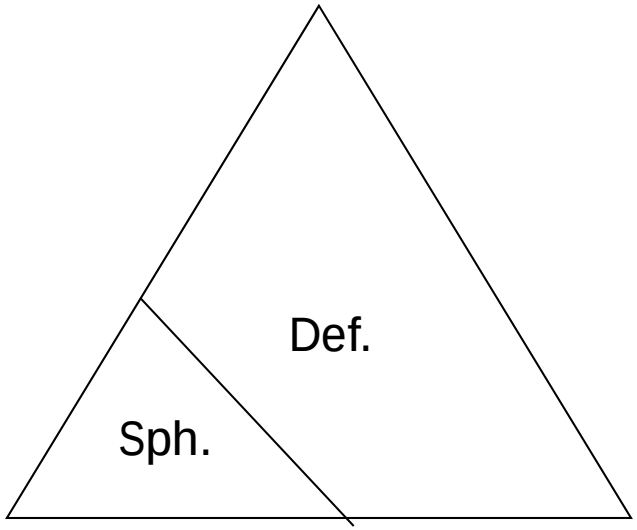


Magical group theory stuff happens here

U(5)
vibrator

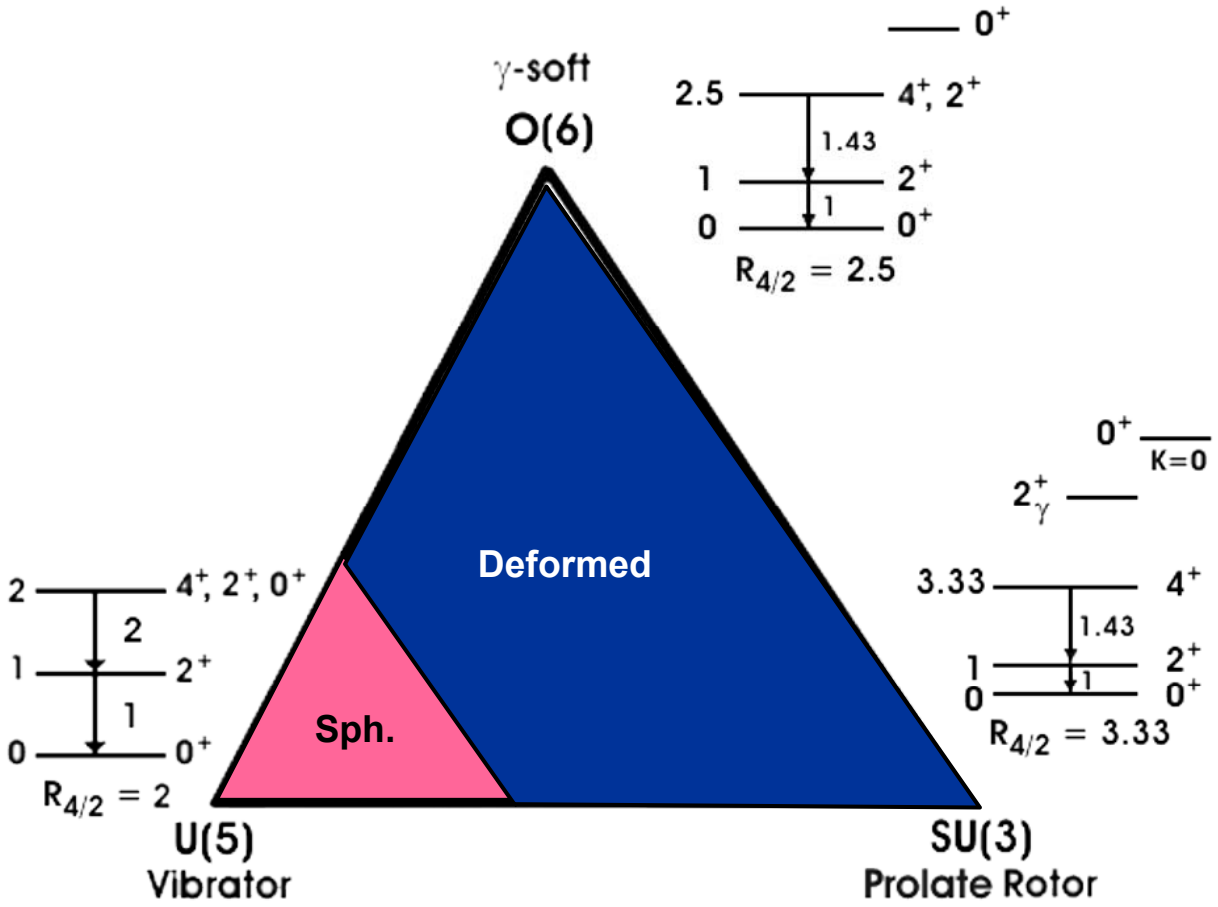
SU(3)
rotor

O(6)
γ-soft



Symmetry Triangle of the IBA

Classifying Structure -- The Symmetry Triangle



Most nuclei do not exhibit the idealized symmetries but rather lie in transitional regions. Mapping the triangle.

IBA Hamiltonian

Counts the number of d bosons out of N bosons, total.

The last are s-bosons with $\Sigma_s = 0$ since we deal only with excitation energies.

Conserves the number of d bosons. Gives terms in the Hamiltonian where the energies of configurations of 2 d bosons depend on their total combined angular momentum. Allows for anharmonicities in the phonon multiplets.

$d^+ d$

Excitation energies depend ONLY on the number of d-bosons. E(0) = 0, E(1) = ε , E(2) = 2 ε.

$$H = \varepsilon_d n_d$$

Mixes d and s components of the wave functions

Most general IBA Hamiltonian in terms with up to four boson operators (given N)

U(5)

Spherical, vibrational nuclei

Simplest Possible IBA Hamiltonian – given by energies of the bosons with NO interactions

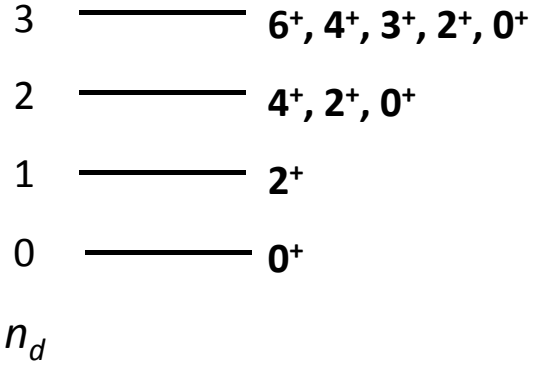
$$H = \varepsilon_d \, n_d + \varepsilon_s \, n_s = \text{E of d bosons + E of s bosons}$$

$$= \varepsilon_d \left(d^\dagger \, d \right) + \varepsilon_s \, s^\dagger \, s$$

Excitation energies so, set $\varepsilon_s = 0$, and drop subscript *d* on ε_d

$$\boxed{H = \varepsilon \, n_d}$$

What is spectrum? Equally spaced levels defined by number of *d* bosons



What *J*'s? M-scheme

Look familiar? Same as quadrupole vibrator.

U(5) also includes anharmonic spectra

*E*2 Transitions in the IBA

Key to most tests

Very sensitive to structure

*E*2 Operator: Creates or destroys an *s* or *d* boson or recouples two *d* bosons.
Must conserve *N*

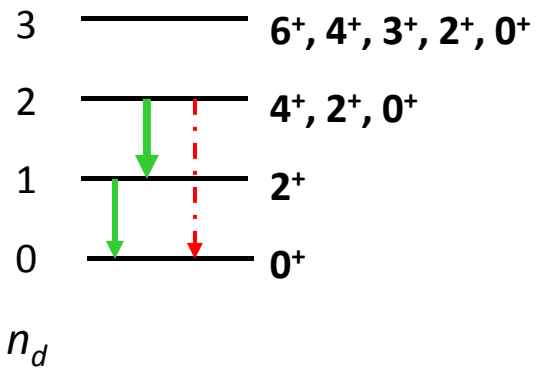
$$\mathbf{T = e\,Q = e[s^{\dagger}\tilde{d} + d^{\dagger}s + \chi\,(d^{\dagger}\,\tilde{d}\,)^{(2)}]}$$

Specifies relative strength of this term

E2 transitions in U(5)

- **$\chi = 0$**
- **That is:** $T = e[s^{\dagger}\tilde{d} + d^{\dagger}s]$

- Why? So that it can create or destroy a single d boson, that is, a single phonon.



Creation and destruction operators as

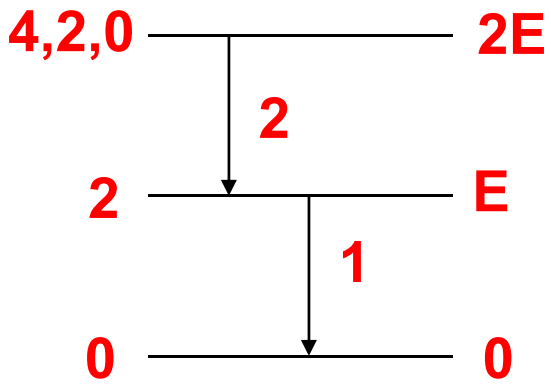
"Ignorance operators"

Example: Consider the case we have just discussed – the spherical vibrator.

Why is the B(E2: 4 – 2) = 2 x B(E2: 2– 0) ??

Difficult to see with Shell Model wave functions with 1000's of components

However, as we have seen, it is trivial using destruction operators WITHOUT EVER KNOWING ANYTHING ABOUT THE DETAILED STRUCTURE OF THESE VIBRATIONS !!!! These operators give the relationships between states.

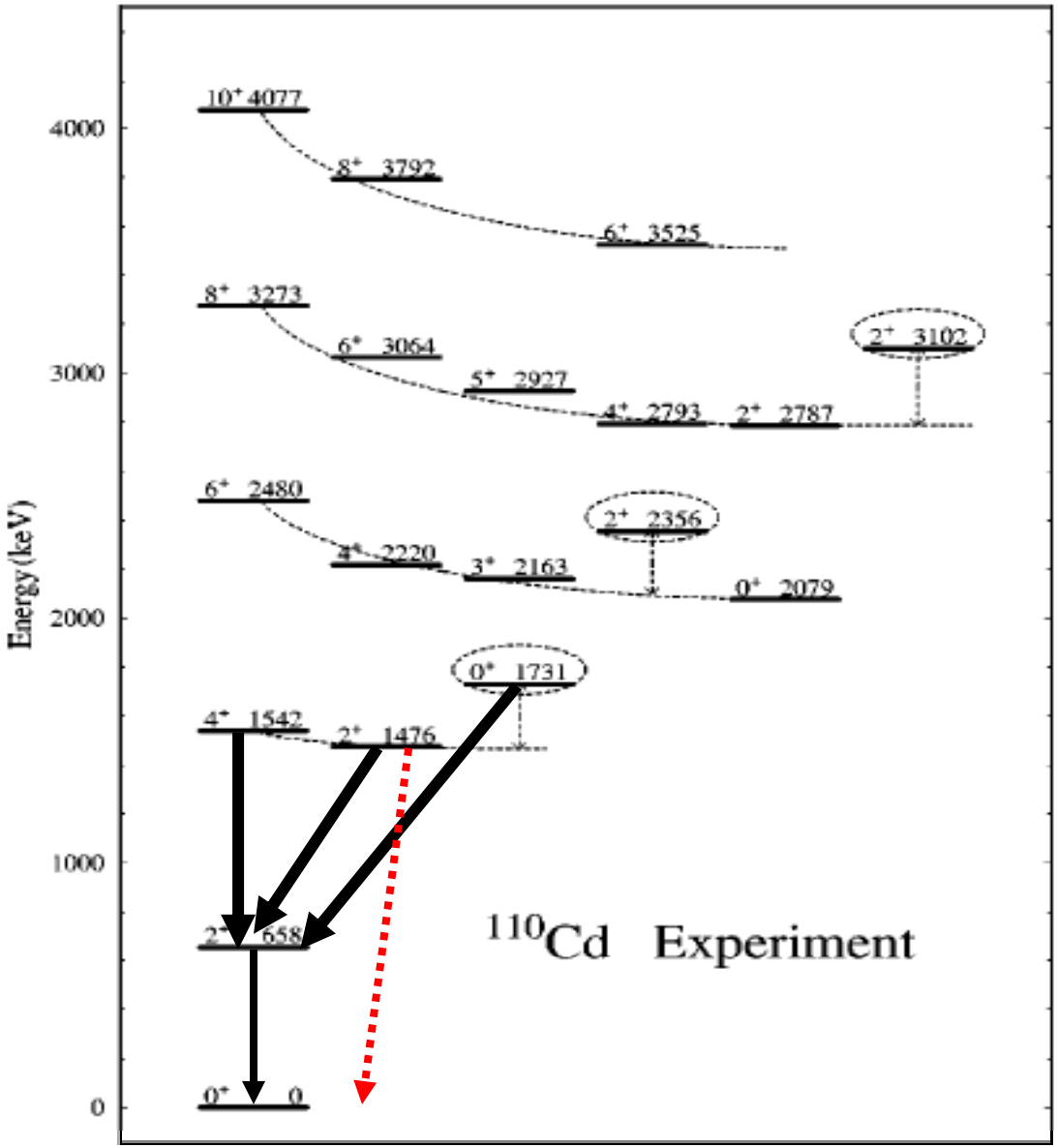


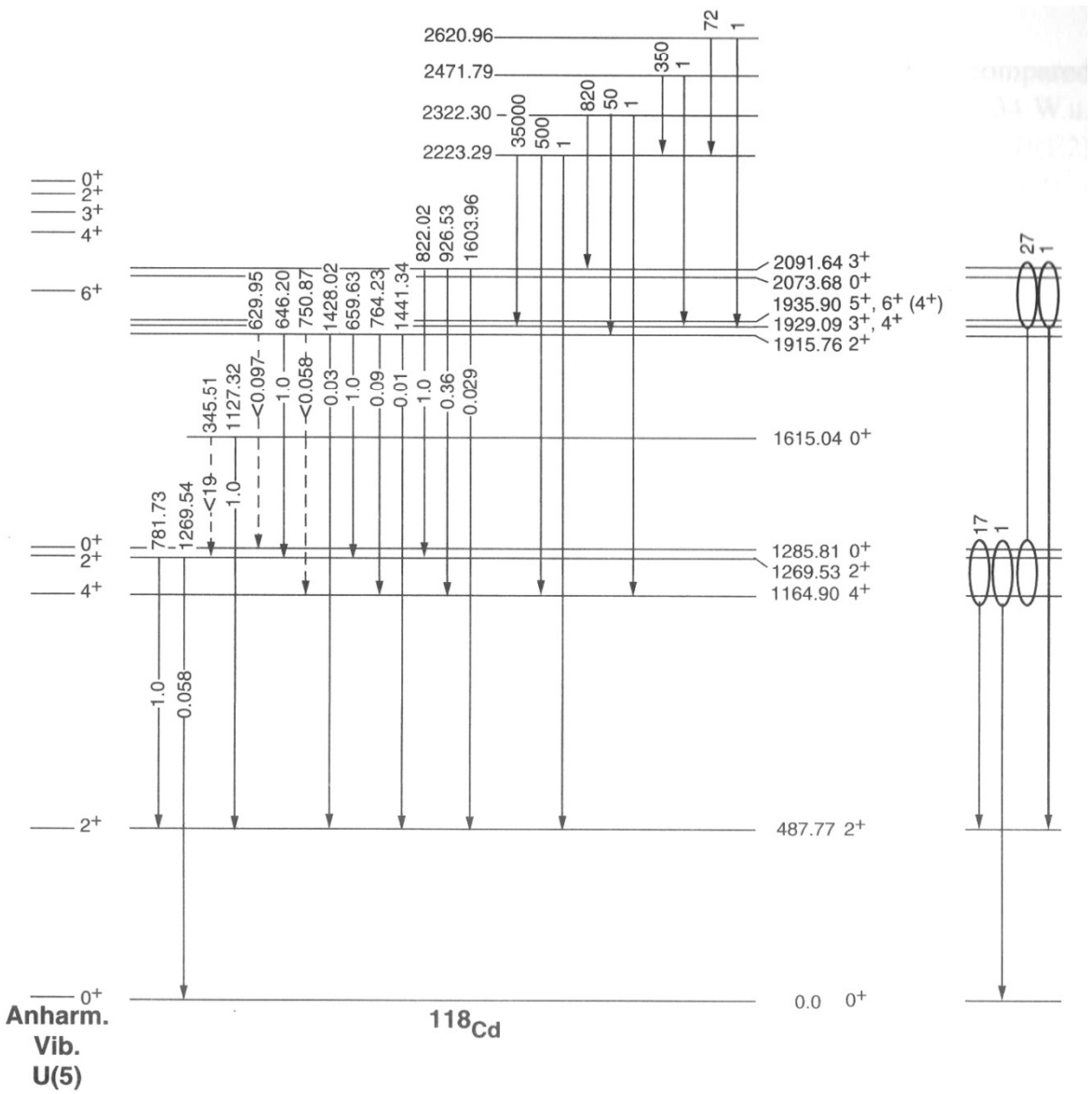
Vibrator

Vibrator (H.O.)

$E(I) = n\,(\hbar\,\omega_0)$

$R_{4/2} = 2.0$




2620.96
2471.79
2322.30
2223.29
72
1
350
1
820
50
1
35000
500
1
822.02
926.53
1603.96
1441.34
764.23
659.63
1428.02
750.87
646.20
629.95
345.51
1127.32
781.73
1269.54
0+
2+
3+
4+
6+
0+
2+
4+
2+
0+
2091.64 3+
2073.64 0+
1935.90 5+, 6+ (4+)
1929.09 3+, 4+
1915.76 2+
1615.04 0+
1285.81 0+
1269.53 2+
1164.90 4+
487.77 2+
0.0 0+
<0.097
1.0
<0.058
0.03
1.0
0.09
0.01
1.0
0.36
0.029
<19
1.0
1.0
0.058
27
1
17
1
Anharm.
Vib.
U(5)
118Cd

Deformed nuclei

Use the same Hamiltonian but re-write it in more convenient and physically intuitive form

IBA Hamiltonian

$$H = \varepsilon' n_d + \frac{1}{2}\sum_l C_l\,(d^\dagger d^\dagger)^{(l)}\cdot(\tilde{d}\tilde{d})^{(l)}$$
$$+\frac{v_2}{\sqrt{10}}\left[(d^\dagger d^\dagger)^{(2)}\cdot\tilde{d}s + H.c.\right] + \frac{v_0}{\sqrt{5}}\left(d^{\dagger 2}s^2 + H.c.\right)$$

Complicated and, for many calculations, not really necessary to use all these terms and all 6 parameters

Truncated form with just two parameters – RE-GROUP and keep some of the terms above.

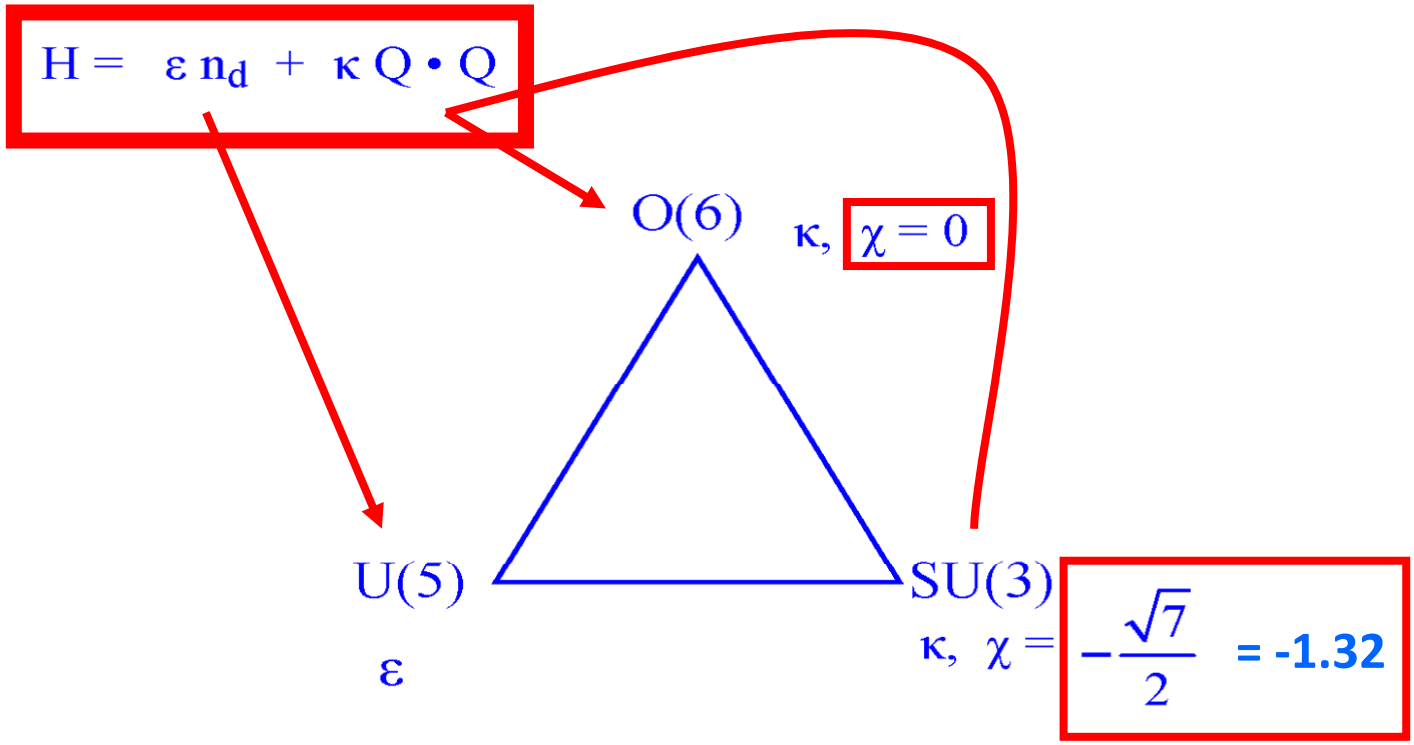
$$H = \varepsilon\, n_d - \kappa\, Q\cdot Q \qquad Q = e[s^\dagger\tilde{d} + d^\dagger s + \chi\,(d^\dagger\tilde{d})^{(2)}]$$

Competition:

$\varepsilon\, n_d$	term gives vibrator.
$\kappa\, Q\cdot Q$	term gives deformed nuclei.

More complicated forms exist but this is the form we will use. It works extremely well in most cases.

Relation of IBA Hamiltonian to Group Structure



We will see later that this same Hamiltonian allows us to calculate the properties of a nucleus ANYWHERE in the triangle simply by choosing appropriate values of the parameters

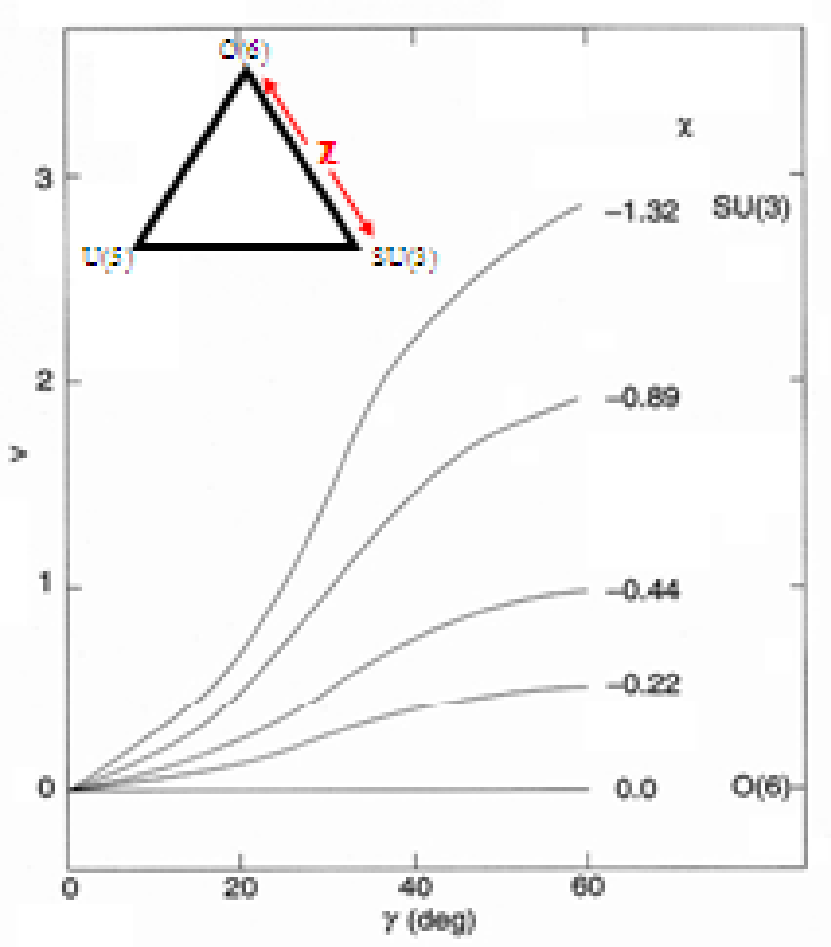
What is the physical meaning of χ

Only minimum is at $\gamma = 0°$

All γ excursions due to dynamical fluctuations in γ (γ-softness), not to rigid asymmetric shapes. This is confirmed experimentally !!!

$V(\gamma)$ vs. χ

$H = -\kappa\, Q \cdot Q$



If you think about zero point motion in a potential like this, it is clear that

<γ> depends on χ. For a flat potential the nucleus oscillates back and forth from 0 to 60 degrees so <γ> = 30 deg. For SU(3), <γ> will be small – nucleus is axially symmetric.

In a region of increasing γ softness, can simulate simply by decreasing $|\chi|$ towards zero

SU(3)

Deformed nuclei

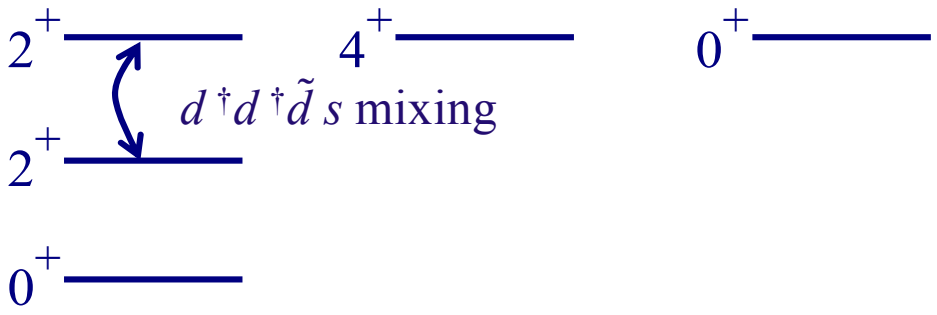
Wave functions in SU(3): Consider non-diagonal effects of the QQ term in H on n_d components in the wave functions

Q operator: $Q = (s^\dagger d + d^\dagger s) + \frac{\sqrt{7}}{2}(d^\dagger d)^{(2)}$

$QQ = [\{(s^\dagger d + d^\dagger s) + \chi(d^\dagger d)^{(2)}\} \times \{(s^\dagger d + d^\dagger s) + \frac{\sqrt{7}}{2}(d^\dagger d)^{(2)}\}]$

$\sim \quad s^\dagger d\, s^\dagger d + s^\dagger d\, d^\dagger s + s^\dagger d\, d^\dagger d \ldots$

$\Delta n_d = \quad -2 \qquad 0 \qquad -1 \qquad \ldots \quad 2,\ 0,\ 1$



Any calculation deviating from U(5) gives wave functions where n_d is no longer a good quantum number. If the wave function is expressed in a U(5) – vibrator – basis, then it contains a mixture of terms. Understanding these admixtures is crucial to understanding IBA calculations

SU(3) Rotor
(specific kind)

Need Quad interaction

$$H = -\kappa\, Q \bullet Q$$

States of SU(3) are labeled by 3 q. #'s: λ, μ, J

SU(3): U(6) ⊃ SU(3) ⊃ O(3)

N (λ, μ) J M

Major families of levels labeled by (λ,μ)

States within families labeled by J "K"

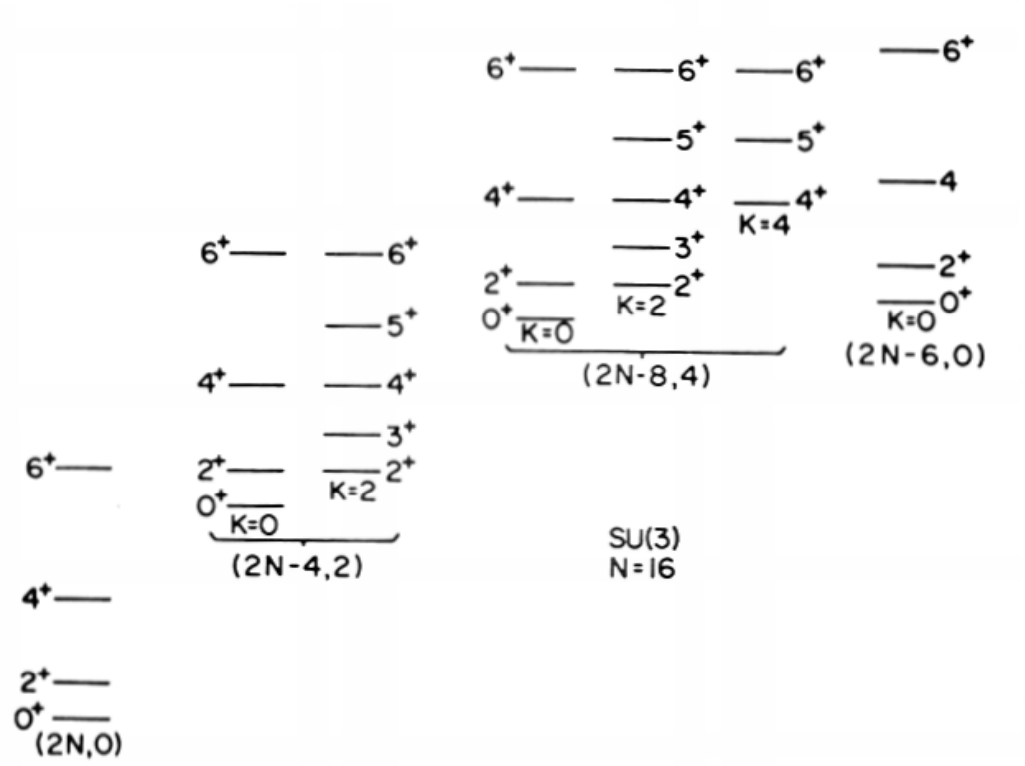
λ: 2N, 2N – X, ... $\lambda_{max} = 2N$

μ: 0, 2, 4, ... $\mu_{min} = 0$

$\mu_{max} = \lambda$

μ even only

Typical SU(3) Scheme



Characteristic signatures:

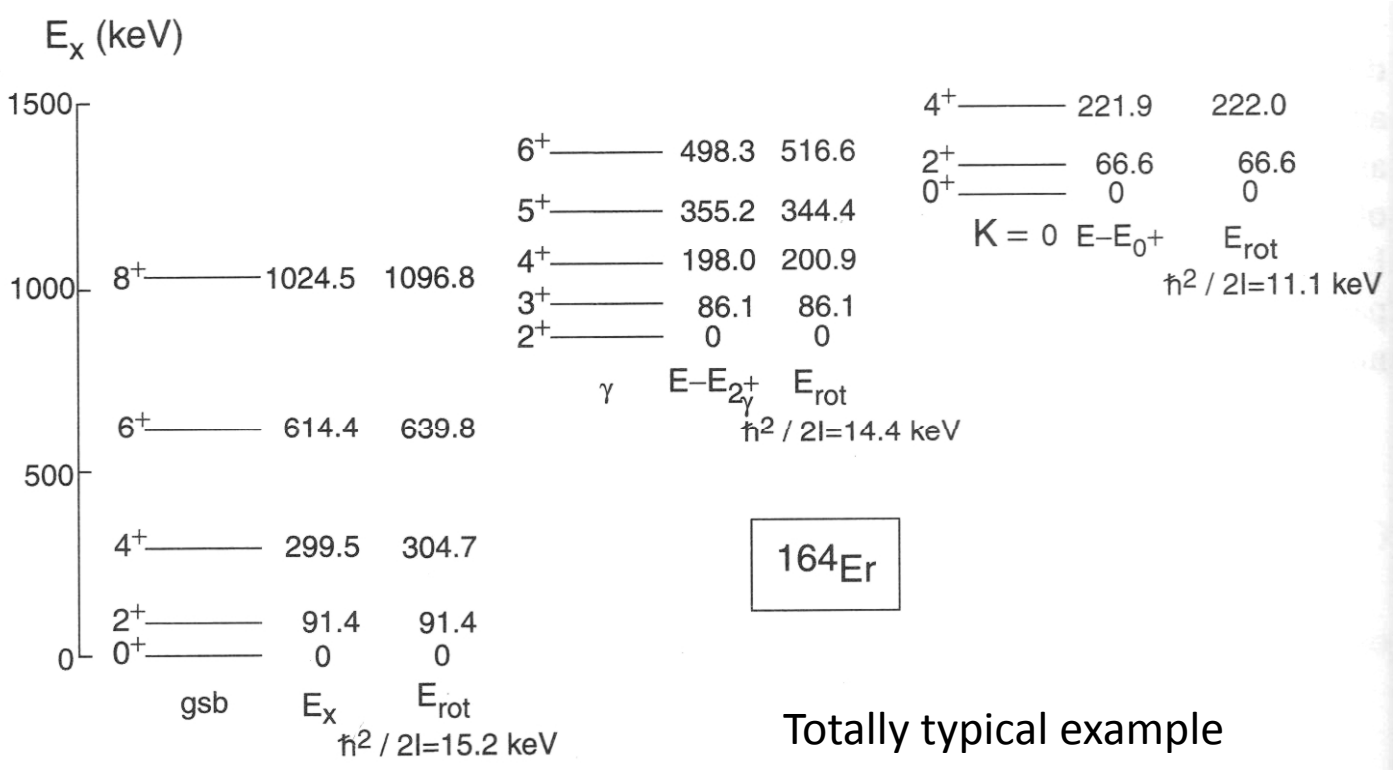
- **Degenerate bands within a group**
- **Vanishing B(E2) values between groups**
- **Allowed transitions between bands within a group**

Where? N~ 104, Yb, Hf

$$E(\lambda,\mu,J) = A[\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda+\mu)] + BJ(J+1)$$

SU(3) ⊃ **O(3)**

***K* bands in (●, ❍) :** $K = 0, 2, 4, - - - -$ ❍



Similar in many ways to SU(3).

But note that the two excited excitations are not degenerate as they should be in SU(3). While SU(3) describes an axially symmetric rotor, not all rotors are described by SU(3) – see later discussion

O(6)

Axially asymmetric nuclei
(gamma-soft)

O(6) γ-soft Rotor

$$U(6) \supset O(6) \supset O(5) \supset O(3)$$
$$N \qquad \sigma \qquad \tau, \nu \qquad J$$

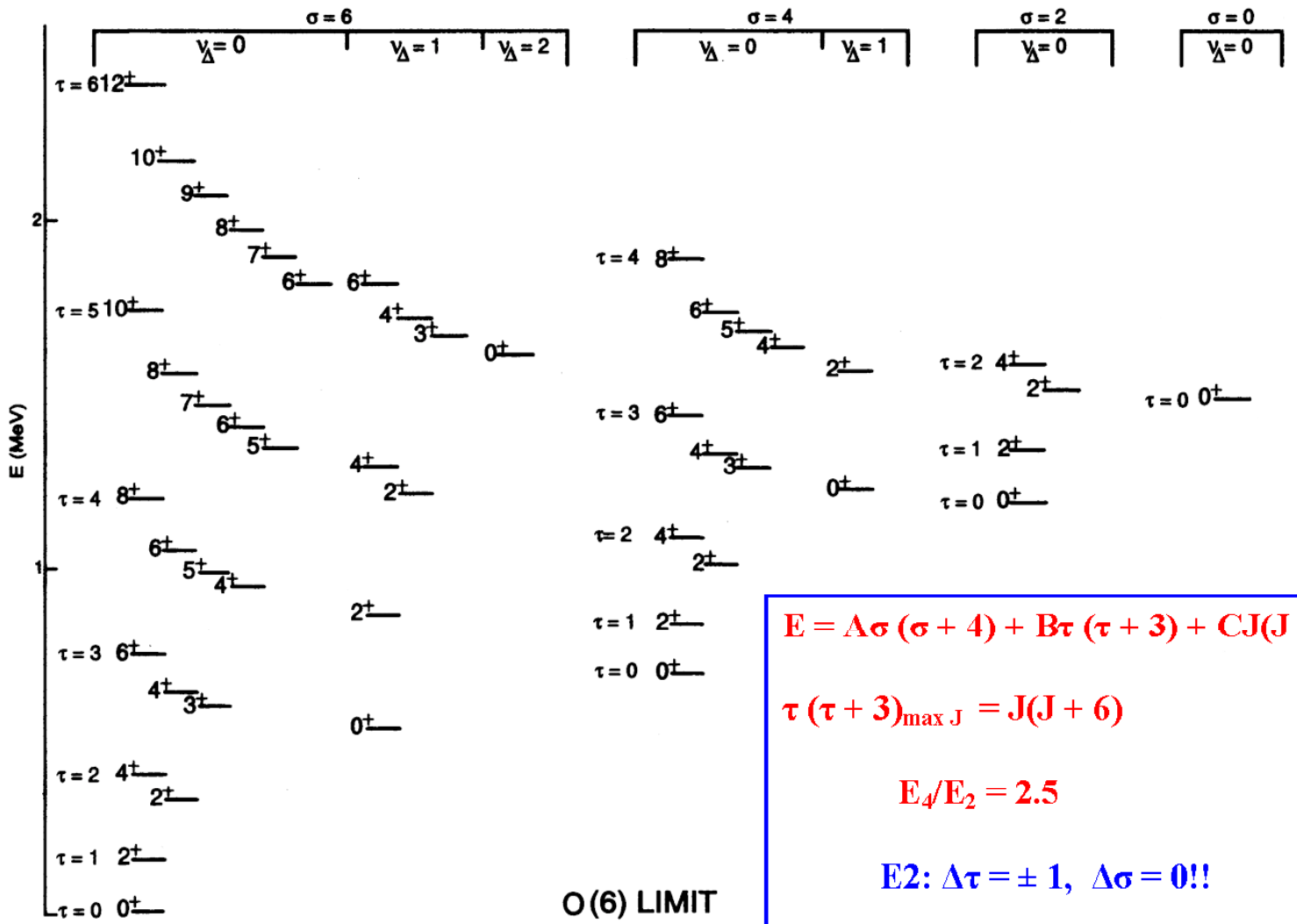
$$E(J) = A\sigma(\sigma + 4) + B\tau(\tau + 3) + CJ(J + 1)$$

major families — "phonons" — J

When proposed (1977-78) this was a new symmetry, not recognized experimentally, that was predicted by the IBA

^{196}Pt

O(6) Symmetry



$$E = A\sigma(\sigma + 4) + B\tau(\tau + 3) + CJ(J + 1)$$

$$\tau(\tau + 3)_{\max J} = J(J + 6)$$

$$E_4/E_2 = 2.5$$

E2: $\Delta\tau = \pm 1$, $\Delta\sigma = 0$!!

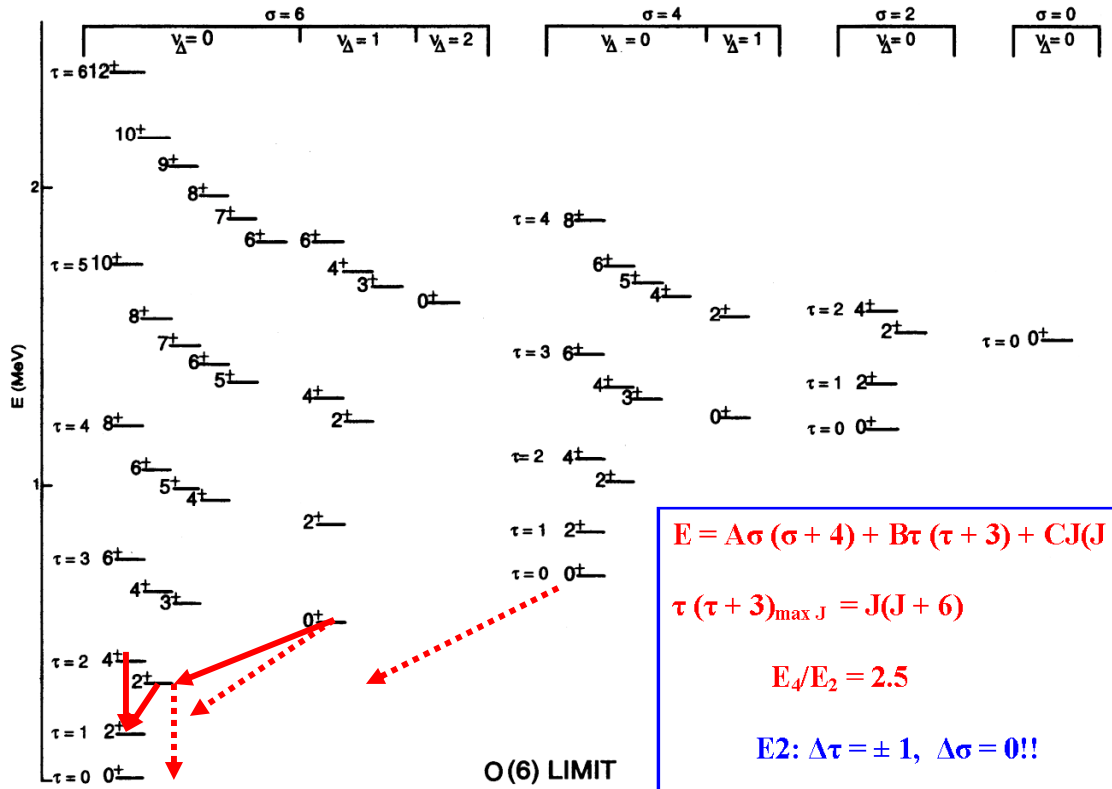
Transition Rates

$T(E2) = e_B Q$ $\qquad$ $Q = (s^\dagger \tilde{d} + d^\dagger s)$

Note: Uses $\chi = 0$

O(6) $\qquad$ E2 $\quad \Delta\sigma = 0 \quad \Delta\tau = 1$

O(6) Symmetry



$E = A\sigma(\sigma + 4) + B\tau(\tau + 3) + CJ(J + 1)$

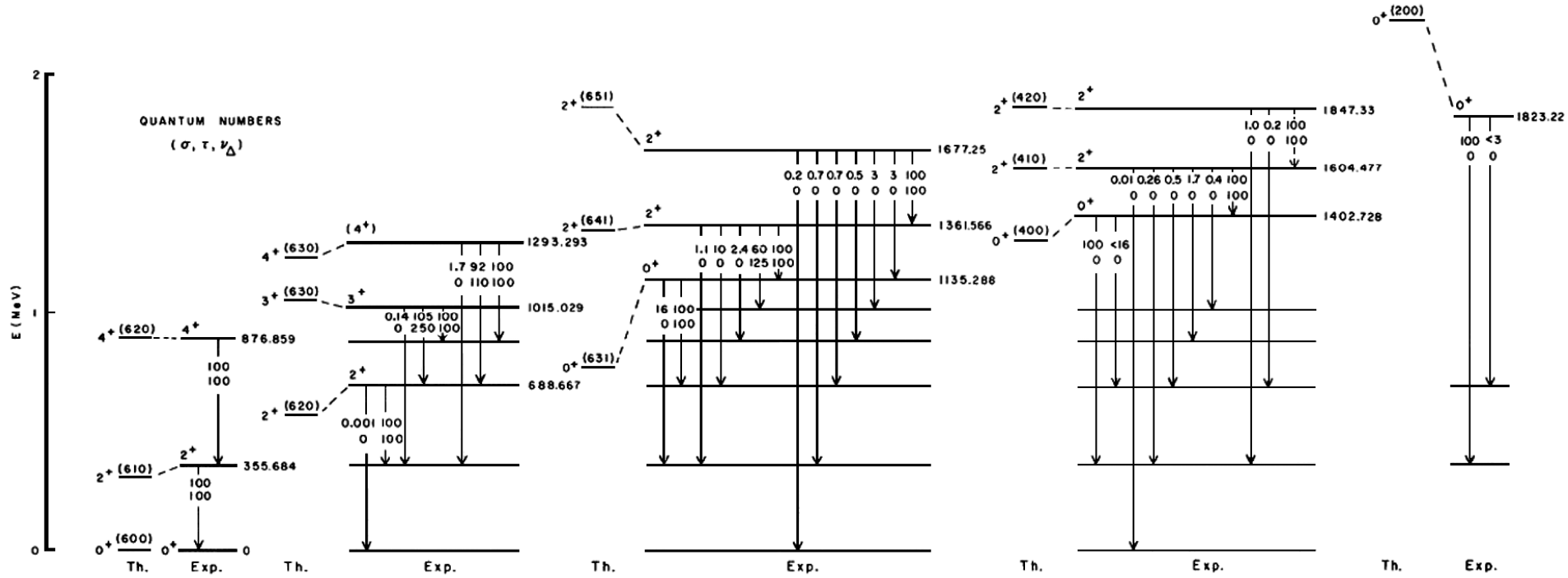
$\tau(\tau + 3)_{\max J} = J(J + 6)$

$E_4/E_2 = 2.5$

E2: $\Delta\tau = \pm 1$, $\Delta\sigma = 0$!!

^{196}Pt: Best (first) O(6) nucleus

γ-soft

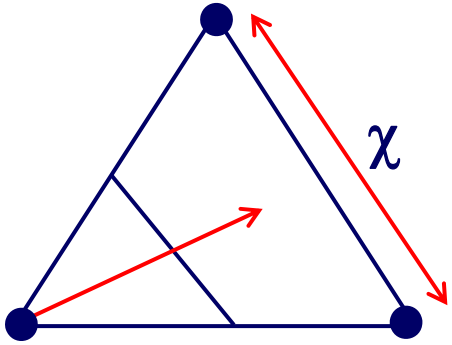


More General IBA calculations

Thus far, we have only dealt with nuclei corresponding to one of the three dynamical symmetries. Probably <1% of nuclei do that.

So, how do we treat the others? That is, how do we calculate with the IBA AWAY from the vertices of the symmetry triangle?

A couple of interesting examples first, then a general approach -- - The technique of Orthogonal Crossing Contours (OCC)



CQF along the O(6) – SU(3) leg

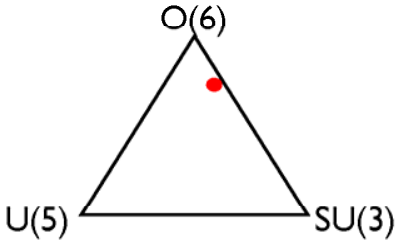
$H = -\kappa\, Q \bullet Q$

Only a single parameter, χ

We will discuss 1) and 4). The others are included in the slides in the Appendix

Examples

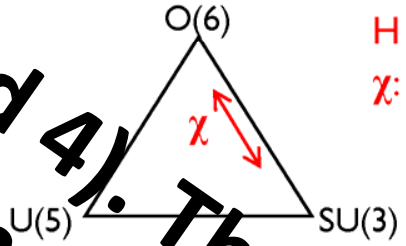
1) Well deformed nucleus – ^{168}Er



$H \sim a_2\, Q \bullet Q$ χ between 0 and $-\sqrt{7}/2$

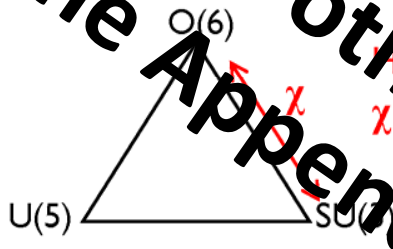
Calculations with $\chi \sim -0.4$ work well.
Fix a_2 from $E(2_1^+)$

2) Os isotopes O(6) → SU(3)



$H = \sim a_2\, Q \bullet Q$
χ: 0 to ~ -0.4

3) Universal Calculations O(6) → SU(3)



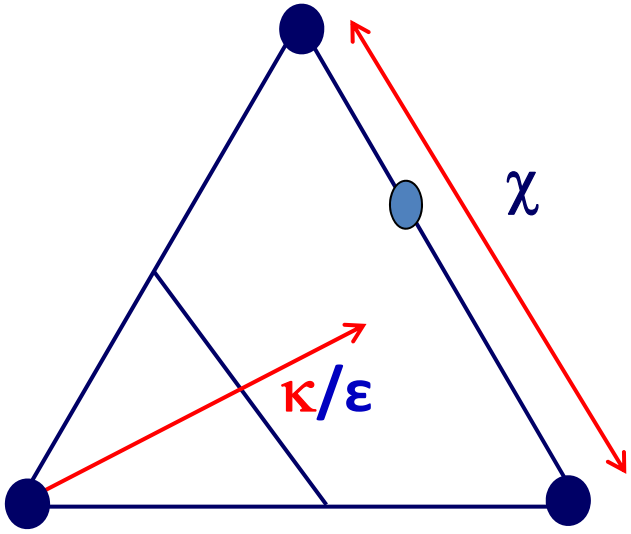
$H \sim a_2\, Q \bullet Q$
χ: 0 to -1.32

$H = \varepsilon\, n_d - \kappa\, Q \cdot Q$
Two parameters
ε/κ and χ

4) Mapping the triangle.
Technique of Orthogonal Crossing Contours

168-Er very simple 1-parameter calculation

$$H = \varepsilon n_d - \kappa Q \cdot Q$$

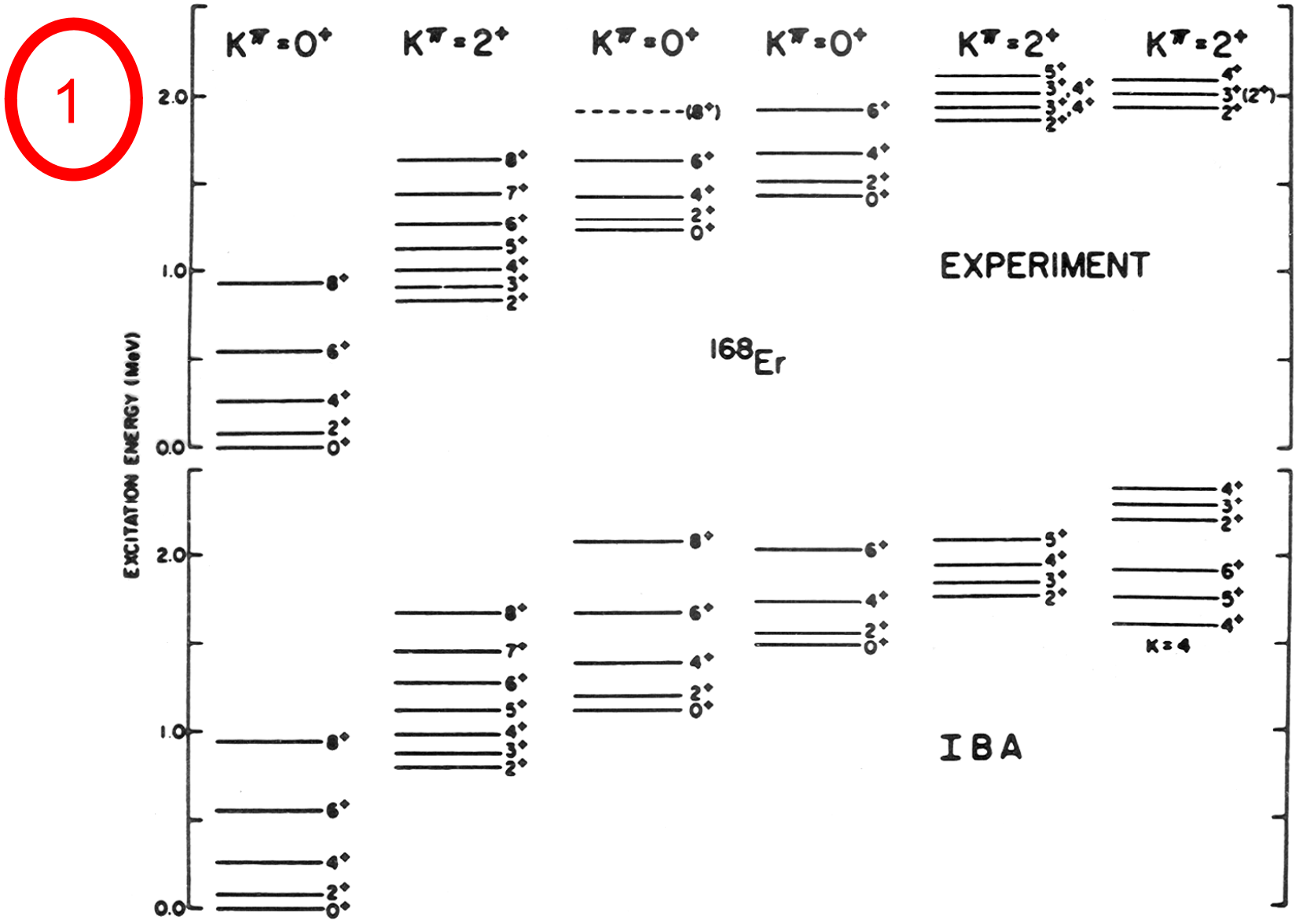


$$\varepsilon = 0$$

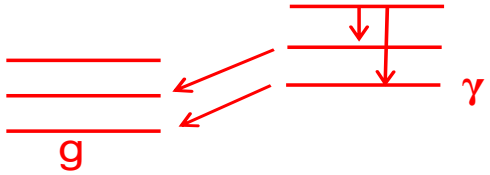
$$H = -\kappa Q \cdot Q$$

κ is just scale factor

So, only parameter is χ


1
$K^\pi = 0^+$
$K^\pi = 2^+$
$K^\pi = 0^+$
$K^\pi = 0^+$
$K^\pi = 2^+$
$K^\pi = 2^+$
EXCITATION ENERGY (MeV)
2.0
1.0
0.0
^{168}Er
EXPERIMENT
IBA
K=4

IBA_{CQF} Predictions for ^{168}Er



J_i	J_f, K_f	Alaga	Exp	IBA(CQF)
2	0,0	70	54.0	54
	2,0	100	100	100
	4,0	5	6.8	7.6
3	2,0	2.6	2.6	2.6
	4,0	1.0	1.7	1.8
	2,2		100	100
4	2,0	2.7	1.6	1.7
	4,0	8.1	8.1	9.6
	6,0	0.8	1.1	1.5
	2,2		100	100
5	4,0	2.9	2.9	3.5
	6,0	1.5	3.6	4.4
	3,2		100	100
	4,2		122	95
6	4,0	1.0	0.44	0.44
	6,0	3.8	3.8	4.9
	8,0	0.4	1.4	1.0
	4,2		100	100
	5,2		69	57
7	6,0		0.7	1.9
	5,2		100	100
	6,2		59	36

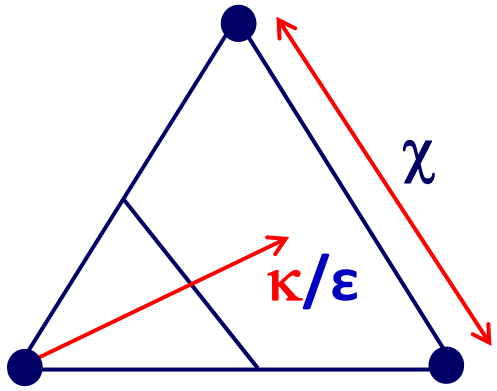
Mapping the Entire Triangle

$$H = \varepsilon n_d - \kappa Q \cdot Q$$

Parameters: κ/ε , χ **(within Q)**

2 parameters

2-D surface



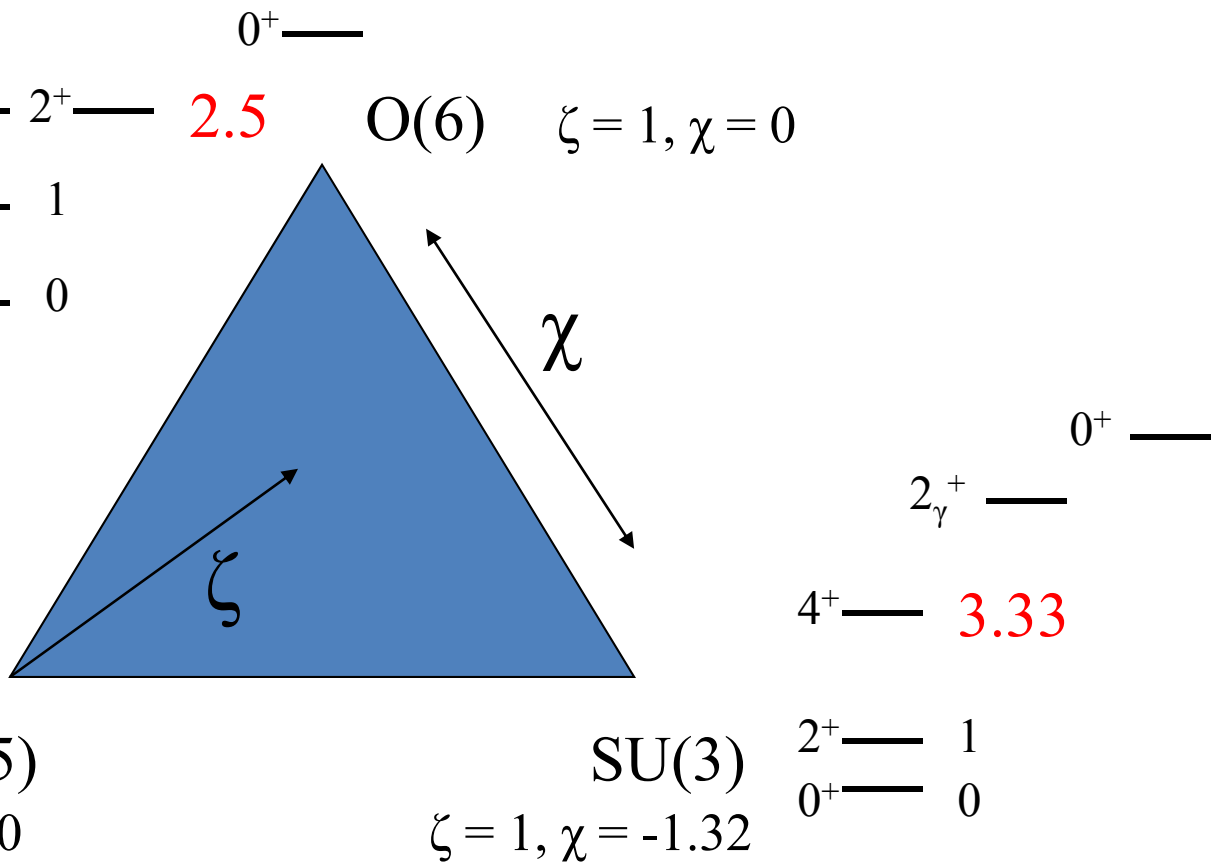
Problem: κ/ε **varies from zero to infinity: Awkward.**

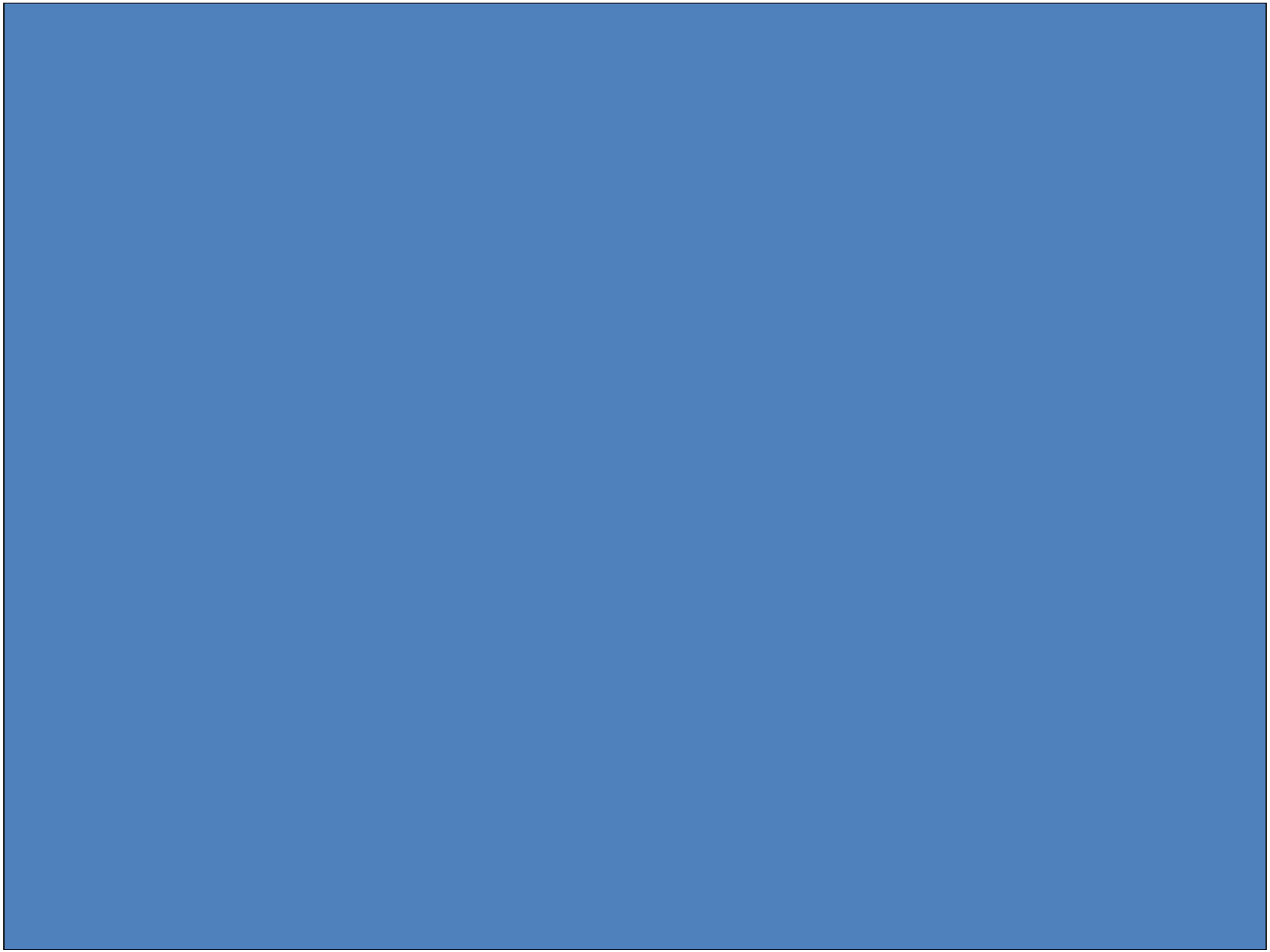
So, introduce a simple change of variables

Spanning the Triangle

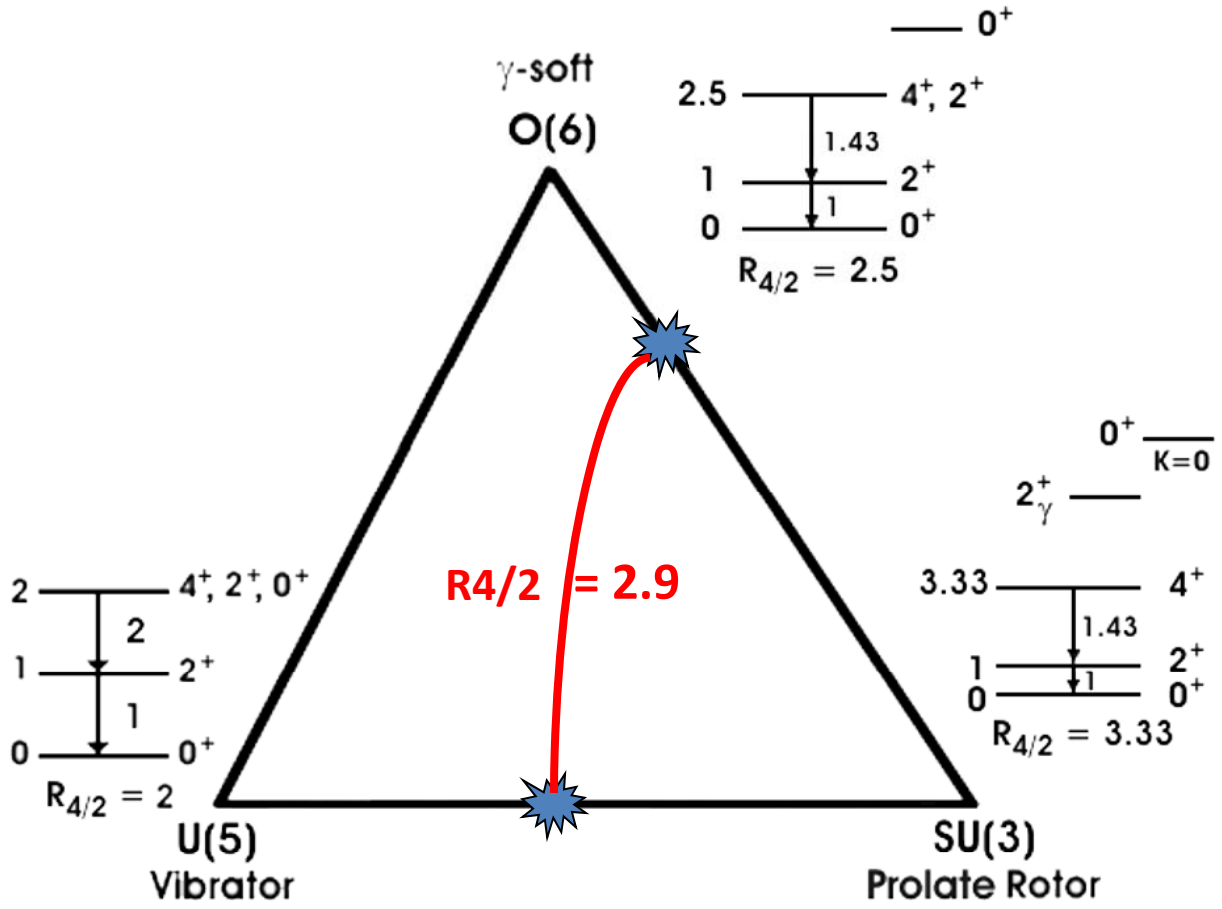
$$H = c \left[\quad \right] - \left[\quad \right]$$

$$\varepsilon/\kappa' = 4N \frac{1-\zeta}{\zeta}$$





H has two parameters. A given observable can only specify one of them. What does this imply? An observable gives a contour of constant values within the triangle



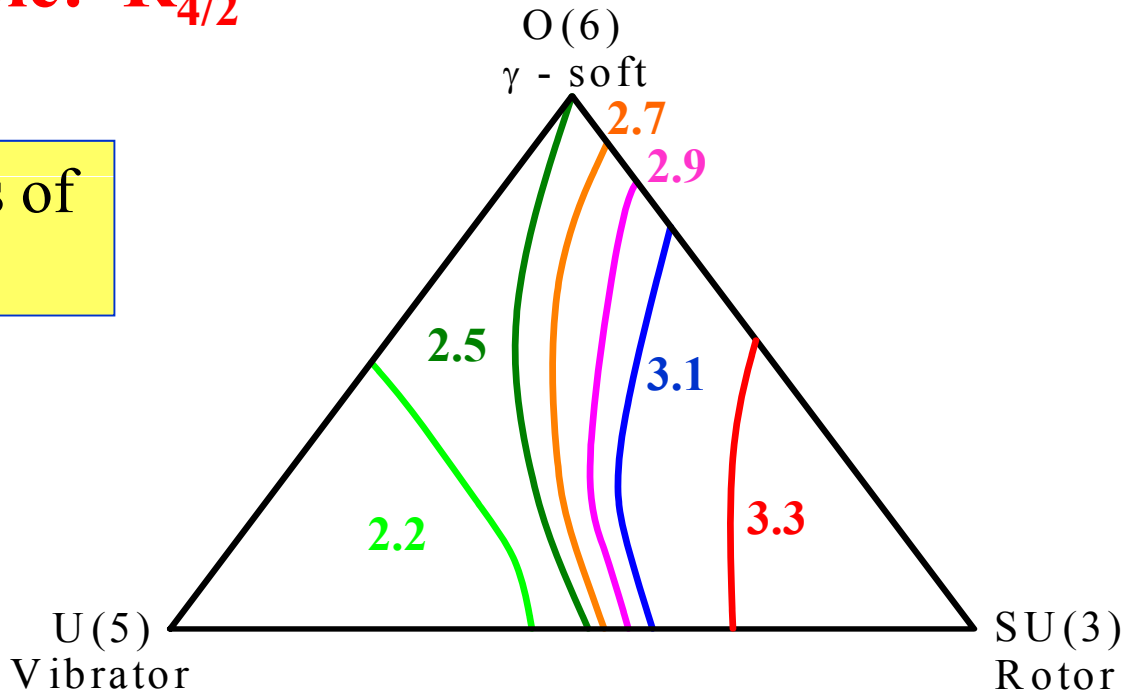
A simple way to pinpoint structure.

Technique of Orthogonal Crossing Contours (OCC)

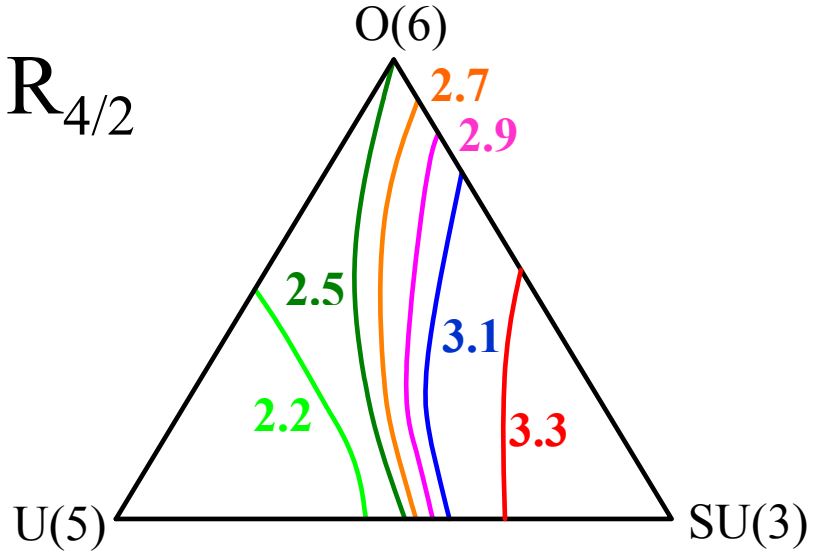
- At the basic level : 2 observables (to map any point in the symmetry triangle)
- Preferably with perpendicular trajectories in the triangle

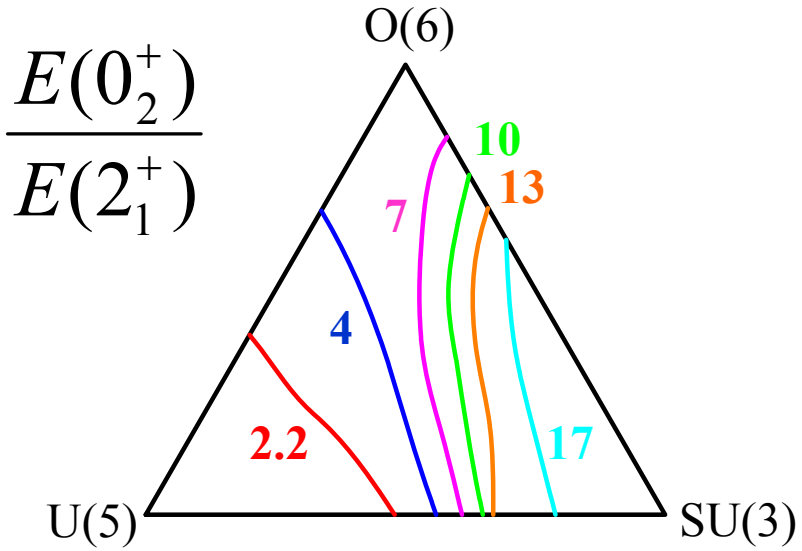
Simplest Observable: $R_{4/2}$

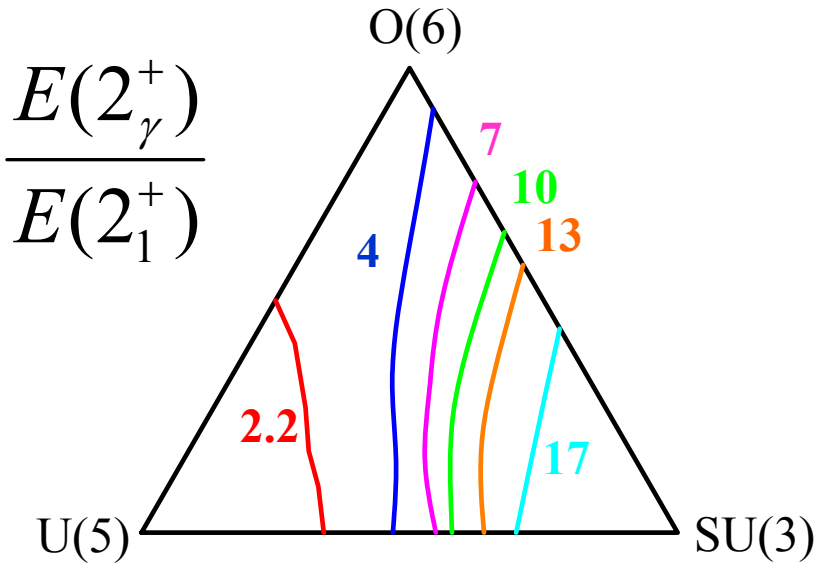
Only provides a locus of structure

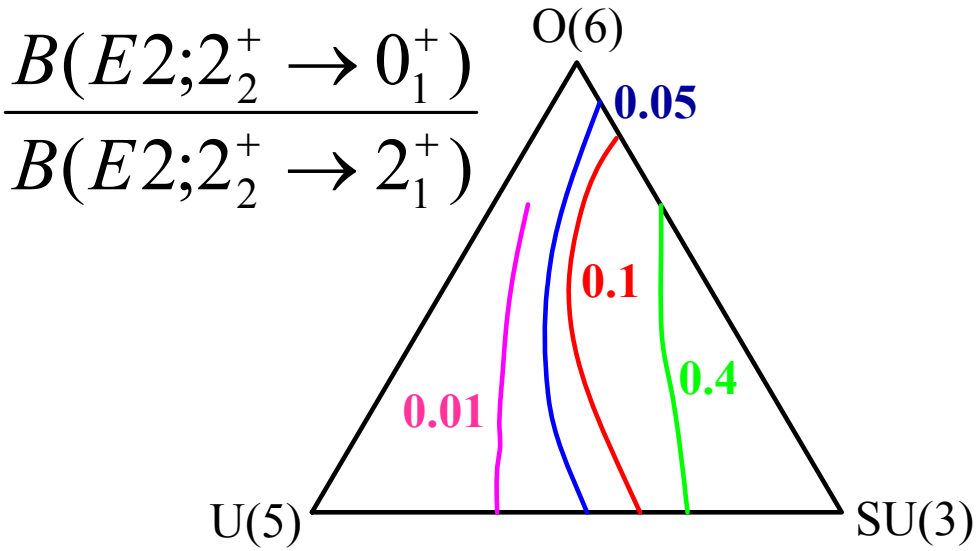


Contour Plots in the Triangle





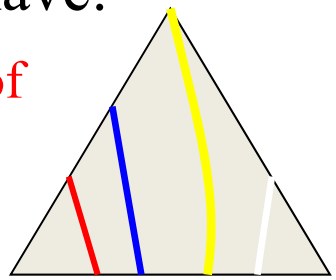




We have a problem

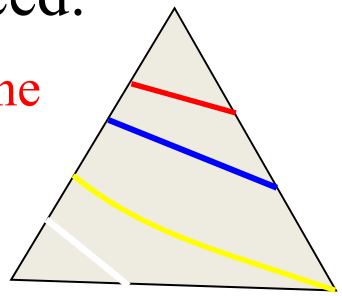
What we have:

Lots of

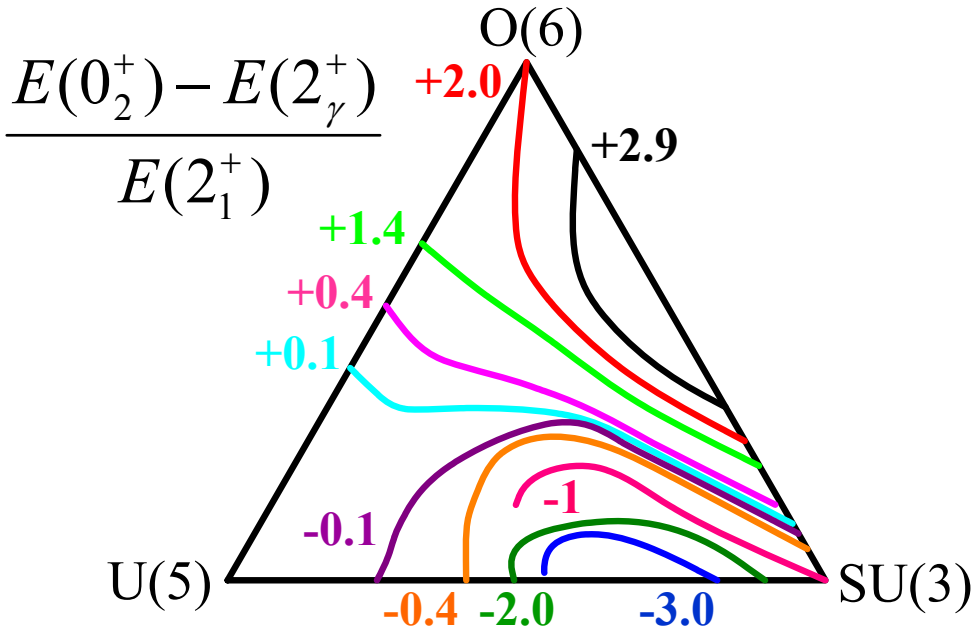


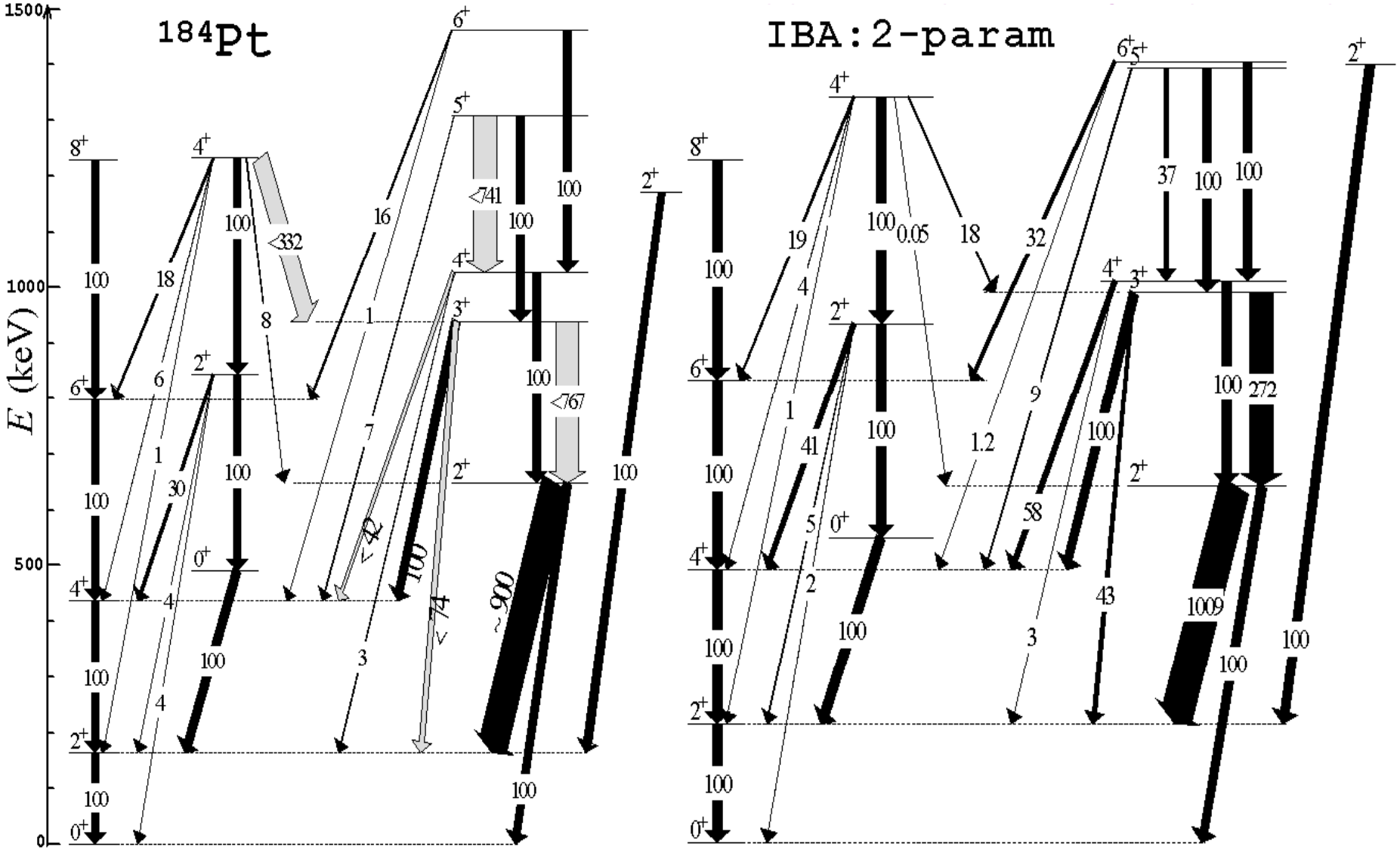
What we need:

Just one



Fortunately:

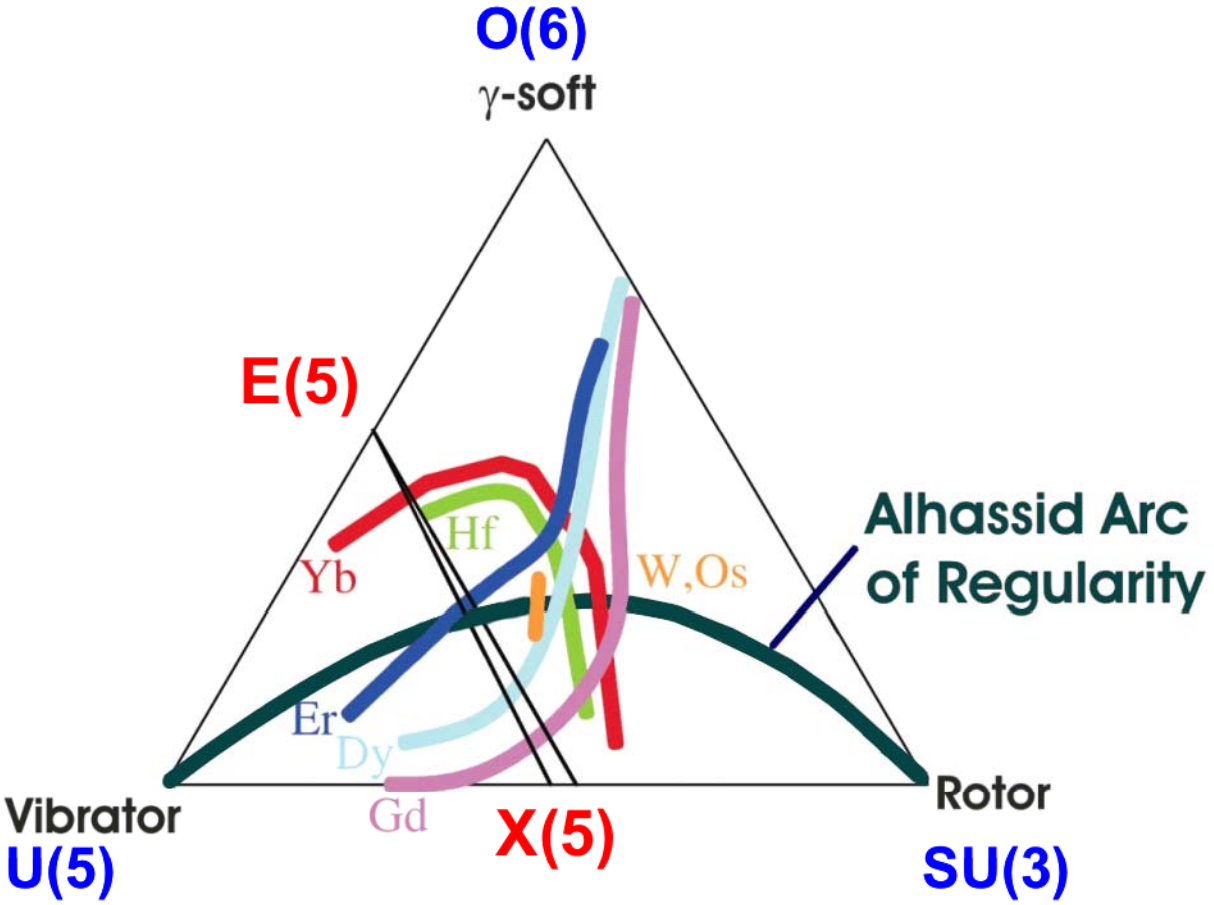




Burcu Cakirli *et al.*

Beta decay exp. + IBA calcs.

Evolution of Structure



Complementarity of macroscopic and microscopic approaches. Why do certain nuclei exhibit specific symmetries? Why these evolutionary trajectories?

What will happen far from stability in regions of proton-neutron asymmetry and/or weak binding?

Special Thanks to:

- **Franco Iachello**
- **Akito Arima**
- **Igal Talmi**
- **Dave Warner**
- **Peter von Brentano**
- **Victor Zamfir**
- **Jolie Cizewski**
- **Hans Borner**
- **Jan Jolie**
- **Burcu Cakirli**
- **Piet Van Isacker**
- **Kris Heyde**
- **Many others**

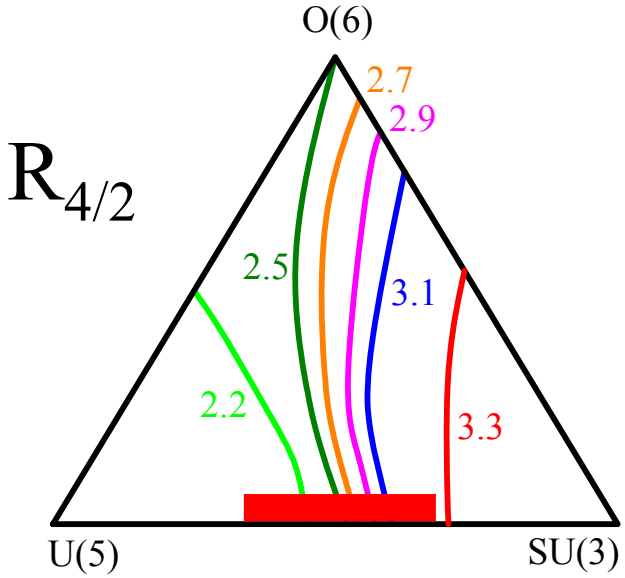
Appendix:

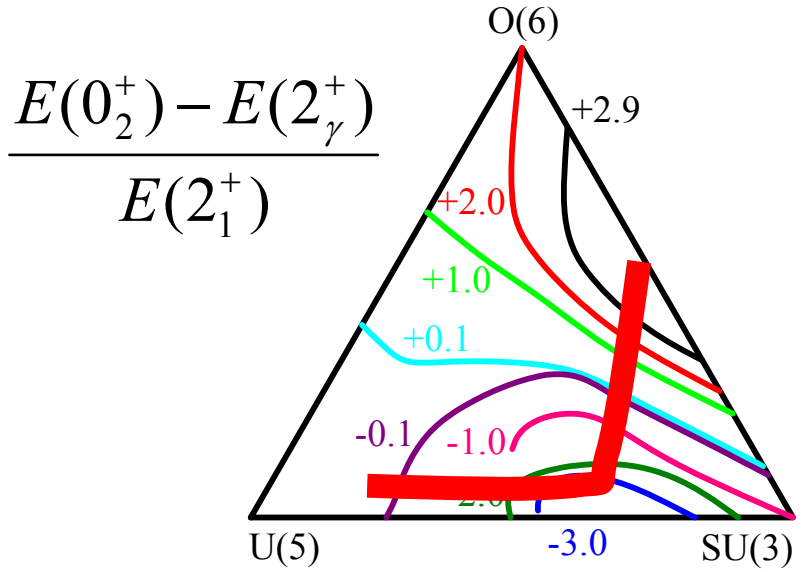
Trajectories-by-eye

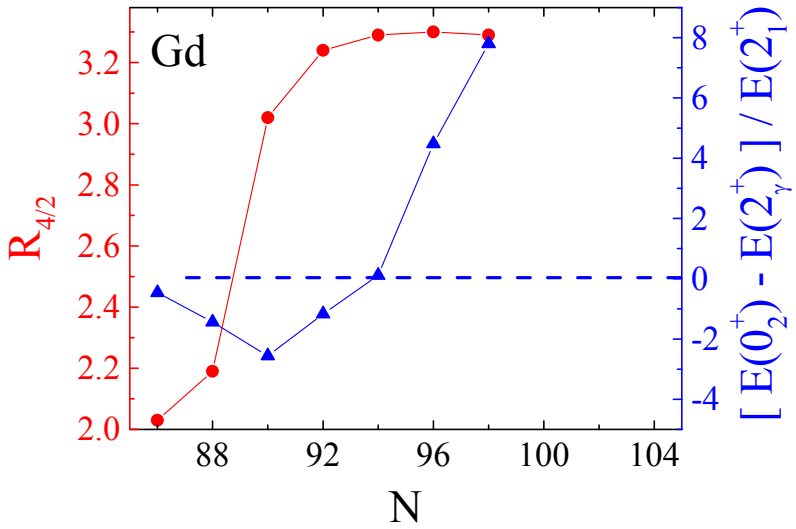
Running the IBA program using the Titan computer at Yale

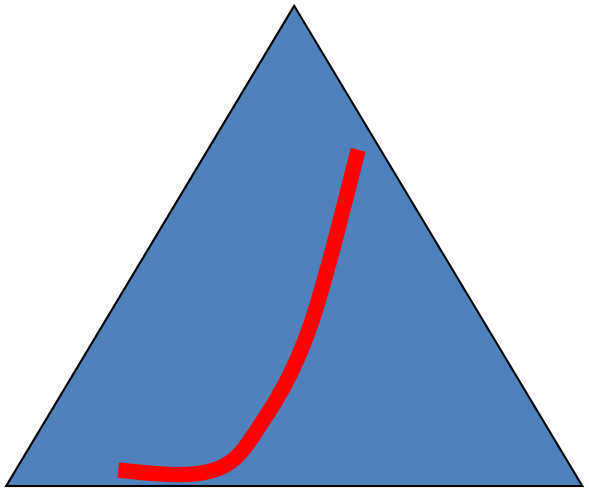
Examples “2” and “3” skipped earlier of the use of the CQF form of the IBA

Trajectories at a Glance









Nuclear Model Codes at Yale
Computer name: Titan

Connecting to SSH: Quick connect

Host name: titan.physics.yale.edu

User name: phy664

Port Number 22

Password: nuclear_codes

---

cd phintm

pico filename.in (ctrl x, yes, return)

runphintm filename (w/o extension)

pico filename.out (ctrl x, return)

U(5)

Input

```
 $diag
 eps = 0.20, kappa = 0.00, chi =-0.00,
 nphmax = 6,
 iai = 0, iam = 6, neig = 3,
 mult=.t.,ell=0.0,pair=0.0,oct=0.0,ippm=1,print=.t.
$
$em
 E2SD=1.0, E2DD=-0.00
$
SLCT 2 2+  0+  2

99999
```

Output

```
----------------------------

L P =  0+

            Basis vectors
              |NR> = |ND,NB,NC,LD,NF,L P>
             ----------------------------
             | 1> = | 0, 0, 0, 0, 0, 0+>
             | 2> = | 2, 1, 0, 0, 0, 0+>
             | 3> = | 3, 0, 1, 0, 0, 0+>
             | 4> = | 4, 2, 0, 0, 0, 0+>
             | 5> = | 5, 1, 1, 0, 0, 0+>
             | 6> = | 6, 0, 2, 0, 0, 0+>
             | 7> = | 6, 3, 0, 0, 0, 0+>

  Energies
    0.0000   0.4000   0.6000   0.8000   1.0000   1.2000   1.2000

  Eigenvectors

    1:   1.000  0.000  0.000
    2:   0.000  1.000  0.000
    3:   0.000  0.000  1.000
    4:   0.000  0.000  0.000
    5:   0.000  0.000  0.000
    6:   0.000  0.000  0.000
    7:   0.000  0.000  0.000
```

Basis

Energies

Pert. Wave Fcts.

```
----------------------------
L P =  1+
No states
----------------------------
L P =  2+

  Energies
    0.2000   0.4000   0.6000   0.8000   0.8000   1.0000   1.0000   1.2000   1.2000
----------------------------
L P =  3+

  Energies
    0.6000   1.0000   1.2000
----------------------------
L P =  4+

 Energies
    0.4000   0.6000   0.8000   0.8000   1.0000   1.0000   1.2000   1.2000   1.2000
----------------------------
L P =  5+

  Energies
    0.8000   1.0000   1.2000
----------------------------
L P =  6+

  Energies
    0.6000   0.8000   1.0000   1.0000   1.2000   1.2000   1.2000
----------------------------

Transitions:  2+ ->  0+  (BE2)
 2+,1 -> 0+,1:  6.00000  2+,1 -> 0+,2:  2.00000  2+,1 -> 0+,3:  0.00000
 2+,2 -> 0+,1:  0.00000  2+,2 -> 0+,2:  0.00000  2+,2 -> 0+,3:  2.40000
 2+,3 -> 0+,1:  0.00000  2+,3 -> 0+,2:  5.60000  2+,3 -> 0+,3:  0.00000
  and  0+ ->  2+  (BE2)
 0+,1 -> 2+,1: 30.00000  0+,2 -> 2+,1: 10.00000  0+,3 -> 2+,1:  0.00000
 0+,1 -> 2+,2:  0.00000  0+,2 -> 2+,2:  0.00000  0+,3 -> 2+,2: 12.00000
 0+,1 -> 2+,3:  0.00000  0+,2 -> 2+,3: 28.00000  0+,3 -> 2+,3:  0.00000

Transitions:  4+ ->  2+  (BE2)
 4+,1 -> 2+,1: 10.00000  4+,1 -> 2+,2:  0.00000  4+,1 -> 2+,3:  2.28571
 4+,2 -> 2+,1:  0.00000  4+,2 -> 2+,2:  6.28571  4+,2 -> 2+,3:  0.00000
 4+,3 -> 2+,1:  0.00000  4+,3 -> 2+,2:  0.00000  4+,3 -> 2+,3:  3.85714
```

O(6)

Input

```
 $diag
 eps = 0.0, kappa = 0.02, chi =-0.0,
 nphmax = 6,
 iai = 0, iam = 6, neig = 5,
 mult=.t.,ell=0.0,pair=0.0,oct=0.0,ippm=1,print=.t.
$
$em
 E2SD=1.0, E2DD=-0.00
$
99999
```

Output

L P = 0+

Basis vectors

|NR> = |ND,NB,NC,L,D,NF,L P>

| 1> = | 0, 0, 0, 0, 0, 0+>
| 2> = | 2, 1, 0, 0, 0, 0+>
| 3> = | 3, 0, 1, 0, 0, 0+>
| 4> = | 4, 2, 0, 0, 0, 0+>
| 5> = | 5, 1, 1, 0, 0, 0+>
| 6> = | 6, 0, 2, 0, 0, 0+>
| 7> = | 6, 3, 0, 0, 0, 0+>

Basis

Energies

0.0000 0.3600 0.5600 0.9200 0.9600 1.0800 1.2000

Energies

Eigenvectors

1: -0.433 0.000 0.685 0.000 0.559
2: -0.750 0.000 0.079 0.000 -0.581
3: 0.000 -0.886 0.000 0.463 0.000
4: -0.491 0.000 -0.673 0.000 0.296
5: 0.000 -0.463 0.000 -0.886 0.000
6: 0.000 0.000 0.000 0.000 0.000
7: -0.094 0.000 -0.269 0.000 0.512

Pert. Wave Fcts.

L P = 1+

No states

L P = 2+

Energies

0.0800 0.2000 0.5600 0.6400 0.7600 0.8000 1.0400 1.1200 1.1600

L P = 3+

Energies

0.3600 0.9200 1.0800

L P = 4+

Energies

0.2000 0.3600 0.5600 0.7600 0.8000 0.9200 1.0800 1.1200 1.1600

L P = 5+

Energies

0.5600 0.8000 1.1200

L P = 6+

Energies

0.3600 0.5600 0.8000 0.9200 1.0800 1.0800 1.1200

Binding energy = -0.6000 , eps-eff = -0.1200

SU(3)

```
******************** Input file contents ********************

 $diag
 eps = 0.00, kappa = 0.02, chi =-1.3229,
 nphmax = 6,
 iai = 0, iam = 6, neig = 5,
 mult=.t.,ell=0.0,pair=0.0,oct=0.0,ippm=1,print=.t.
 $
 $em
 E2SD=1.0, E2DD=-2.598

 $
99999
************************************************************
```

L P = 0+

Basis vectors

|NR> = |ND,NB,NC,LD,NF,L P>

| 1> = | 0, 0, 0, 0, 0, 0+>
| 2> = | 2, 1, 0, 0, 0, 0+>
| 3> = | 3, 0, 1, 0, 0, 0+>
| 4> = | 4, 2, 0, 0, 0, 0+>
| 5> = | 5, 1, 1, 0, 0, 0+>
| 6> = | 6, 0, 2, 0, 0, 0+>
| 7> = | 6, 3, 0, 0, 0, 0+>

Energies

0.0000 0.6600 1.0800 1.2600 1.2600 1.5600 1.8000

Eigenvectors

1:	0.134	0.385	-0.524	-0.235	0.398
2:	0.463	0.600	-0.181	0.041	-0.069
3:	-0.404	-0.204	-0.554	-0.557	-0.308
4:	0.606	-0.175	0.030	-0.375	-0.616
5:	-0.422	0.456	-0.114	0.255	-0.432
6:	-0.078	0.146	-0.068	0.245	-0.415
7:	0.233	-0.437	-0.606	0.606	0.057

Wave fcts. in U(5) basis

L P = 1+

No states

L P = 2+

Energies

0.0450 0.7050 0.7050 1.1250 1.1250 1.3050 1.3050 1.6050

L P = 3+

Energies

0.7500 1.1700 1.6500

L P = 4+

Energies

0.1500 0.8100 0.8100 1.2300 1.2300 1.2300 1.4100 1.4100

L P = 5+

Energies

0.8850 1.3050 1.3050

L P = 6+

Energies

0.3150 0.9750 0.9750 1.3950 1.3950 1.5750 1.5750

Binding energy = -1.2000 , eps-eff = -0.1550

2

"Universal" IBA Calculations for the SU(3) – O(6) leg

$H = - \kappa\, Q \bullet Q$

κ is just energy scale factor

Ψ's, $B(E2)$'s independent of κ

Results depend **only** on χ
[and, of course, vary with N_B]

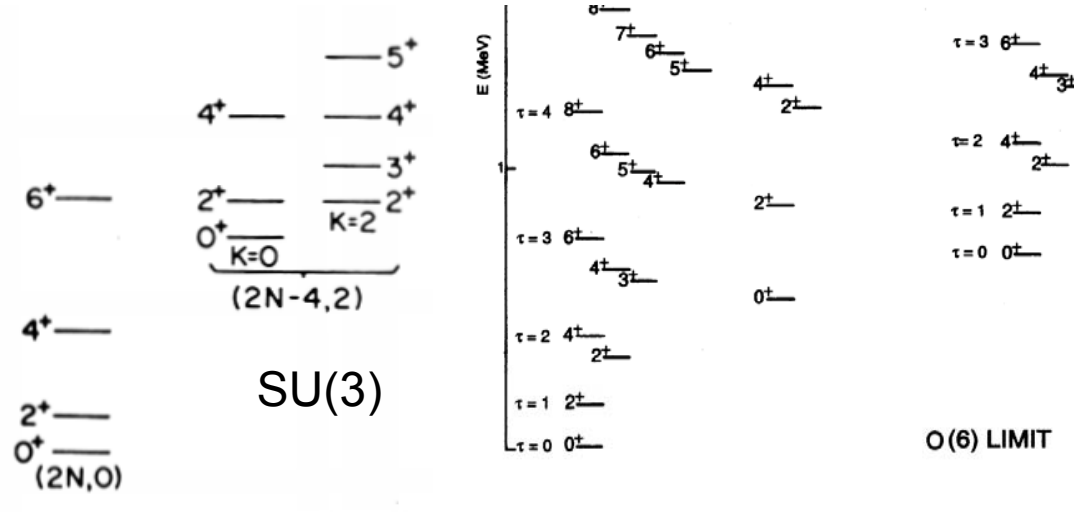
Can plot any observable as a set of contours vs. N_B and χ.

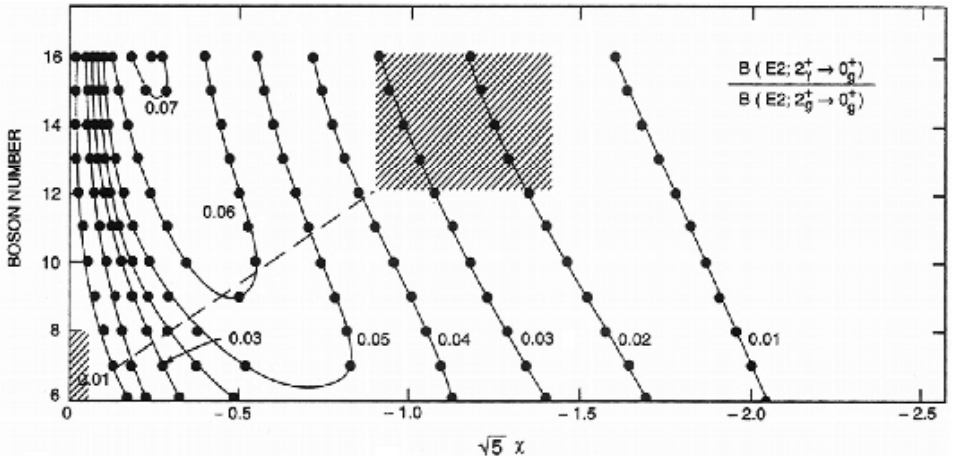
Universal O(6) – SU(3) Contour Plots

$$H = -\kappa\, Q \cdot Q$$

$\chi = 0$ O(6)

$\chi = -1.32$ SU(3)

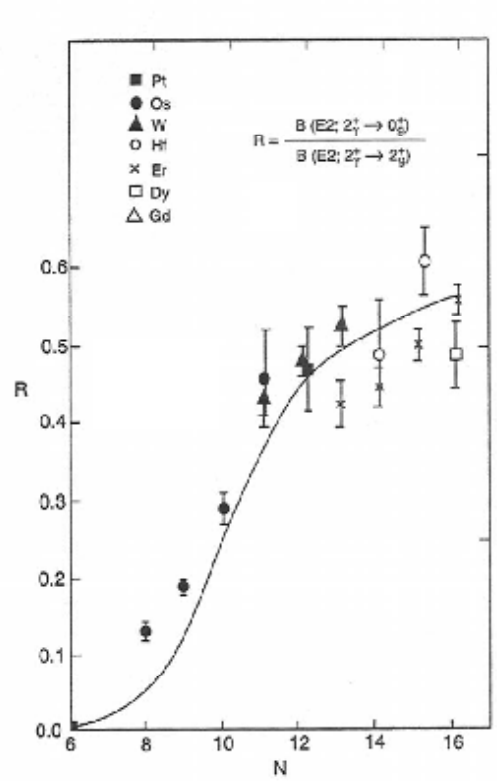


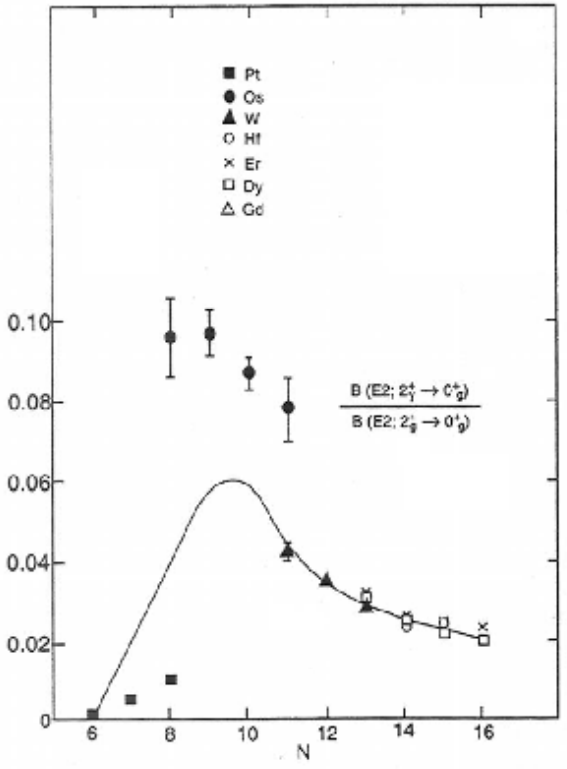


CQF

$$\frac{2_\gamma \rightarrow 0_g}{2_\gamma \rightarrow 2_g}$$

$$\frac{2_\gamma \rightarrow 0_g}{2_g \rightarrow 0_g}$$

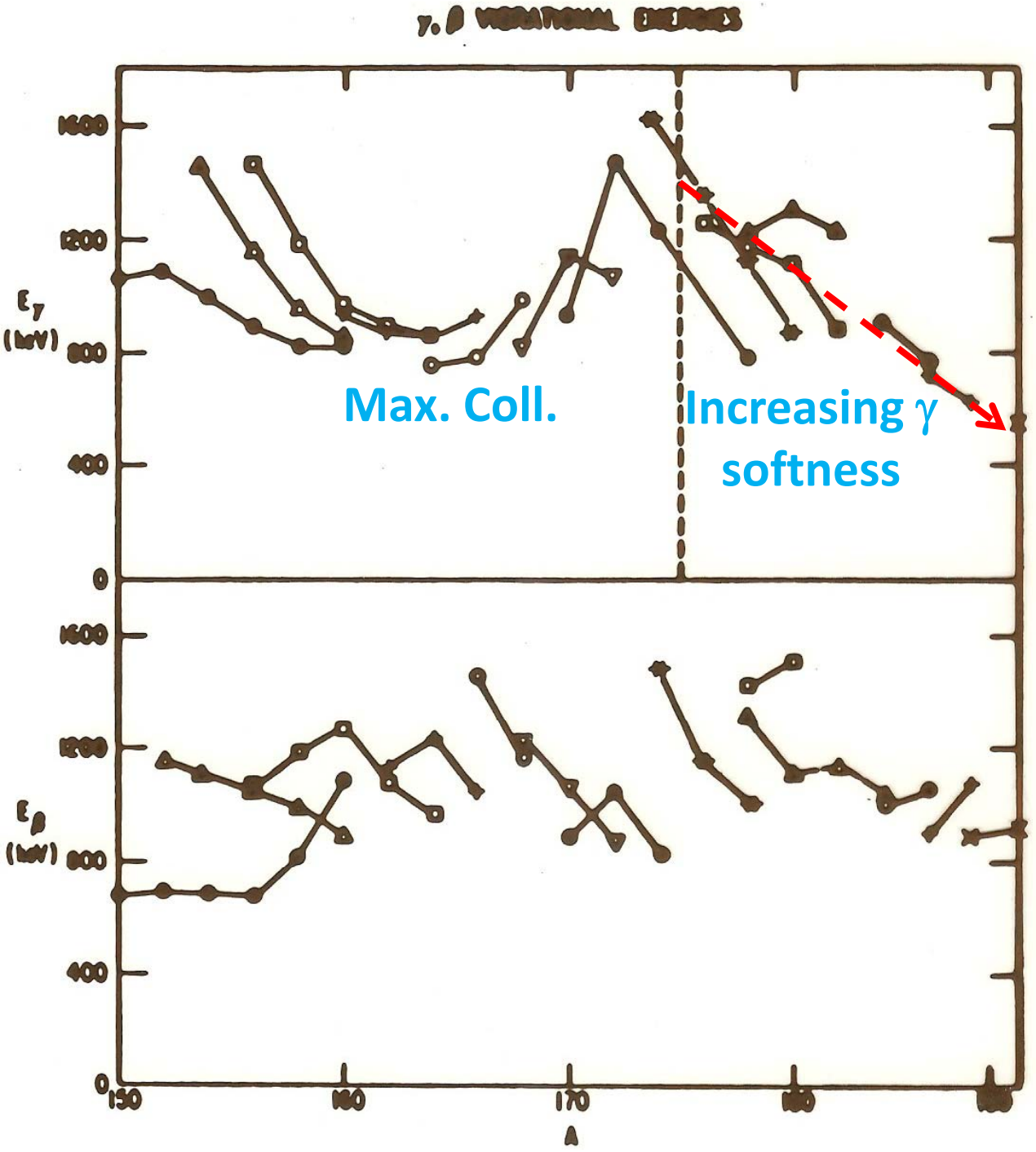




Alaga = 0.7

Vanishes in all 3 symmetries. Finite only in transition regions

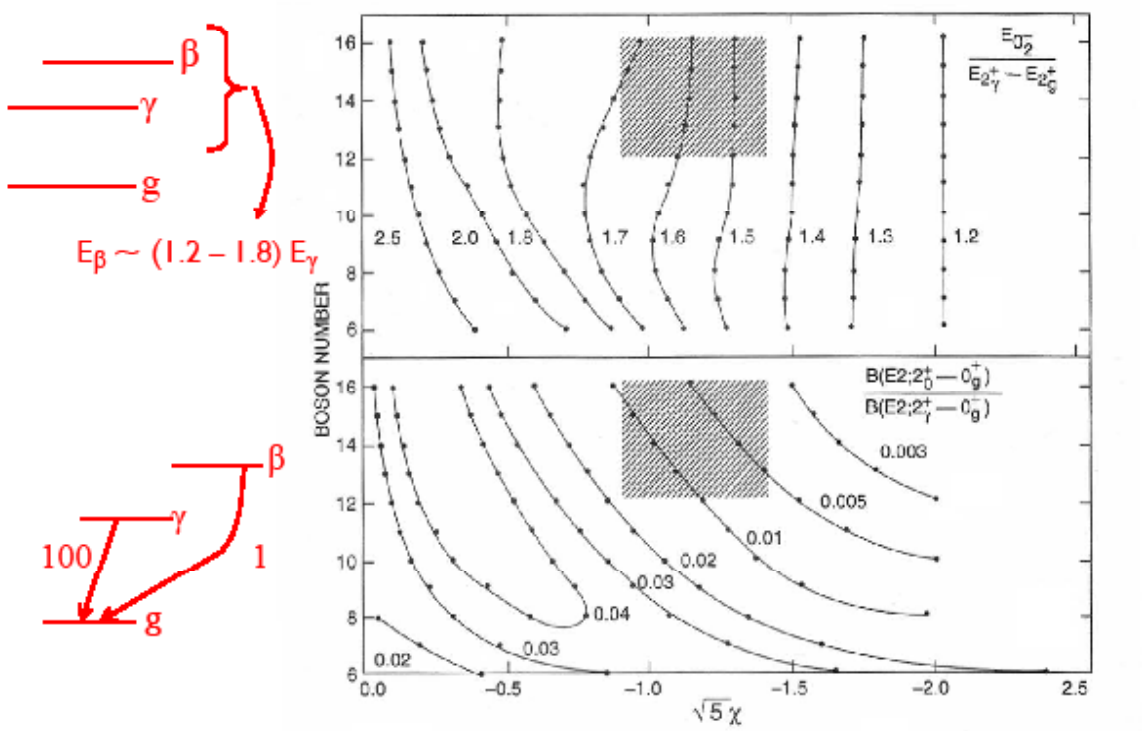
Systematics and collectivity of the lowest vibrational modes in deformed nuclei

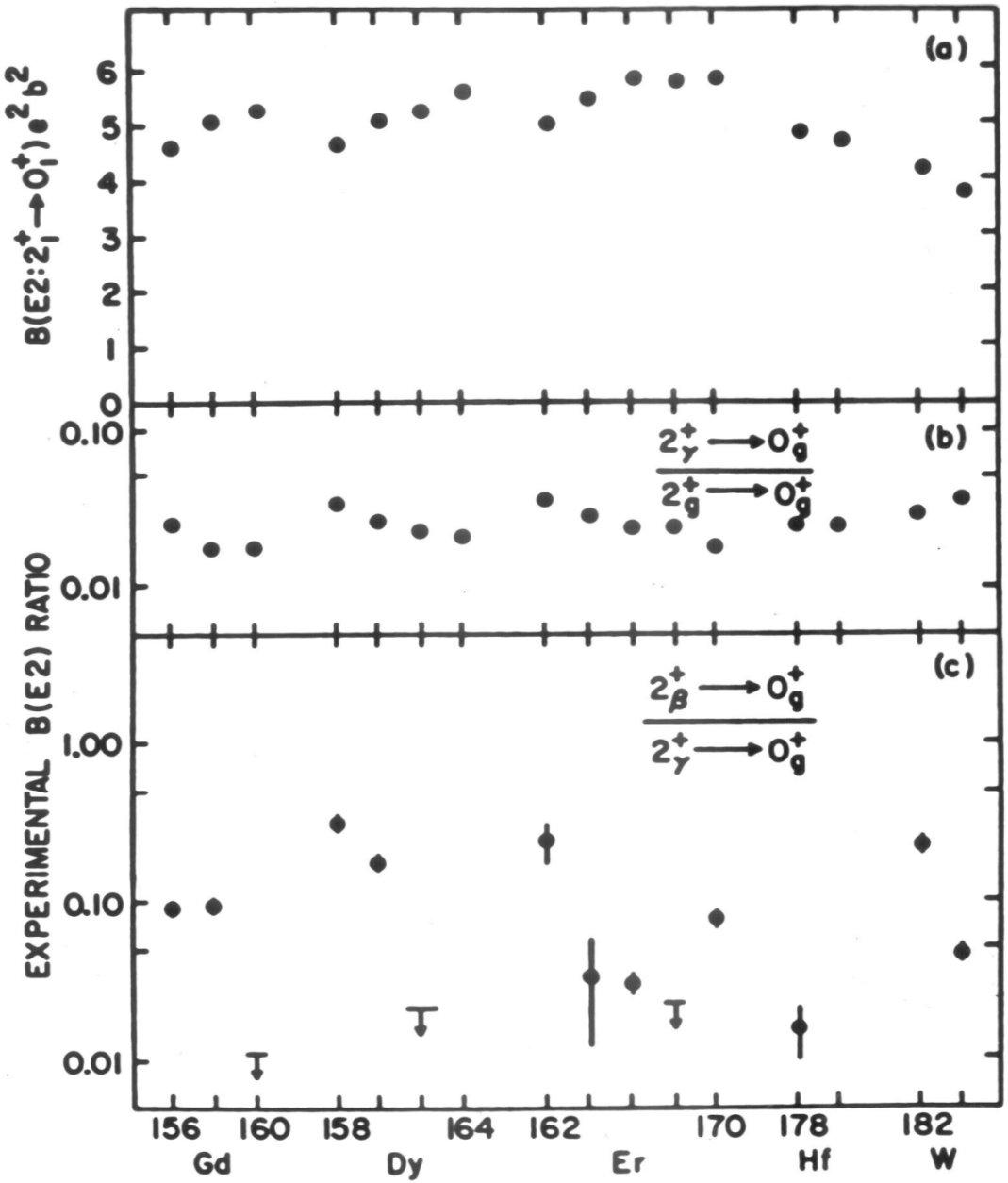


Notice that the the β mode is at higher energies (~ 1.5 times the γ vibration near mid-shell)* and fluctuates more. This points to lower collectivity of the β vibration.

* Remember for later !

Universal Contour Plots: O(6)→ SU(3) H = -κ Q • Q

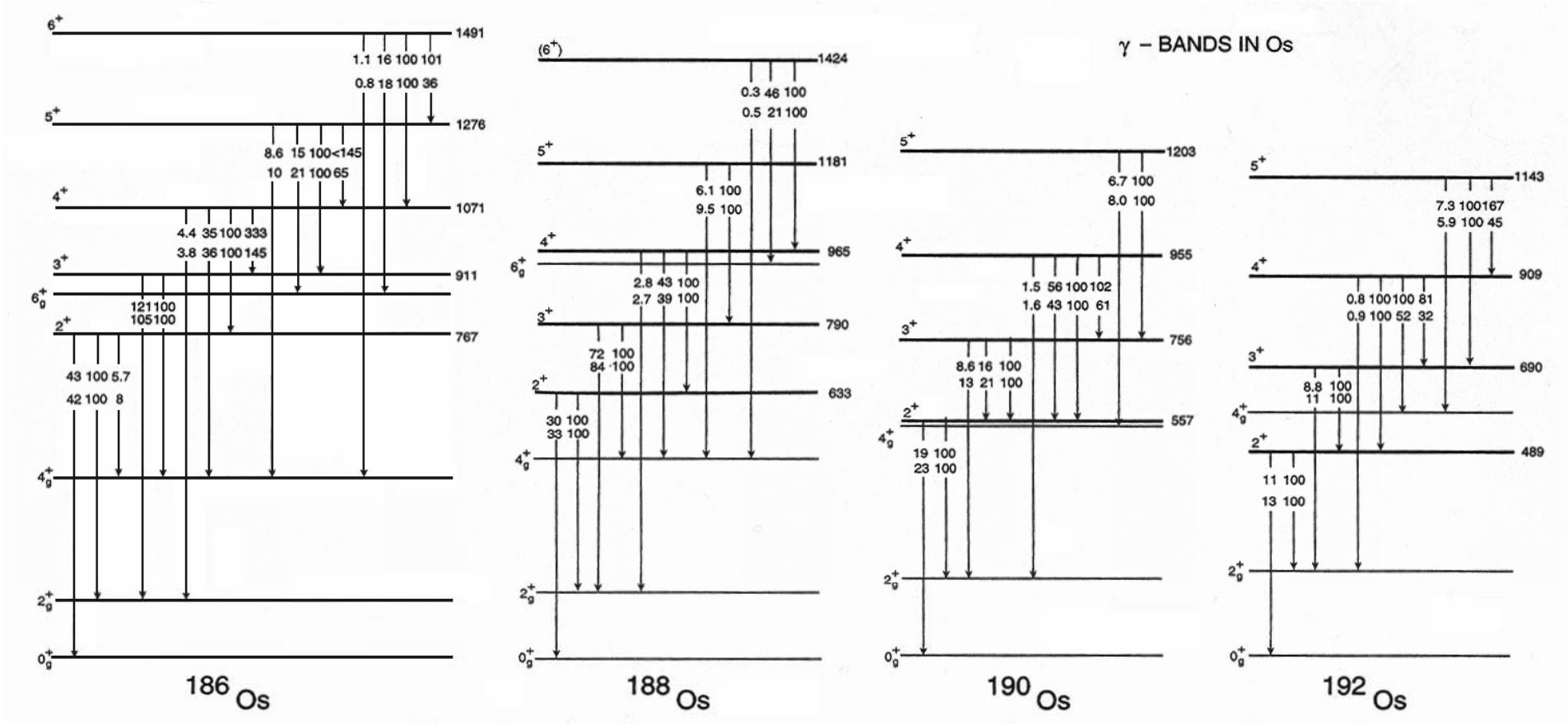



(a)
B(E2:2$_1^+$→0$_1^+$)e^{2}b^2
0
1
2
3
4
5
6
(b)
$\frac{2_\gamma^+ \rightarrow 0_g^+}{2_g^+ \rightarrow 0_g^+}$
0.10
0.01
(c)
$\frac{2_\beta^+ \rightarrow 0_g^+}{2_\gamma^+ \rightarrow 0_g^+}$
1.00
0.10
0.01
EXPERIMENTAL B(E2) RATIO
156 Gd
160
158 Dy
164
162 Er
170
178 Hf
182 W

Os isotopes from A = 186 to 192: Structure varies from a moderately gamma soft rotor to close to the O(6) gamma-independent limit. Describe simply with:

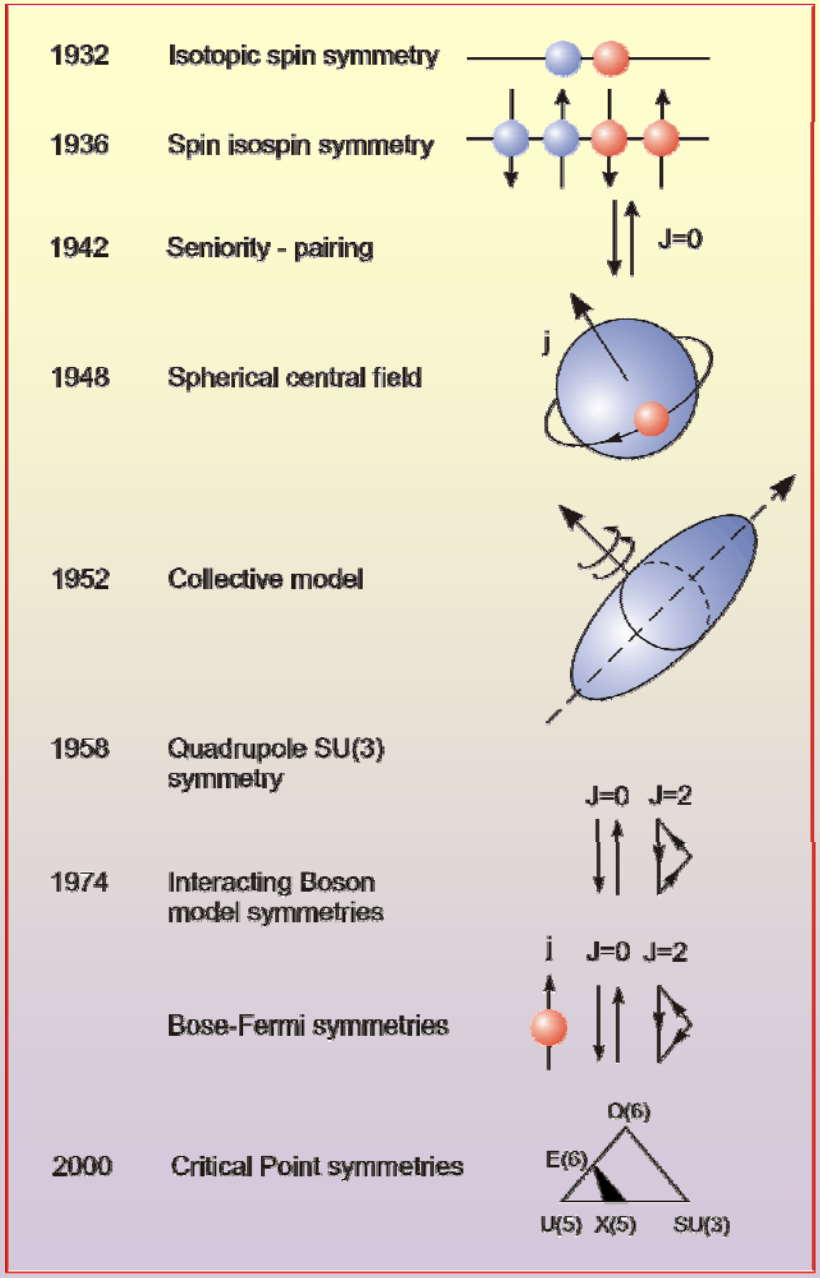
3

$H = -\kappa\, Q \bullet Q$ χ : 0 ☹ small as *A* <u>decreases</u>



End of Appendix

Backups



Symmetry has been used
as a guiding principle
to create order and beauty
in modeling the
nuclear many body-system

K. Heyde, priv. comm.

More than one phonon? What angular momenta? M-scheme for bosons

Table 6.1 *m scheme for two-quadrupole phonon states**

$J_1 = 2$ m_1	$J_2 = 2$ m_2	$M = \sum m_i$	J
2	2	4	
2	1	3	
2	0	2	4
2	−1	1	
2	−2	0	
1	1	2	
1	0	1	2
1	−1	0	
0	0	0	0

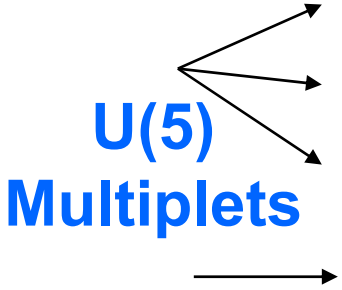
*Only positive total M values are shown: the table is symmetric for $M < 0$. The full set of allowable m_i values giving $M \geq 0$ is obtained by the conditions $m_1 \geq 0, m_2 \leq m_1$.

U(5)

$H = \varepsilon n_d$ + anharmonic terms

n_d = # d bosons in wave function

ε = energy of d boson



4 8^+ (400) 6^+ (400) 5^+ (400) 4^+ (400) 4^+ (410) 2^+ (401) 2^+ (410) 0^+ (420)

3 6^+ (300) 4^+ (300) 3^+ (300) 2^+ (310) 0^+ (301) $(d^\dagger d^\dagger d^\dagger)^L$ sss $|0>$

2 4^+ (200) 2^+ (200) 0^+ (210) $(d^\dagger d^\dagger)^L$ ss $|0>$

1 2^+ (100) $d^\dagger s\,|\,0>$

0 0^+ (000)

U(5)
$(n_d\ n_\beta\ n_\Delta)$

E Harmonic Spectrum

Important as a benchmark of structure, but also since the U(5) states serve as a convenient set of basis states for the IBA

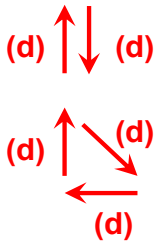
Quantum Numbers

$N = n_s + n_d$ = Total Boson No.

n_d = # d bosons

n_β = # of pairs of d bosons coupled to $J = 0^+$

n_Δ = # of triplets of d bosons coupled to $J = 0^+$



Review of phonon creation and destruction operators

$$\mathbf{b}|n_b\rangle = \sqrt{n_b}\,|n_b - 1\rangle$$

$$\mathbf{b}^\dagger|n_b\rangle = \sqrt{(n_b + 1)}\,|n_b + 1\rangle$$

What is a creation operator? Why useful?

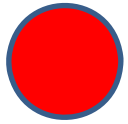
A) Bookkeeping – makes calculations very simple.

B) "Ignorance operator": We don't know the structure of a phonon but, for many predictions, we don't need to know its microscopic basis.

$$\mathbf{b}^\dagger\mathbf{b}|n_b\rangle = \mathbf{b}^\dagger\sqrt{n_b}\,|n_b - 1\rangle = \sqrt{n_b}\sqrt{(n_b - 1) + 1}|n_b\rangle = n_b|n_b\rangle$$

$\mathbf{b}^\dagger\mathbf{b}$ is a **b**-phonon number operator.

For the IBA a boson is the same as a phonon – think of it as a collective excitation with ang. mom. 0 (s) or 2 (d).



Brief, simple, trip into the Group Theory of the IBA

DON'T BE SCARED

You do not need to understand all the details but try to get the idea of the relation of groups to degeneracies of levels and quantum numbers

A more intuitive name for this application of Group Theory is

"Spectrum Generating Algebras"

Concepts of group theory

First, some fancy words with simple meanings: Generators, Casimirs, Representations, conserved quantum numbers, degeneracy splitting

Generators of a group: Set of operators , O_i that close on commutation.

$[O_i , O_j] = O_i O_j - O_j O_i = O_k$ *i.e.*, their commutator gives back 0 or a member of the set

For IBA, the 36 operators $s^\dagger s$, $d^\dagger s$, $s^\dagger d$, $d^\dagger d$ are generators of the group U(6).

ex:

$$\left[d^\dagger s, s^\dagger s\right]\left|n_d n_s\right\rangle = \left(d^\dagger s s^\dagger s - s^\dagger s d^\dagger s\right)\left|n_d n_s\right\rangle$$

$$= d^\dagger s n_s \left|n_d n_s\right\rangle - s^\dagger s d^\dagger s \left|n_d n_s\right\rangle$$

$$= \left(n_s - s^\dagger s\right) d^\dagger s \left|n_d n_s\right\rangle$$

$$= \left(n_s - s^\dagger s\right)\sqrt{n_d+1}\sqrt{n_s}\left|n_d+1, n_s-1\right\rangle$$

$$= \sqrt{n_d+1}\sqrt{n_s}\left[n_s - \left(n_s - 1\right)\right]\left|n_d+1, n_s-1\right\rangle$$

$$= \sqrt{n_d+1}\sqrt{n_s}\left|n_d+1, n_s-1\right\rangle$$

$$= d^\dagger s \left|n_d n_s\right\rangle$$

e.g:

$$\left[N , s^\dagger \tilde{d} \right] \Psi = \left[N s^\dagger \tilde{d} - s^\dagger \tilde{d} N \right] \Psi$$

$$N s^\dagger \tilde{d} \Psi - N s^\dagger \tilde{d} \Psi = 0$$

or:

$$\left[d^\dagger s, s^\dagger s\right] = d^\dagger s$$

Sub-groups:

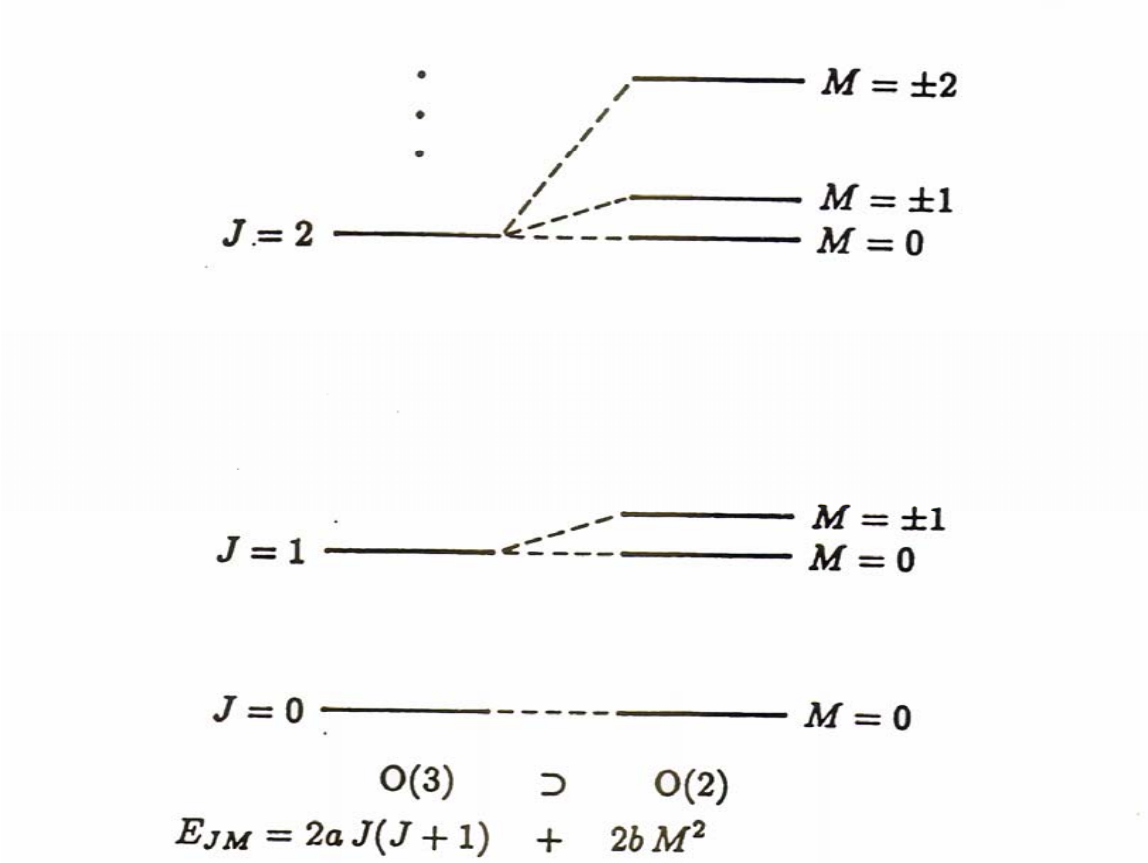
Subsets of generators that commute among themselves.

e.g: $d^{\dagger}d$ 25 generators—span U(5)

They conserve n_d (# *d* bosons)

Set of states with same n_d are the representations of the group [U(5)]

Simple example of dynamical symmetries, group chain, degeneracies



$[H, J^2] = [H, J_Z] = 0$ J, M constants of motion

Let's illustrate group chains and degeneracy-breaking.

Consider a Hamiltonian that is a function ONLY of: $s^\dagger s + d^\dagger d$

That is: $H = a(s^\dagger s + d^\dagger d) = a\,(n_s + n_d) = aN$

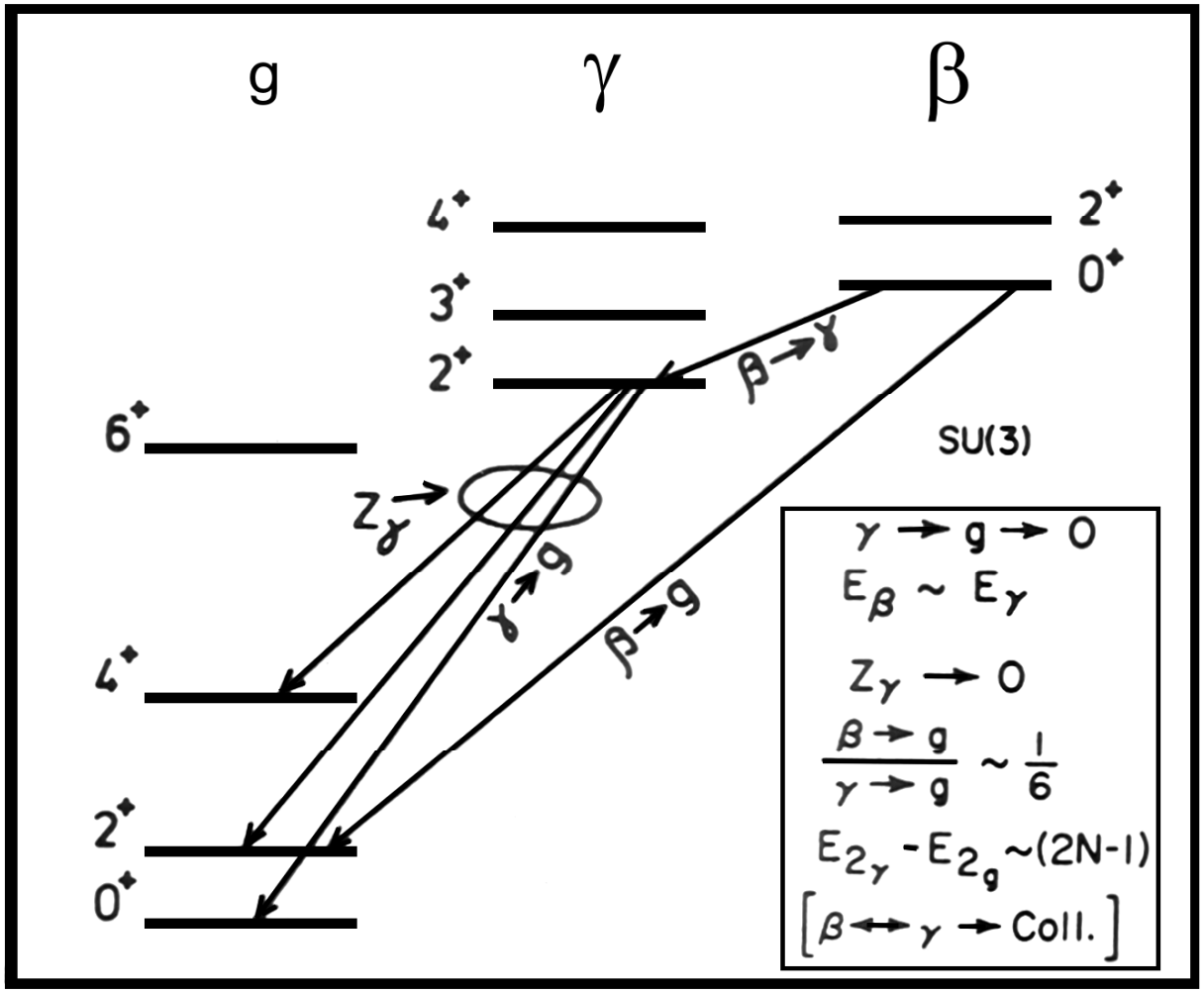
$H' = H \qquad = aN$

Now, add a term to this Hamiltonian:

Now the energies depend not only on N but also on n_d

States of a given n_d are now degenerate. They are "representations" of the group U(5). States with different n_d are not degenerate

Signatures of SU(3)



Signatures of SU(3)

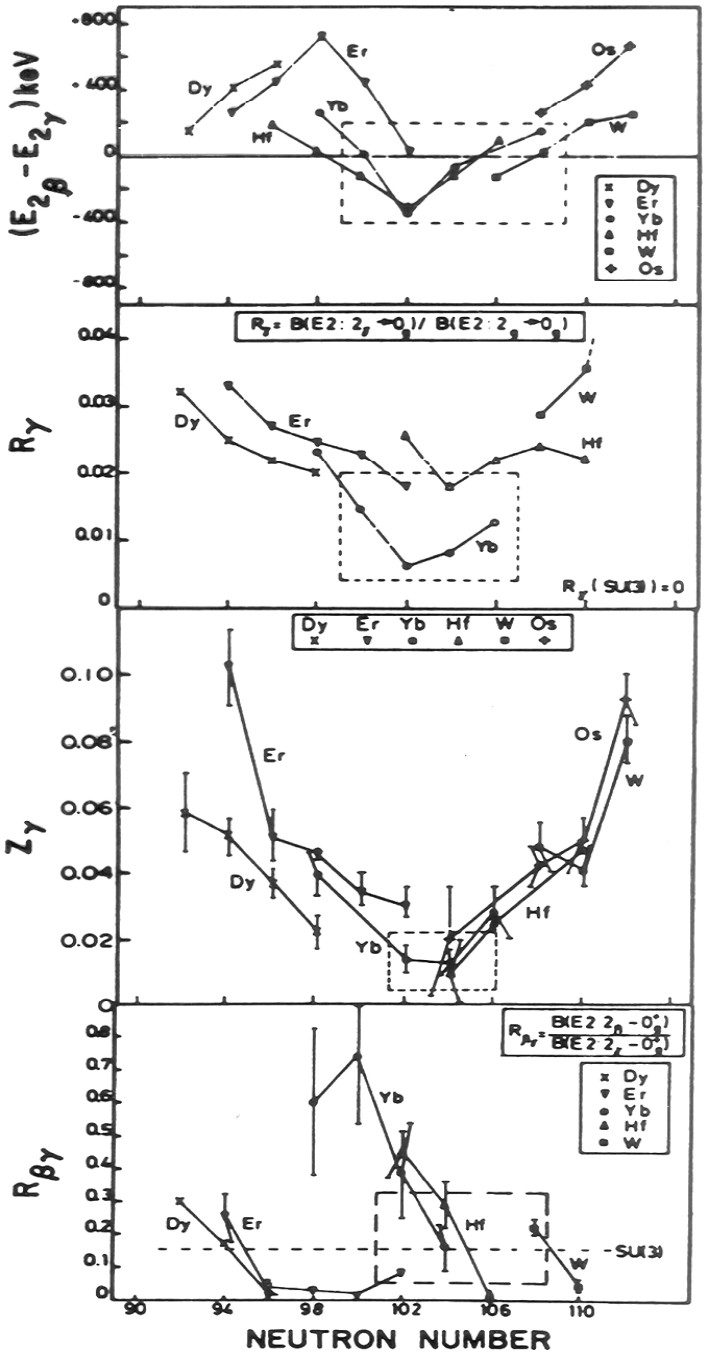
$E_{\square} = E_{\square}$

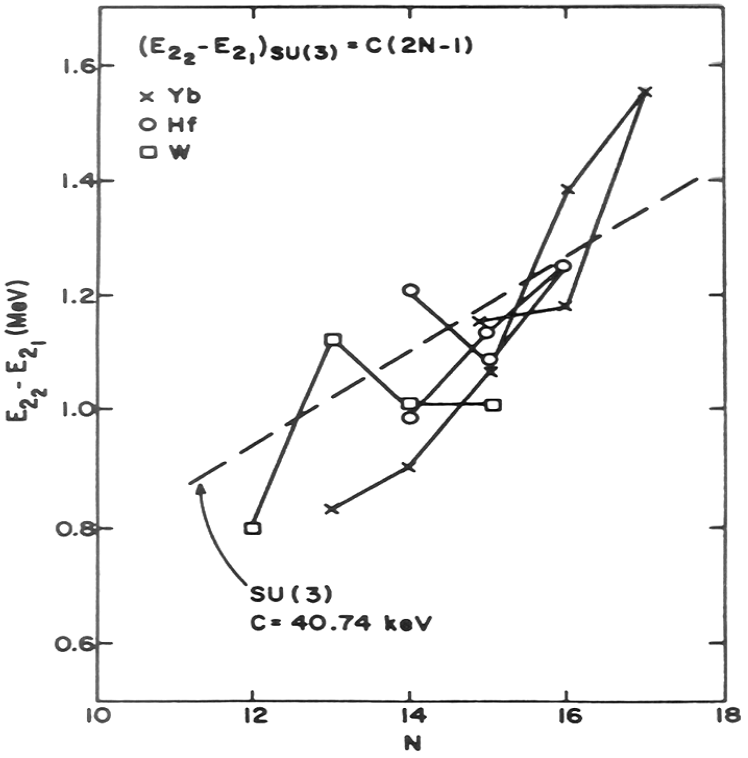
$B(\square \square g) \square 0$

$Z_{\square} \quad \square 0$

$\frac{B(\square \square g)}{B(\square \square g)} \quad \square \ 1/6$

$E(\square\text{-vib}) \quad \square (2N - 1)$





2a N + 2

H’ = aN = *a N*

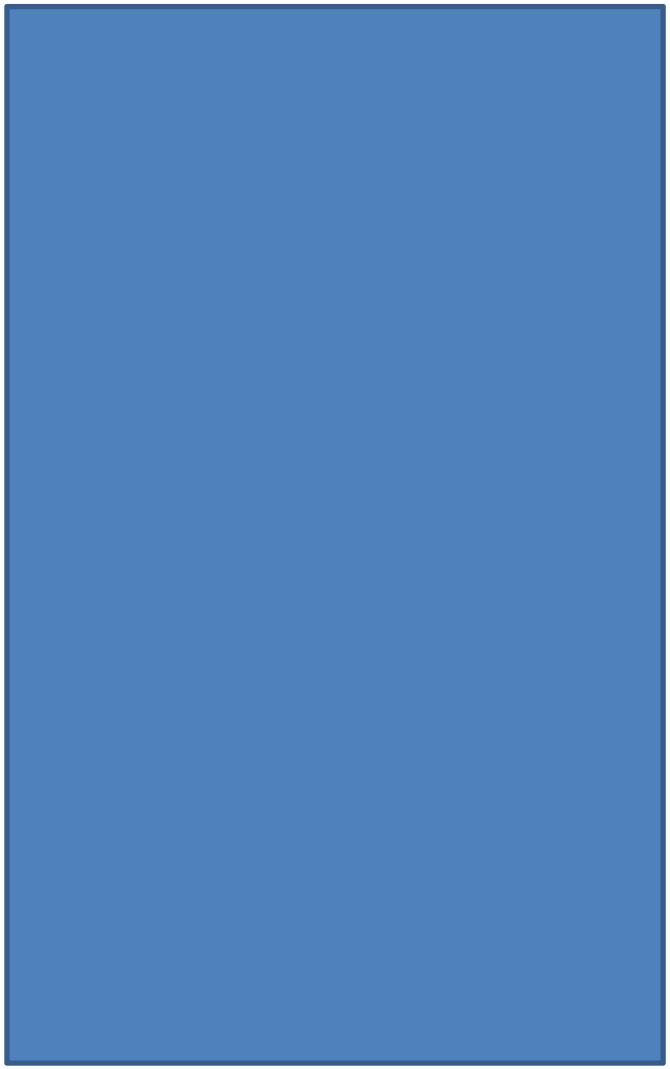
a N + 1

0 N

E

U(6)

H’ = aN



Etc. with further terms

Dynamical Symmetries – The structural benchmarks

- U(5) Vibrator – spherical nucleus that can oscillate in shape
- SU(3) Axial Rotor – can rotate and vibrate
- O(6) Axially asymmetric rotor (“gamma-soft”) – squashed deformed rotor

Classifying Collective Nuclear Structure – The Symmetry Triangle