How nuclei behave: a simple perspective based on symmetry and geometry

(with a discussion of the microscopic drivers of structural evolution)

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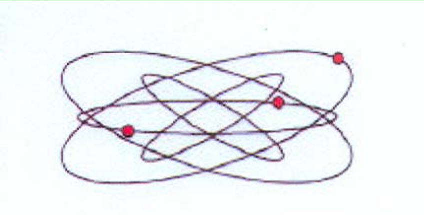
Themes and challenges of Modern Science

- **Complexity out of simplicity -- Microscopic**

  How the world, with all its apparent complexity and diversity can be constructed out of a few elementary building blocks and their interactions

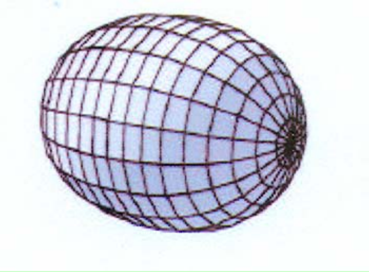
  What is the force that binds nuclei?



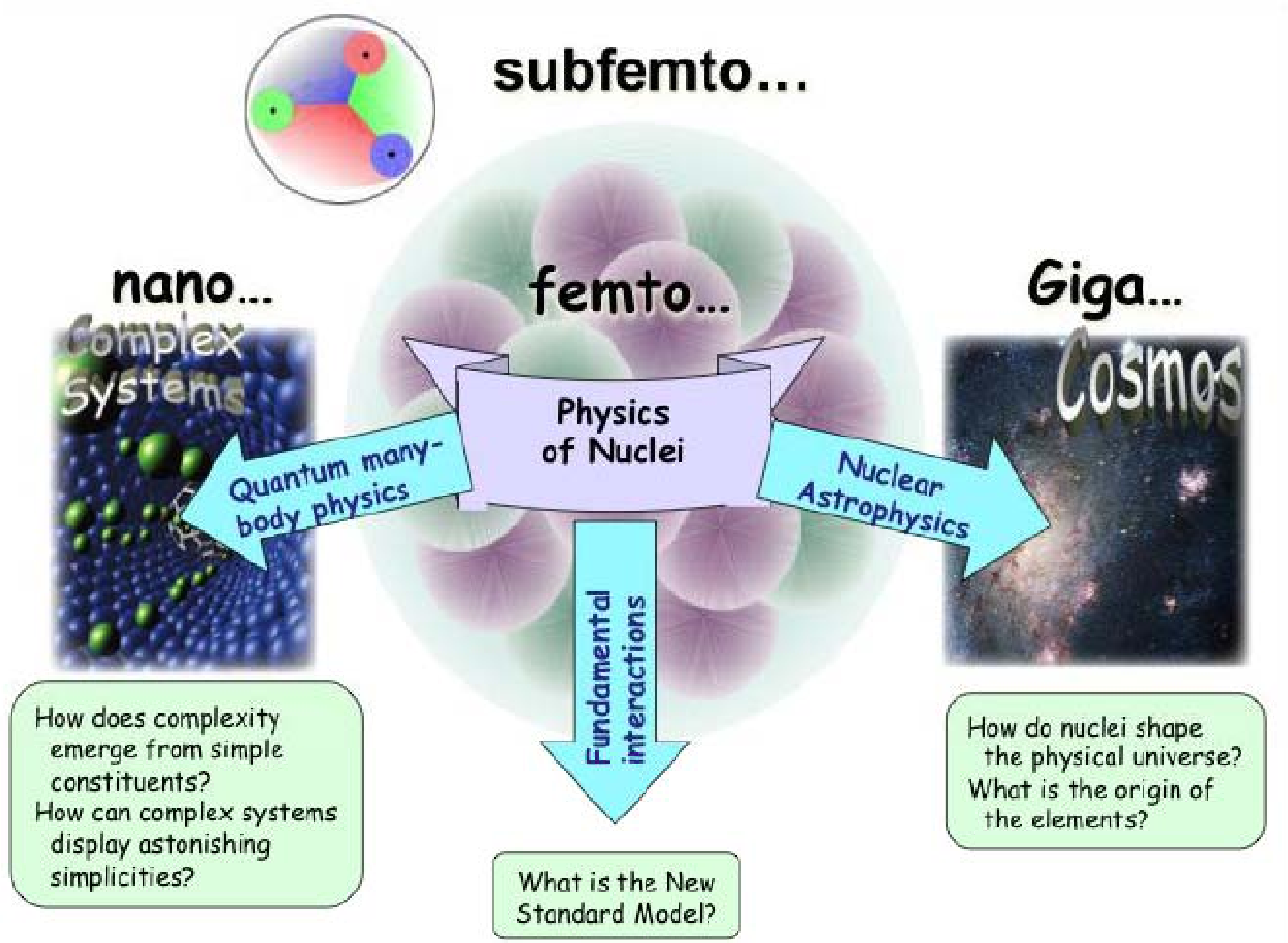
- **Simplicity out of complexity – Macroscopic**

  How the world of complex systems can display such remarkable regularity and simplicity

  What are the simple patterns that nuclei display and what is their origin ?



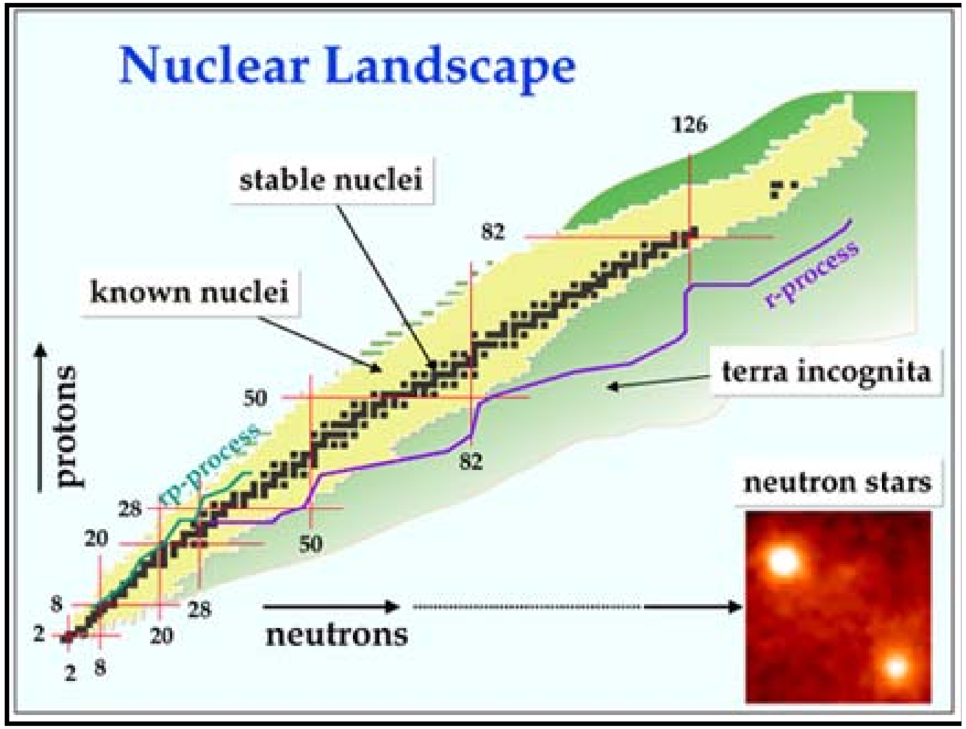
Where do nuclei fit into the overall picture?



The scope of Nuclear Structure Physics

The Four Frontiers

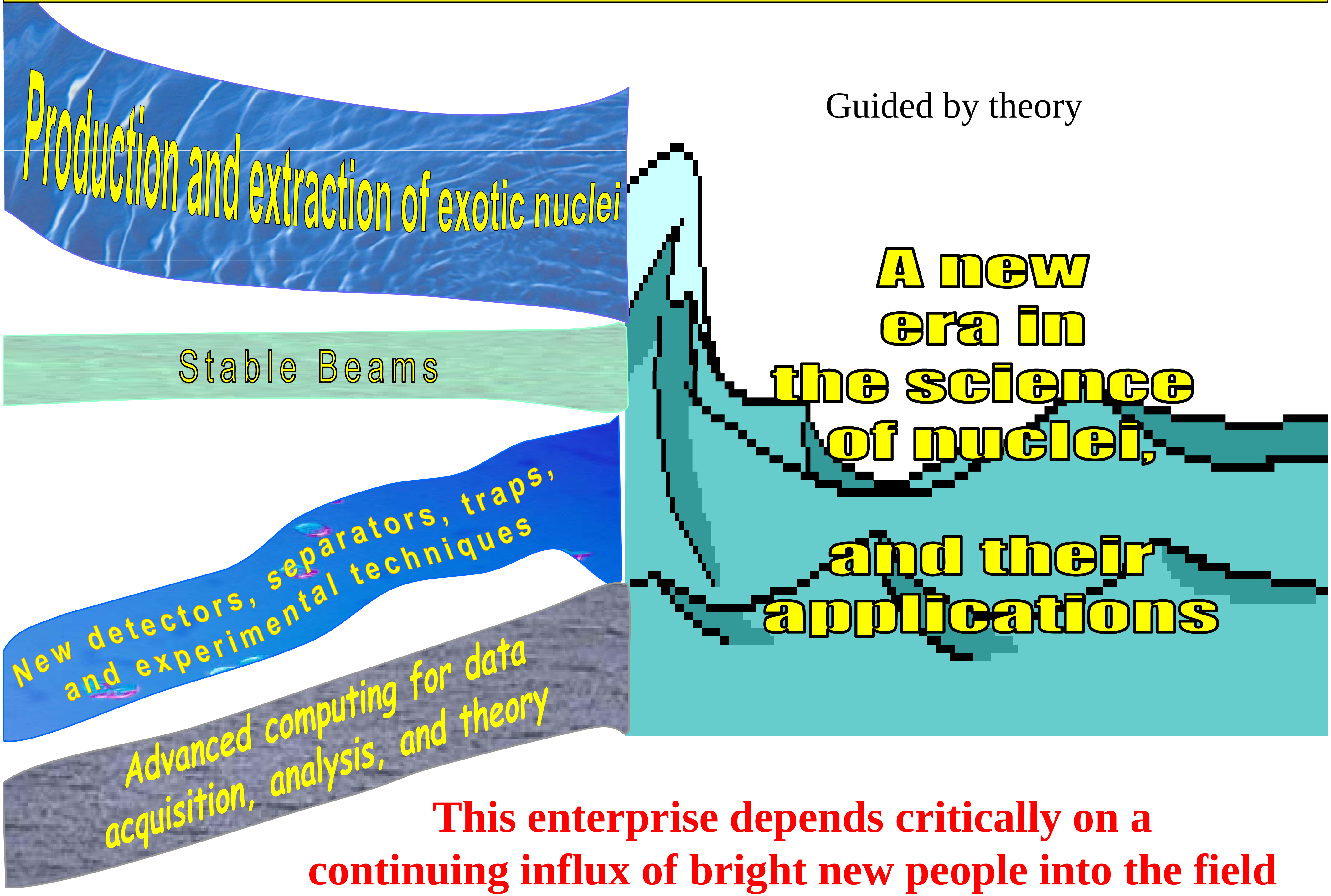
1. Proton Rich Nuclei
2. Neutron Rich Nuclei
3. Heaviest Nuclei
4. *Evolution of structure within these boundaries*



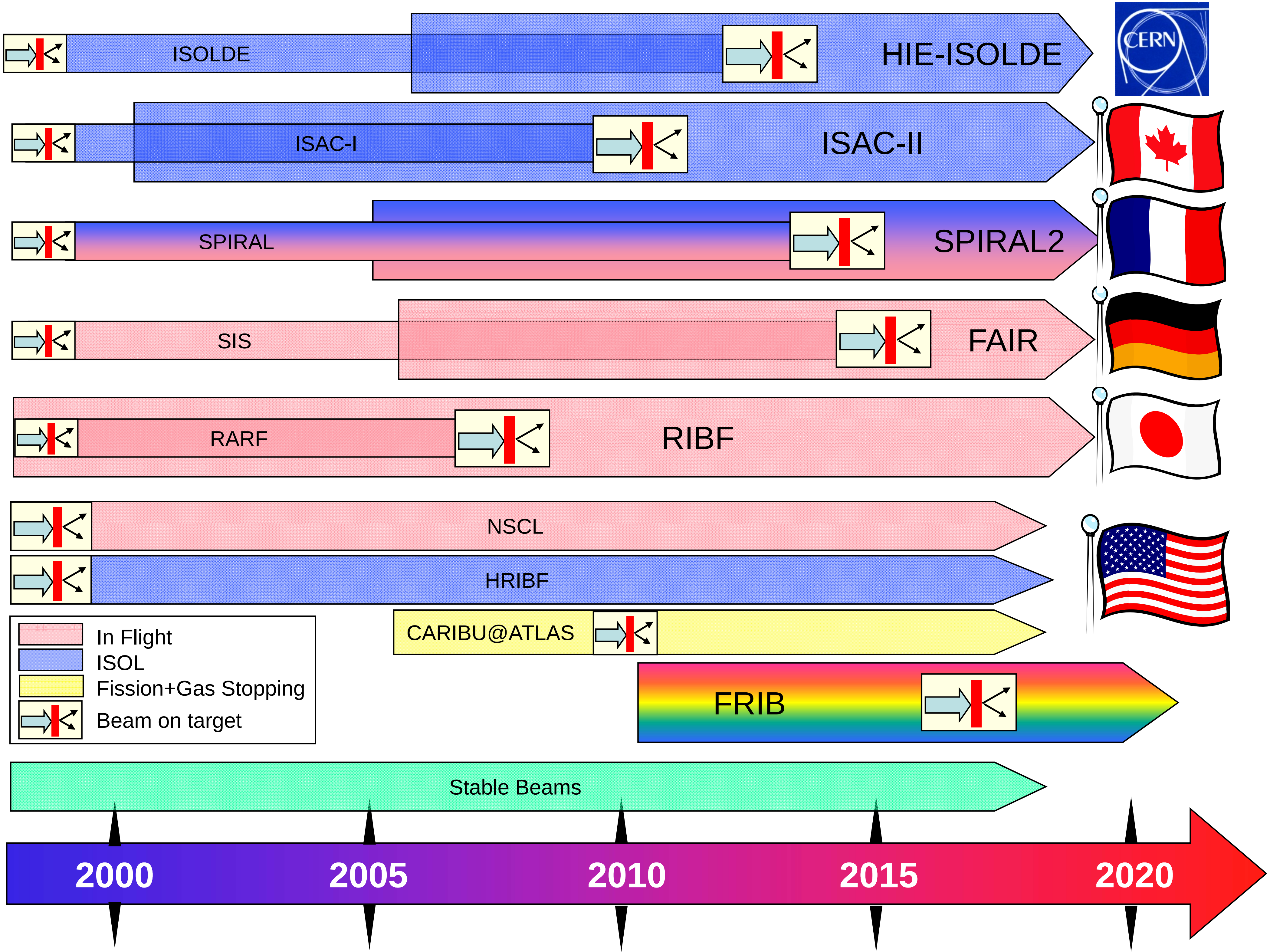
Terra incognita — huge gene pool of new nuclei

We can customize our system – fabricate "designer" nuclei to *isolate and amplify* specific physics or interactions

A confluence of advances leading to a great opportunity for science



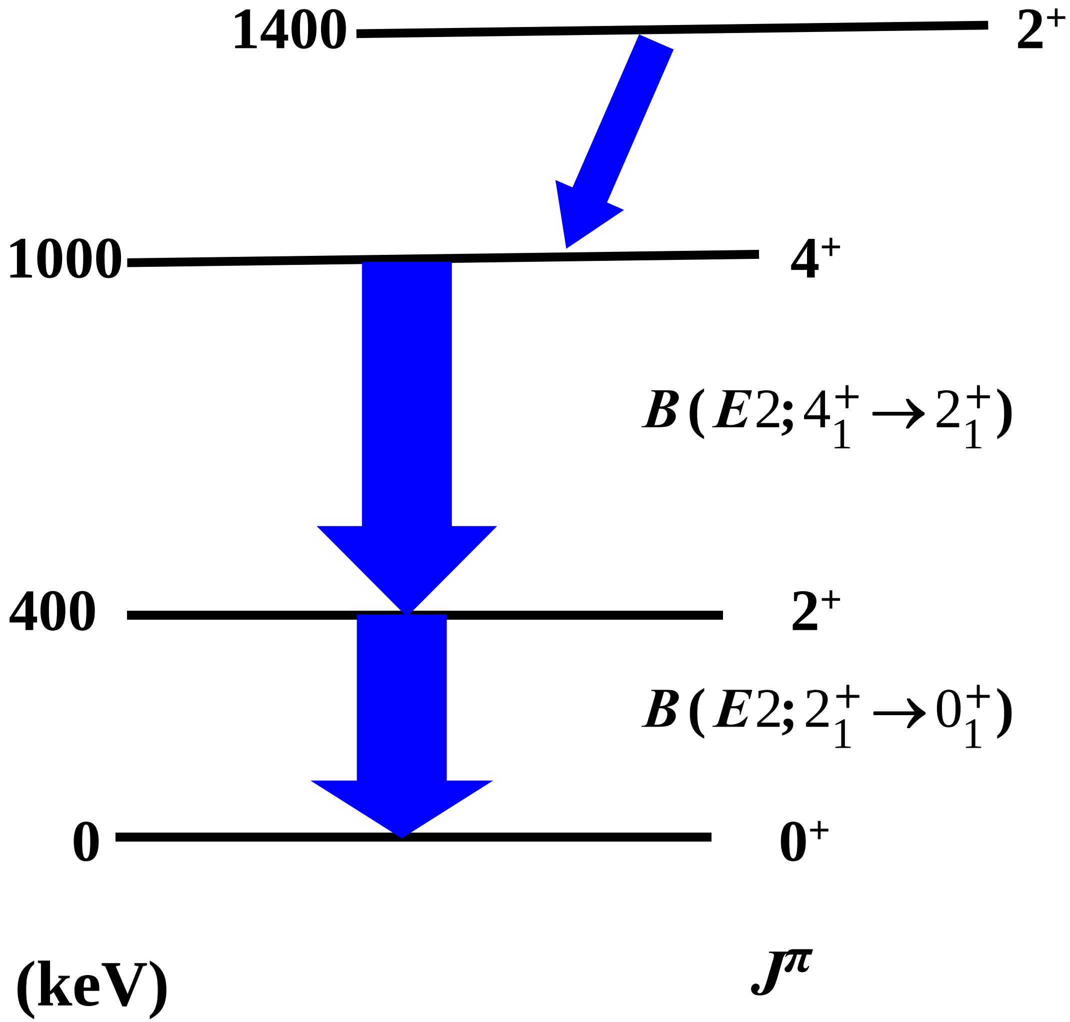
This enterprise depends critically on a continuing influx of bright new people into the field


ISOLDE
HIE-ISOLDE
CERN
ISAC-I
ISAC-II
SPIRAL
SPIRAL2
SIS
FAIR
RARF
RIBF
NSCL
HRIBF
CARIBU@ATLAS
FRIB
In Flight
ISOL
Fission+Gas Stopping
Beam on target
Stable Beams
2000
2005
2010
2015
2020

Simple Observables - Even-Even Nuclei

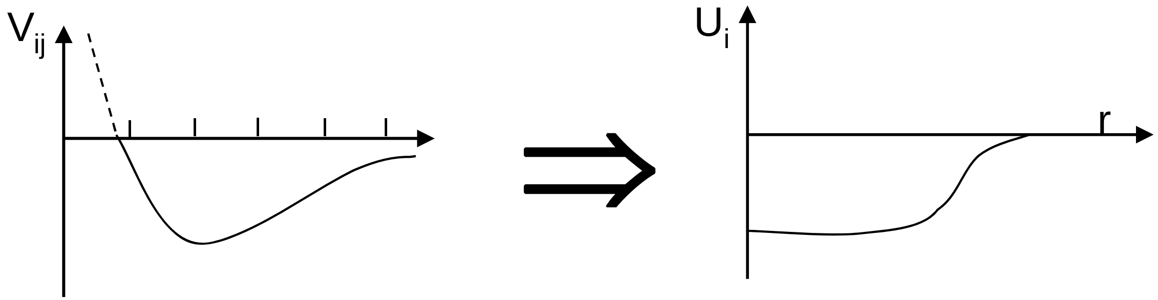
$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$

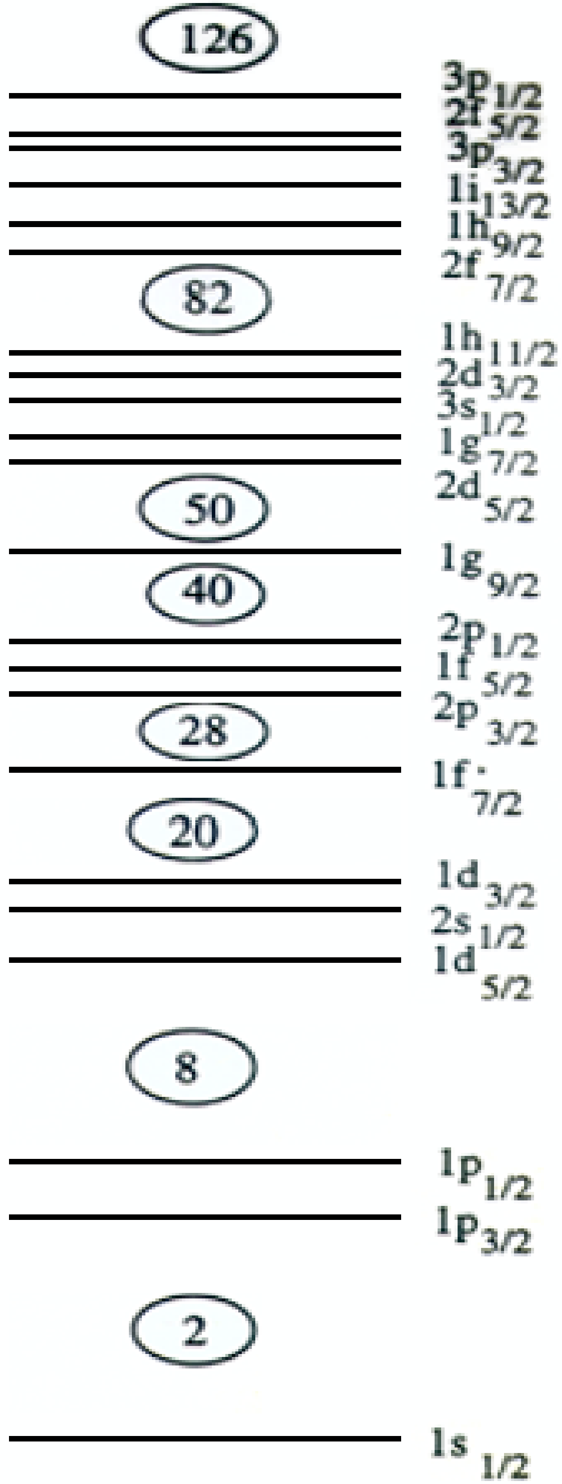
Masses, Radii



$$B(E2;J_i \to J_f) \equiv \frac{1}{2J_i+1} \langle \Psi_i \| E2 \| \Psi_f \rangle^2$$

Reminder slide:
The Independent Particle Model





Clusters of levels + Pauli Principle → magic numbers, inert cores, valence nucleons

Key to structure. Many-body → few-body: each body counts.

(Addition of 2 neutrons in a nucleus with 150 can drastically alter structure)

Residual Interactions

Need to consider a more complete Hamiltonian:

$$H_{\text{Shell Model}} = H_{\text{IPM}} + H_{\text{residual}}$$

H_{residual} reflects interactions not in the single particle potential.

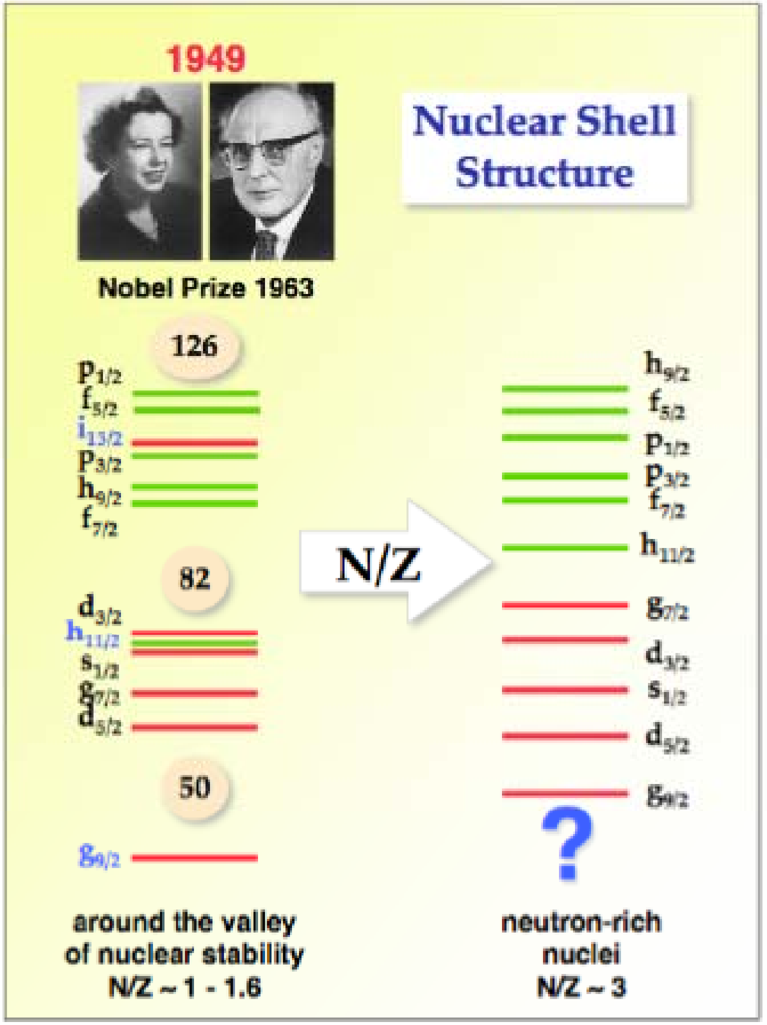
NOT a minor perturbation. In fact, these residual interactions determine almost everything we know about most nuclei.

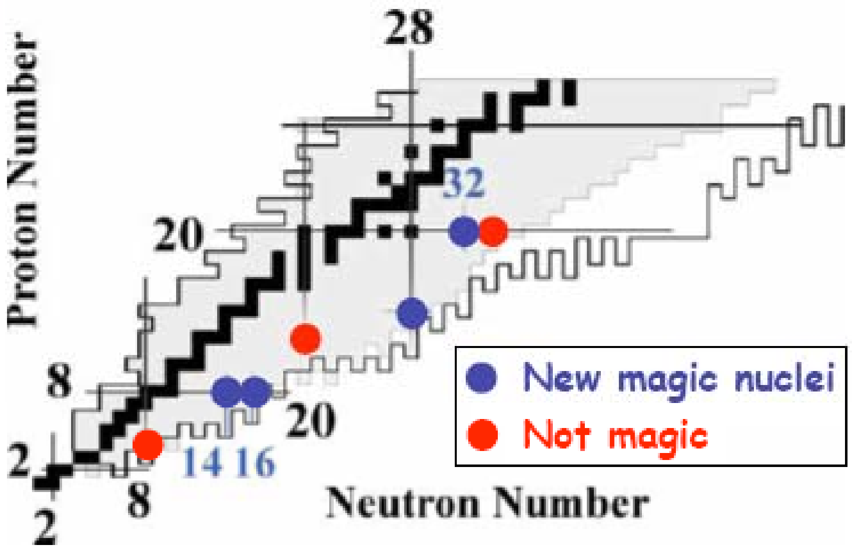
These interactions mix different independent particle model wave functions so that a physical wave function for a given state in the Shell Model is a linear combination of many independent Particle Model configurations.

This mixing is essential to understanding structure and structural evolution.

Caveat slide: Fragility of the Shell Model

Independent Particle Model – Trouble in Paradise





No shell closure for N=8 and 20 for drip-line nuclei; new shells at 14, 16, 32…

How can we see changes in shell structure experimentally.

We will soon see one easy tool: $E(2^+_1)$

Key Nuclear observables and their behavior with N and Z

What nuclei do, how we study them (what observables), and some simple ideas about structure – single particle and collective aspects

Remember: The nuclei are always right !!! Don't impose our preconceptions on them. Let them tell us what they are doing.

Let's start with $R_{4/2}$. How does it vary, and why, and why do we care

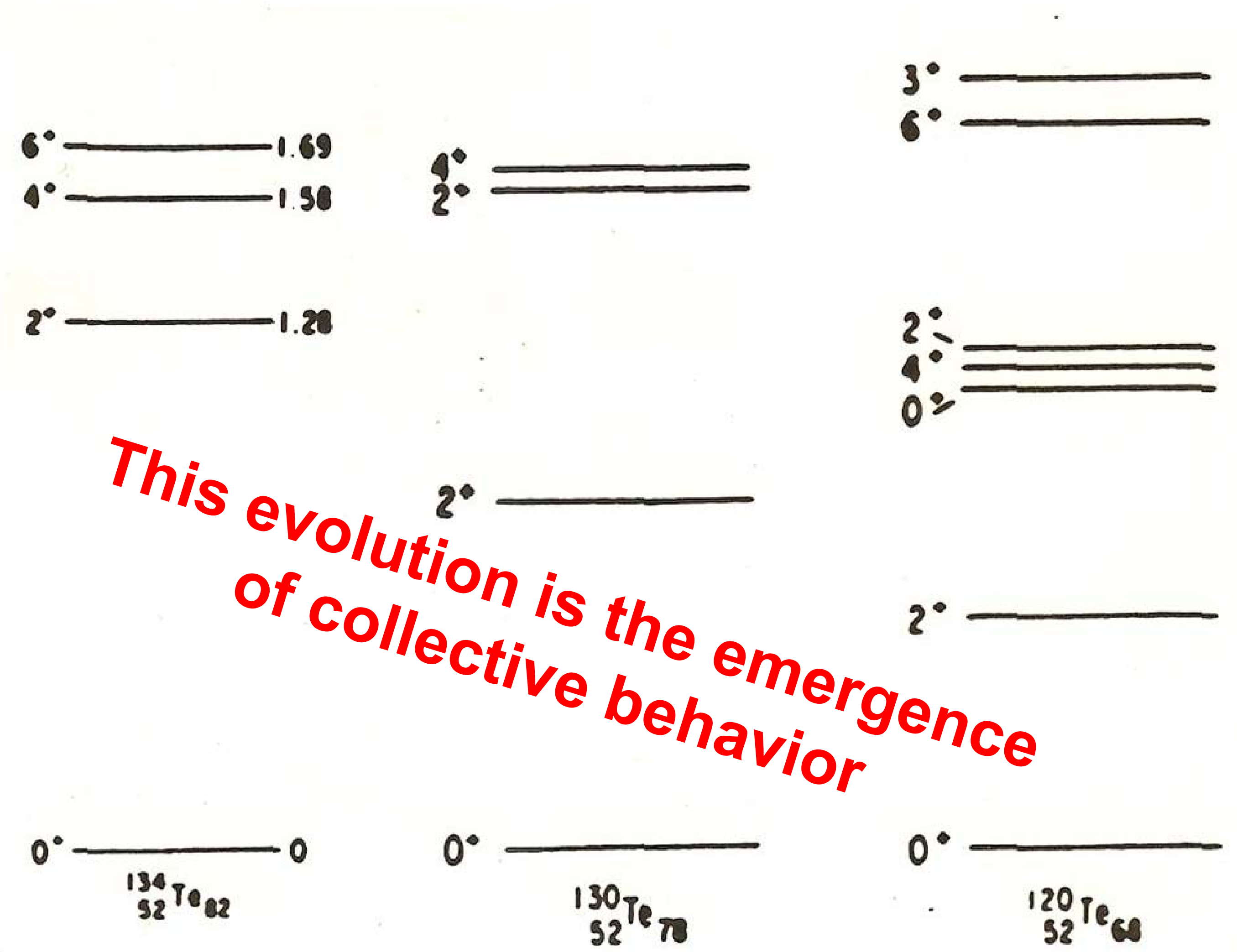
- We care because it is the almost the only observable whose value immediately tells us something (not everything – as we on shall see in the third lecture on the IBA model!!!) about the structure.
- We care because it is easy to measure.
- Other observables, like $E(2_1^+)$ and masses, are measurable even further from stability. They too can give valuable information in the context of regional behavior, but generally not as directly.

Starting from a doubly magic nucleus, what happens as the numbers of valence neutrons and protons **increase**?

Case of **few** valence nucleons:

Lowering of energies, development of multiplets. **$R_{4/2} \rightarrow$ ~2-2.4**

Two nucleons of one type

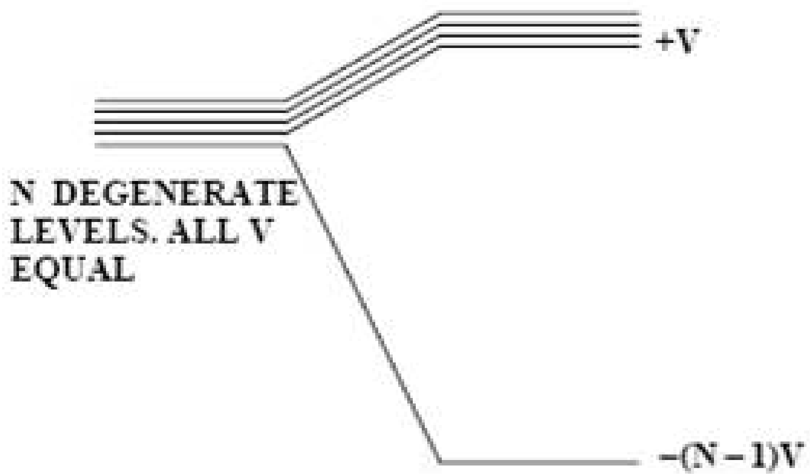


This evolution is the emergence of collective behavior

Few nucleons of both types

Origin of collectivity:
Mixing of many configurations

Consider a toy model: Mixing of degenerate states



$$\Psi_{LOWEST} = \frac{1}{\sqrt{N}}[\phi_1 + \phi_2 + \ldots + \phi_N]$$

This is the origin of collectivity in nuclei.

This is about as important as it gets.

Please remember it and think about it often (and try to develop a deep love for it).

Types of collective structures

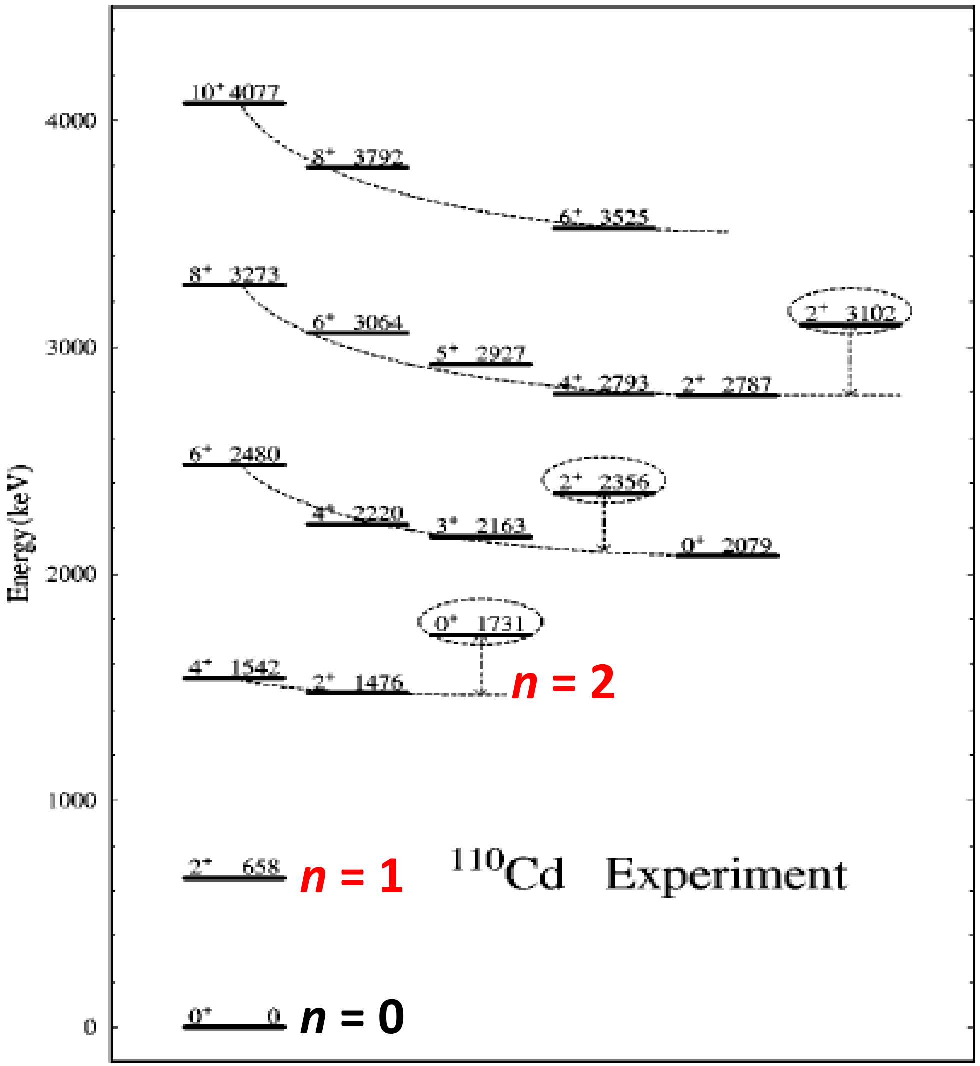
Few valence nucleons of each type:

The spherical vibrator

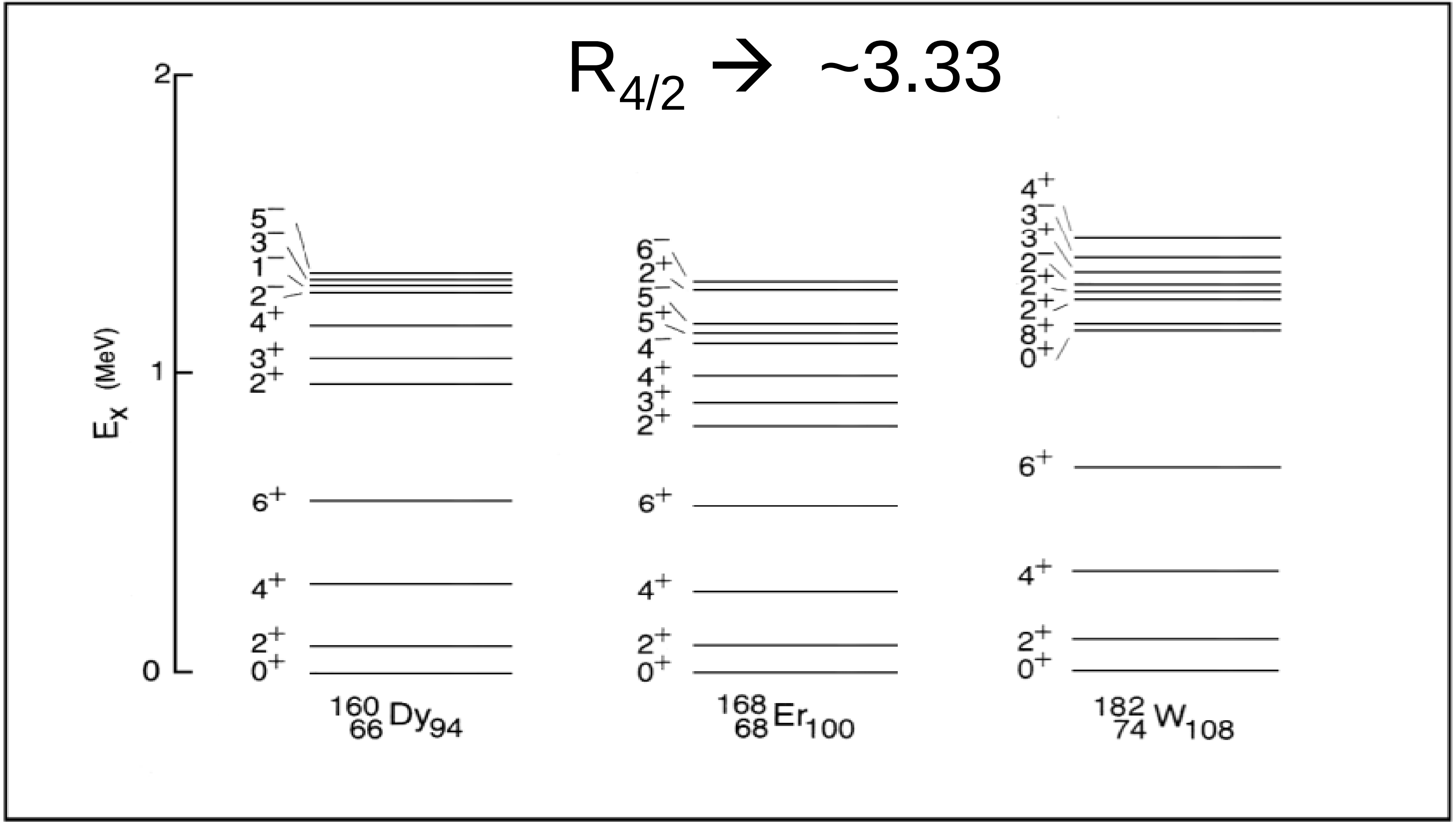
Vibrator (H.O.)

$$E(J) = n\,(\hbar\,\omega_0)$$

$$R_{4/2} = 2.0$$



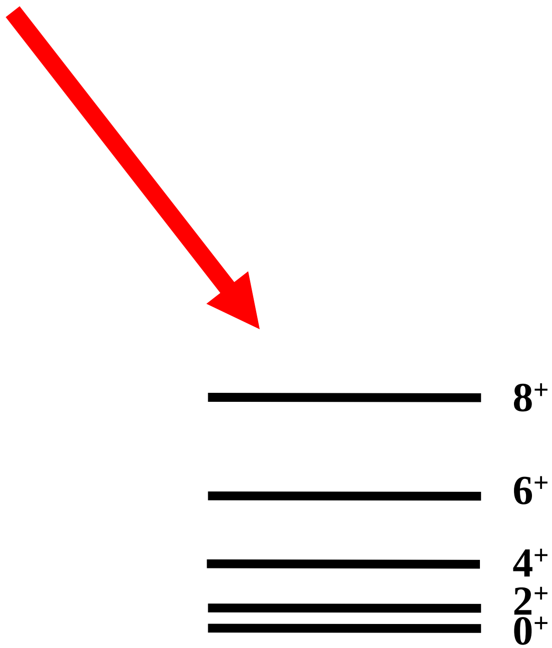
Lots of valence nucleons of **both** types: emergence of deformation and therefore rotation (nuclei live in the world, not in their own solipsistic enclaves)

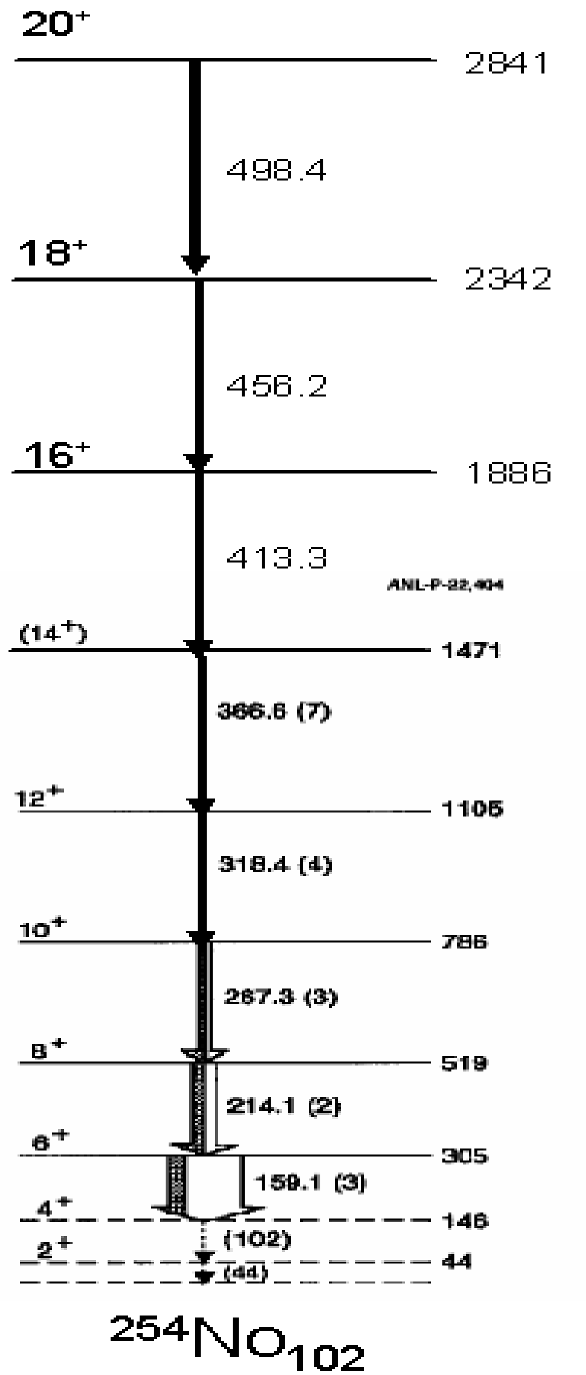


Rotor

$$E(J) \propto \left(\hbar^2/2\mathcal{J} \right) J(J+1)$$

$$R_{4/2} = 3.33$$





Doubly magic plus 2 nucleons

$R_{4/2} < 2.0$

Vibrator (H.O.)

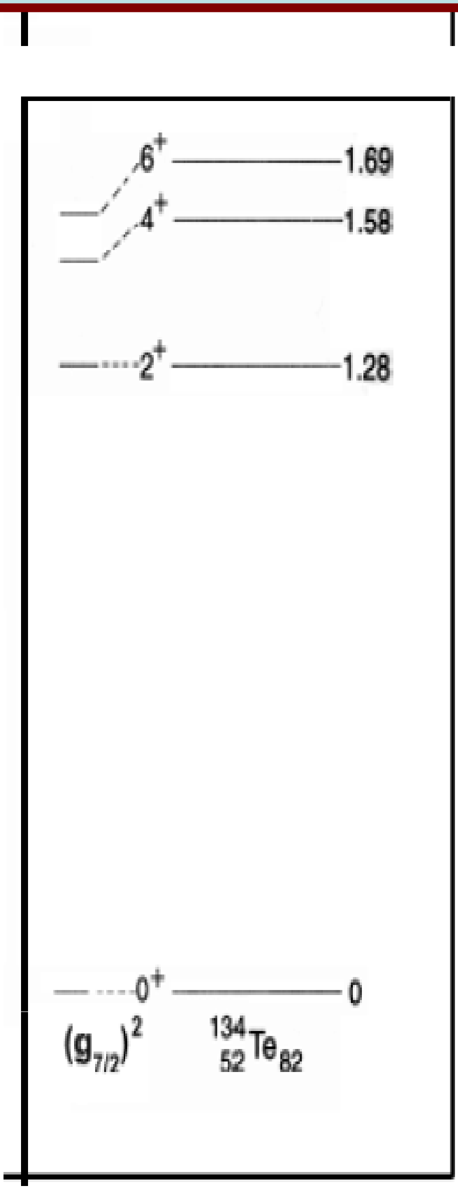
$E(J) = n\,(\hbar\,\omega_0)$

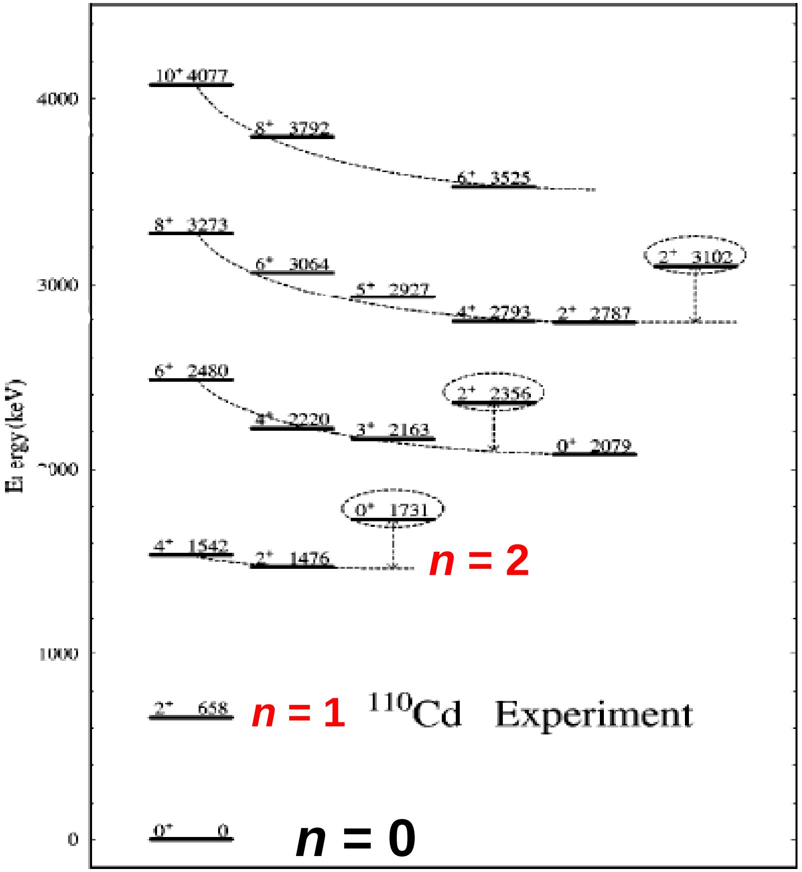
$R_{4/2} = 2.0$

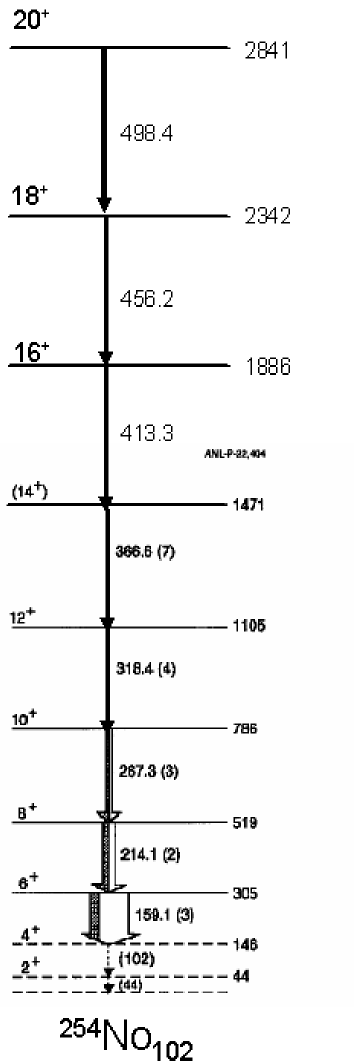
Rotor

$E(J) \propto (\hbar^2/2I)\,J(J+1)$

$R_{4/2} = 3.33$







Value of paradigms

J	E (keV)	Without rotor paradigm	Paradigm Benchmark: Rotor J(J + 1)
6^+	690	?	700
4^+	330		333
2^+	100		100
0^+	0		0

Amplifies structural differences

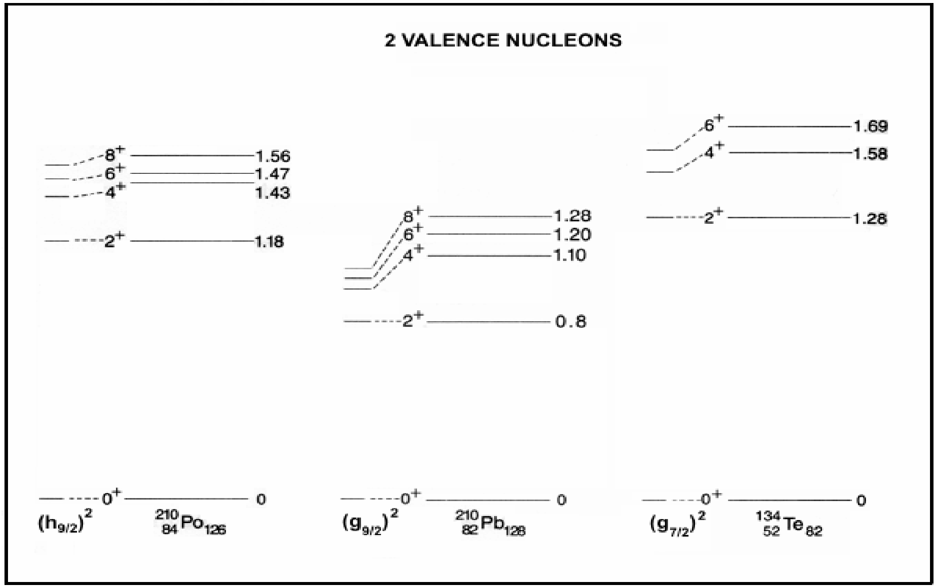
Centrifugal stretching

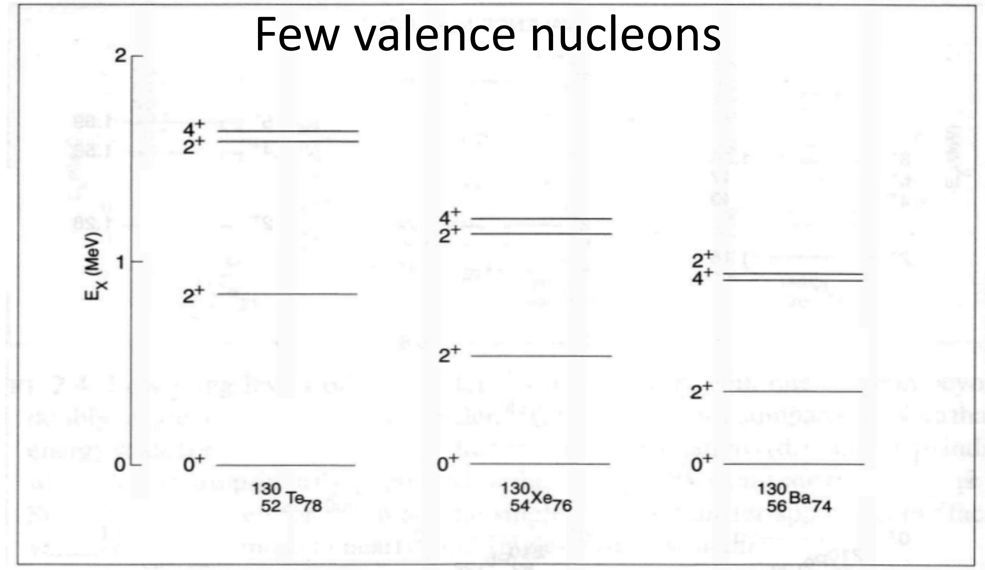
Deviations

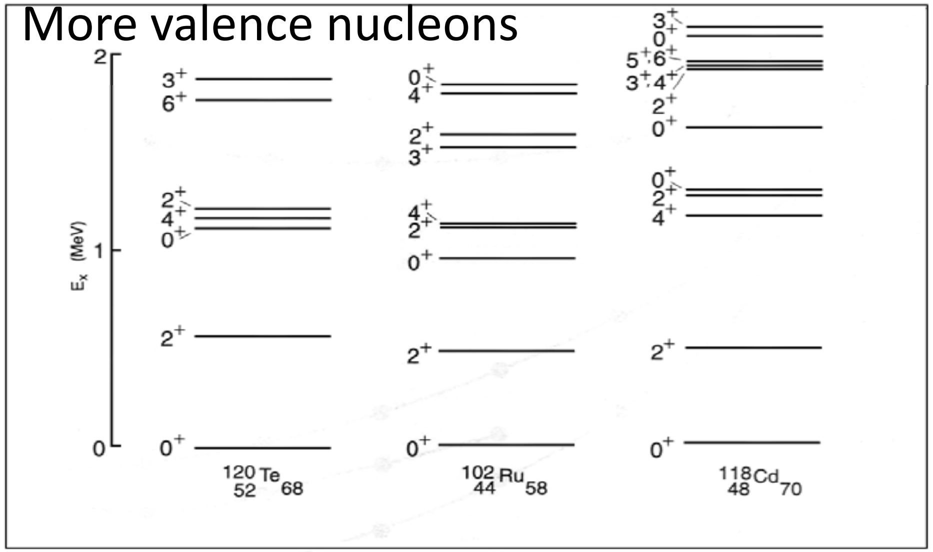
↓

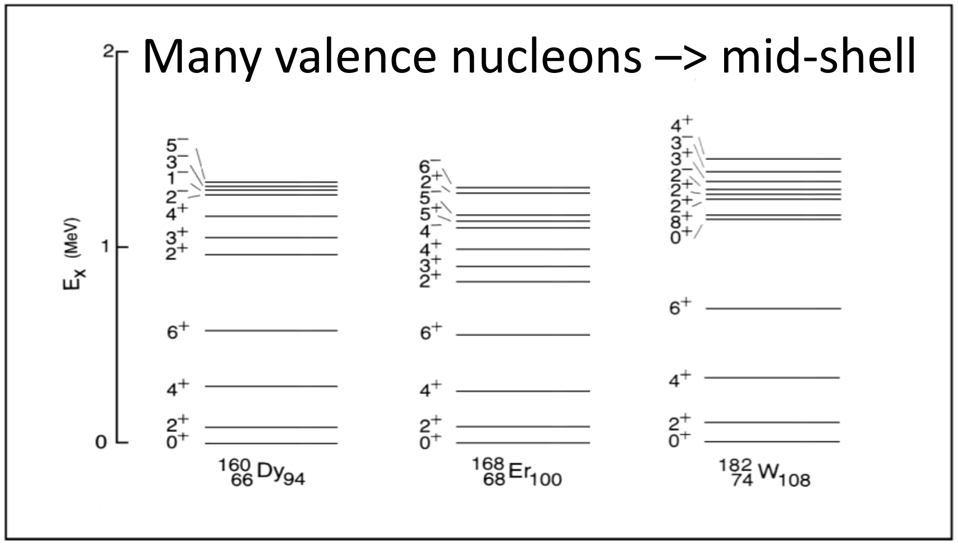
Identify additional degrees of freedom

Reminder of several types of spectra and where they occur

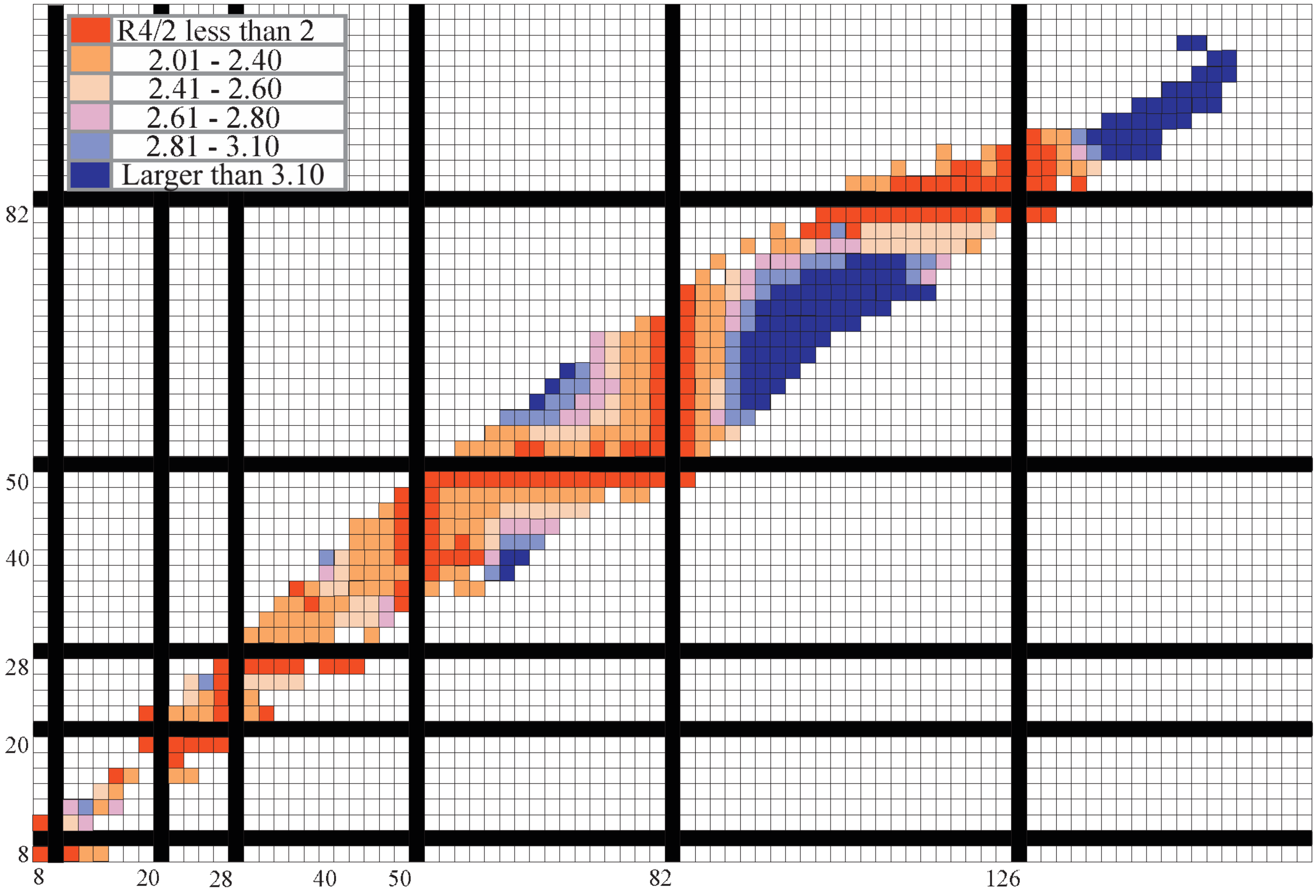




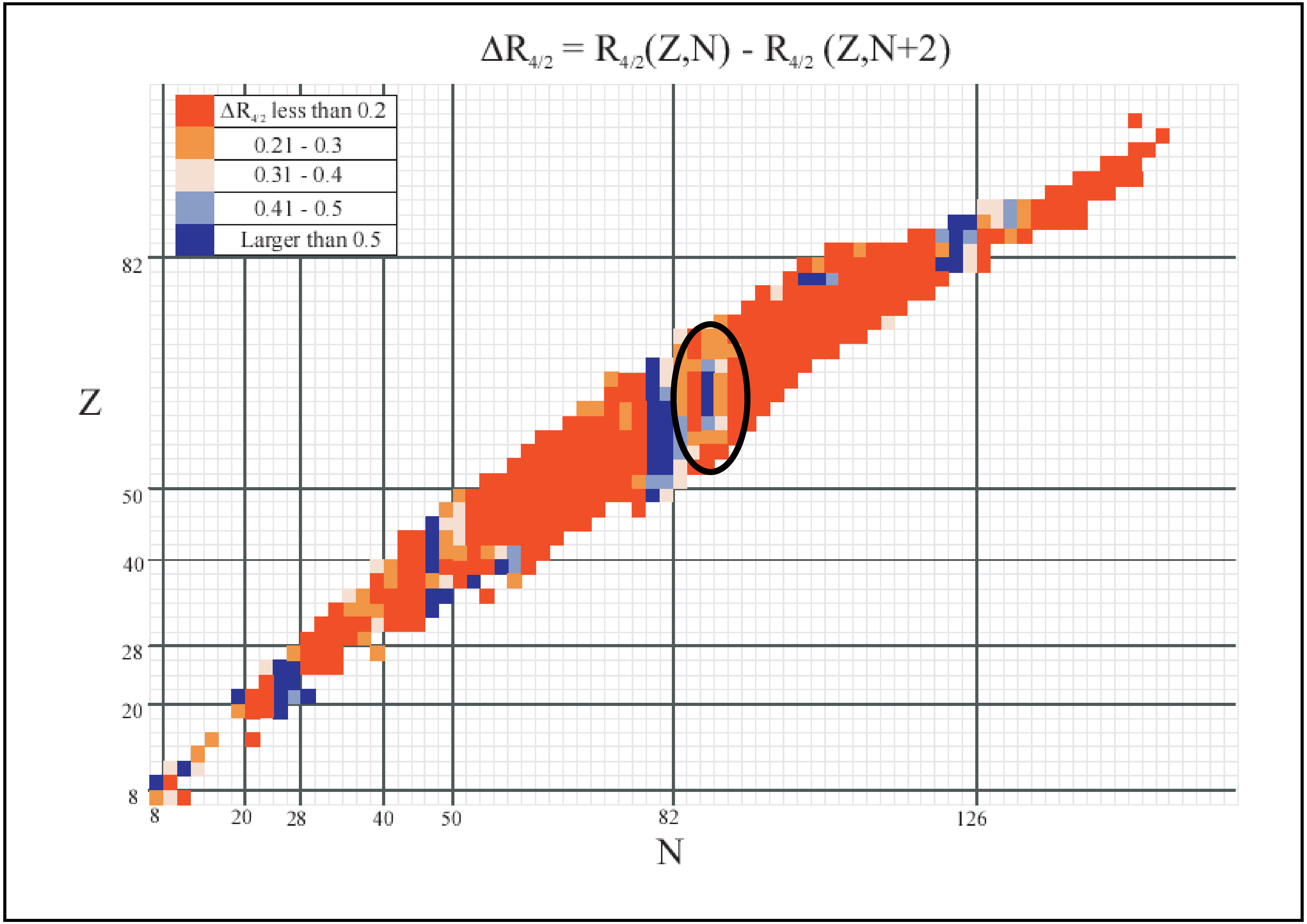




Broad perspective on structural evolution

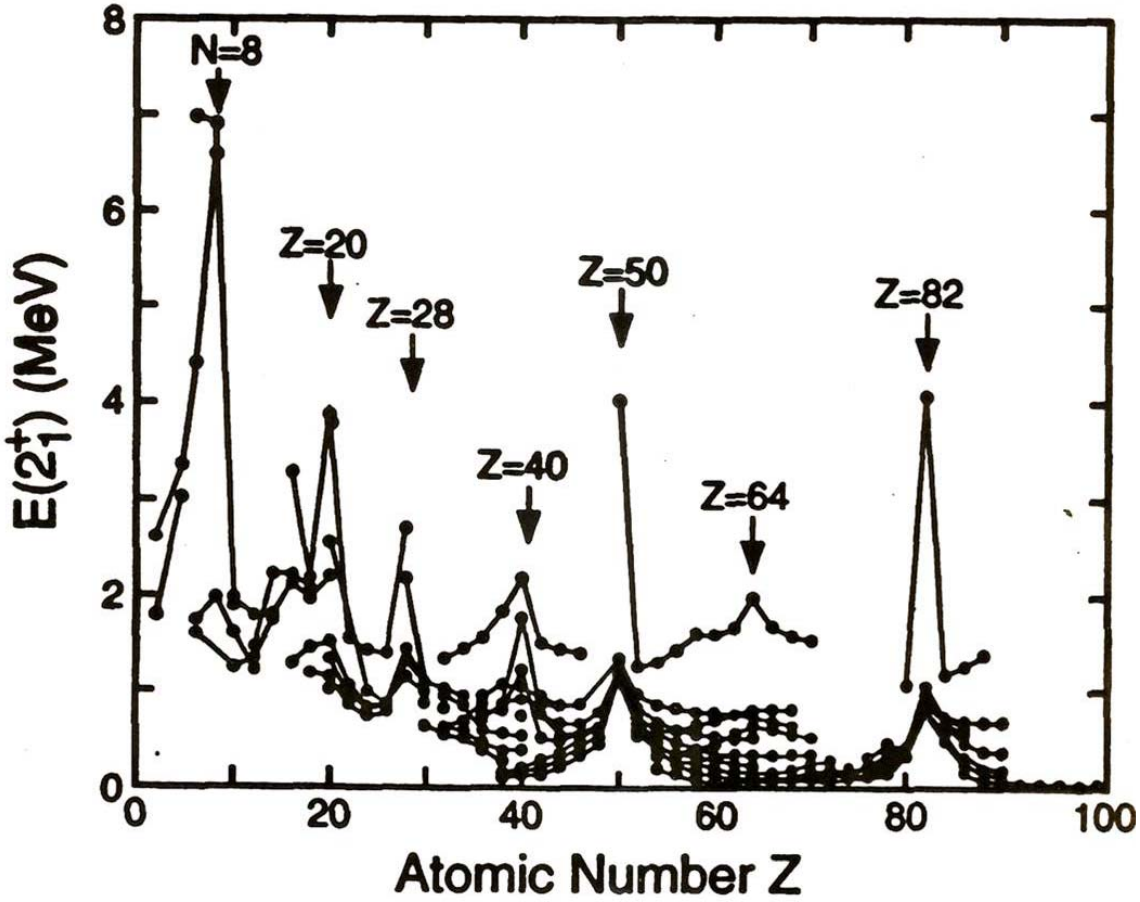


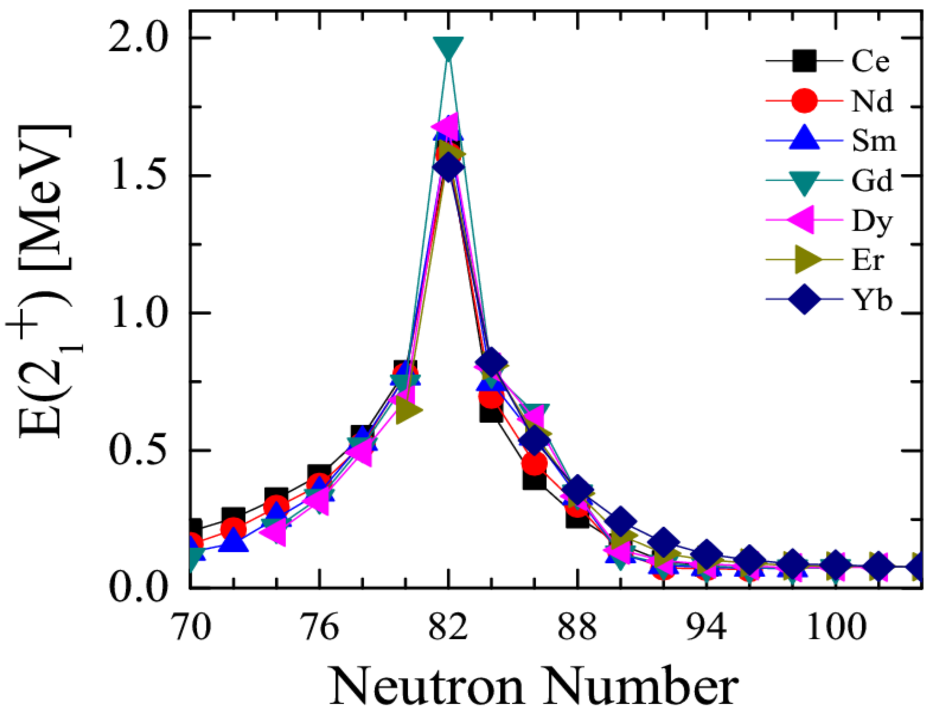
A special phenomenon – rapid structural change



Cakirli

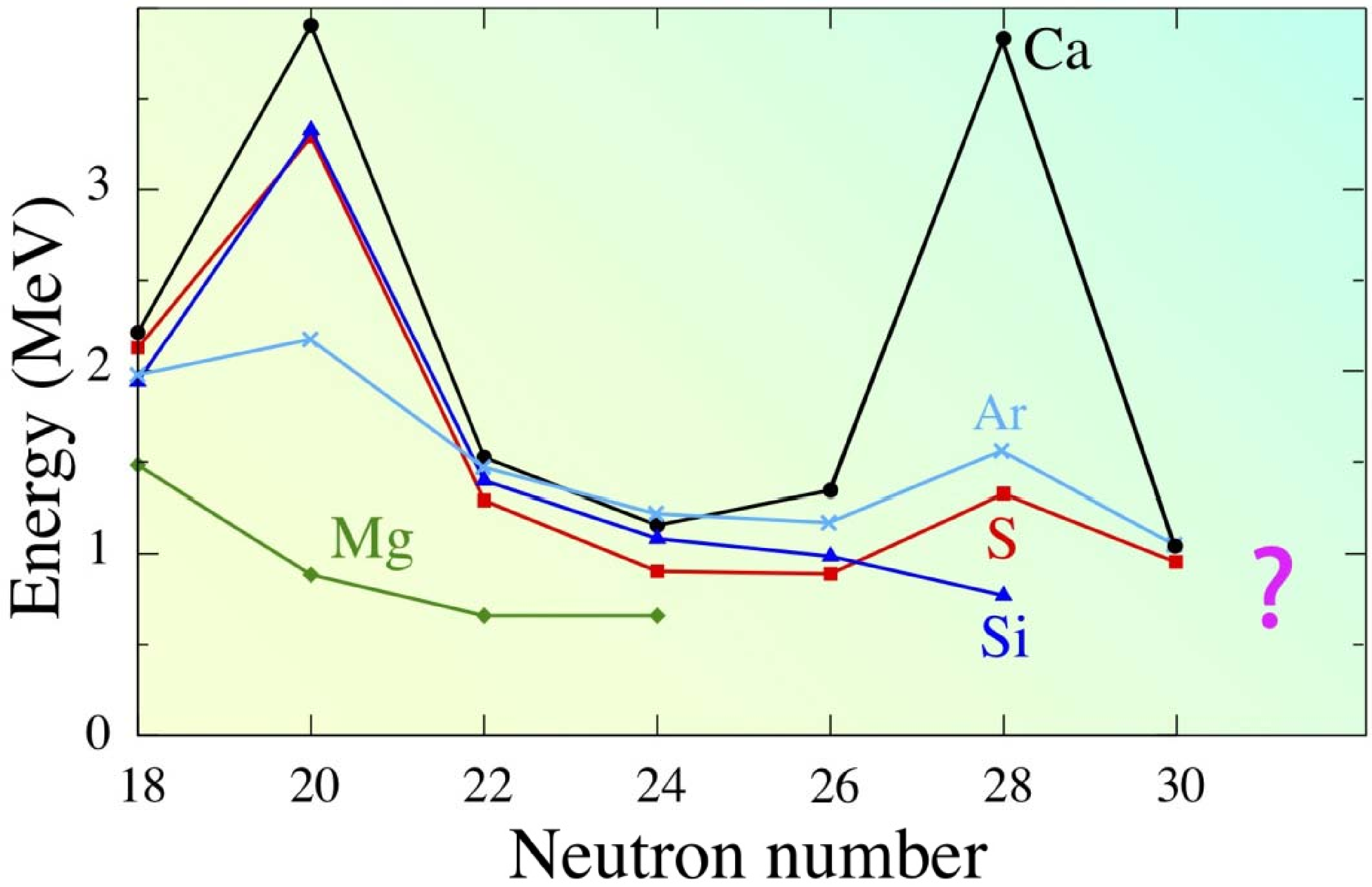
E(2$^+_1$)





E(2$^+_1$)
a simple measure of collectivity

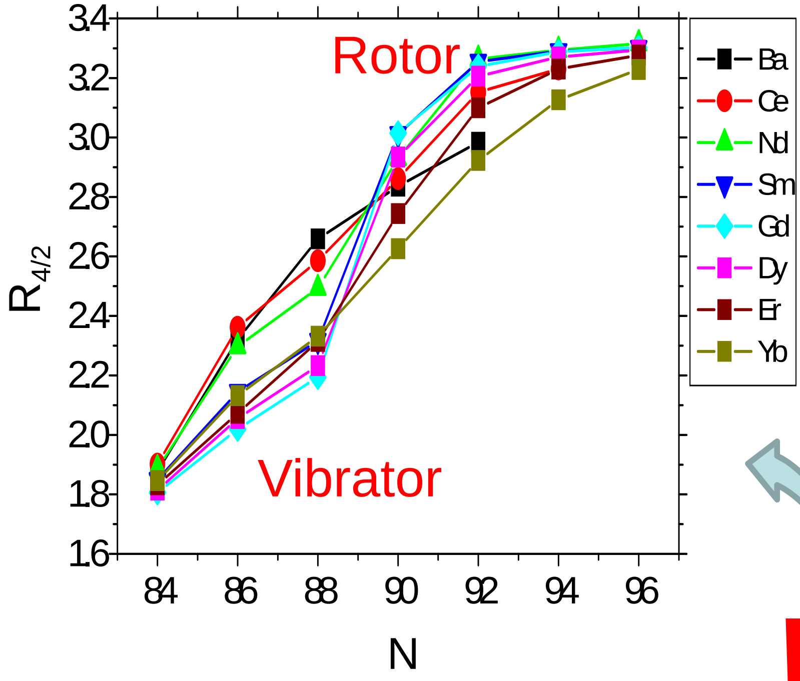
2^+ levels in neutron-rich nuclei



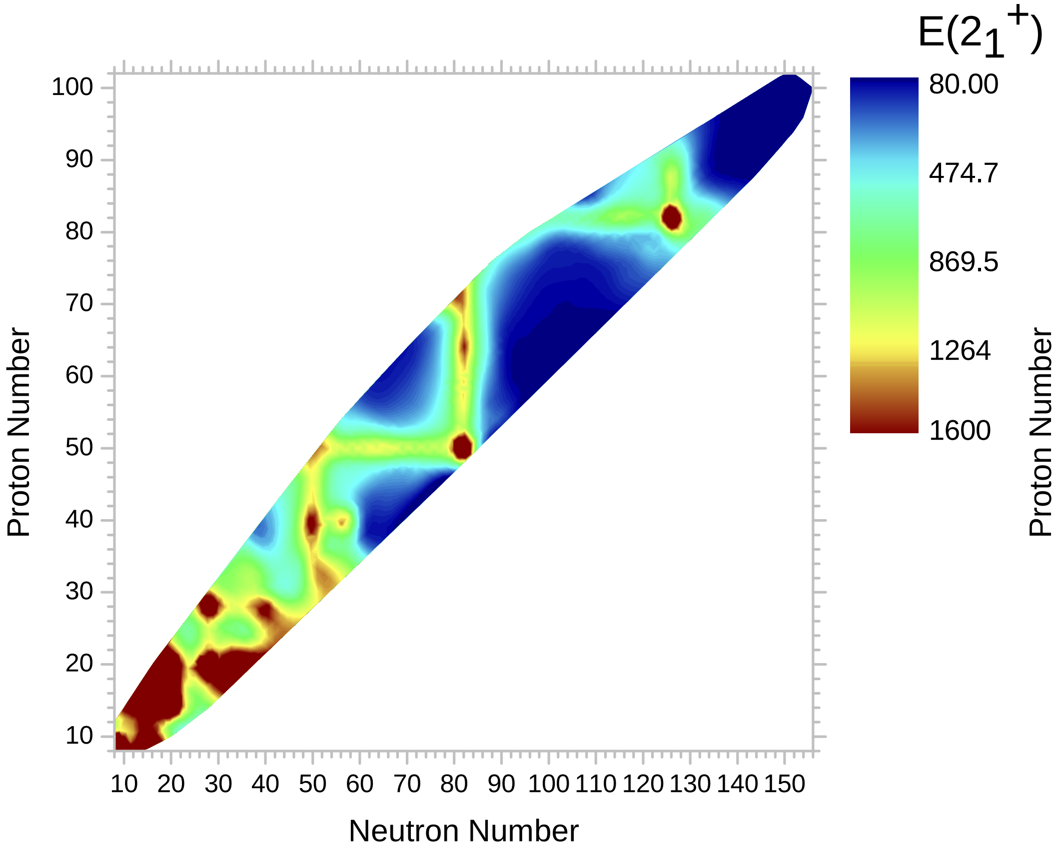
Note that N = 20 is NOT magic for Mg and N = 28 is NOT magic for Si and S !!!! Studying the evolution of shell structure is one of the most active and important areas of nuclear structure research today.

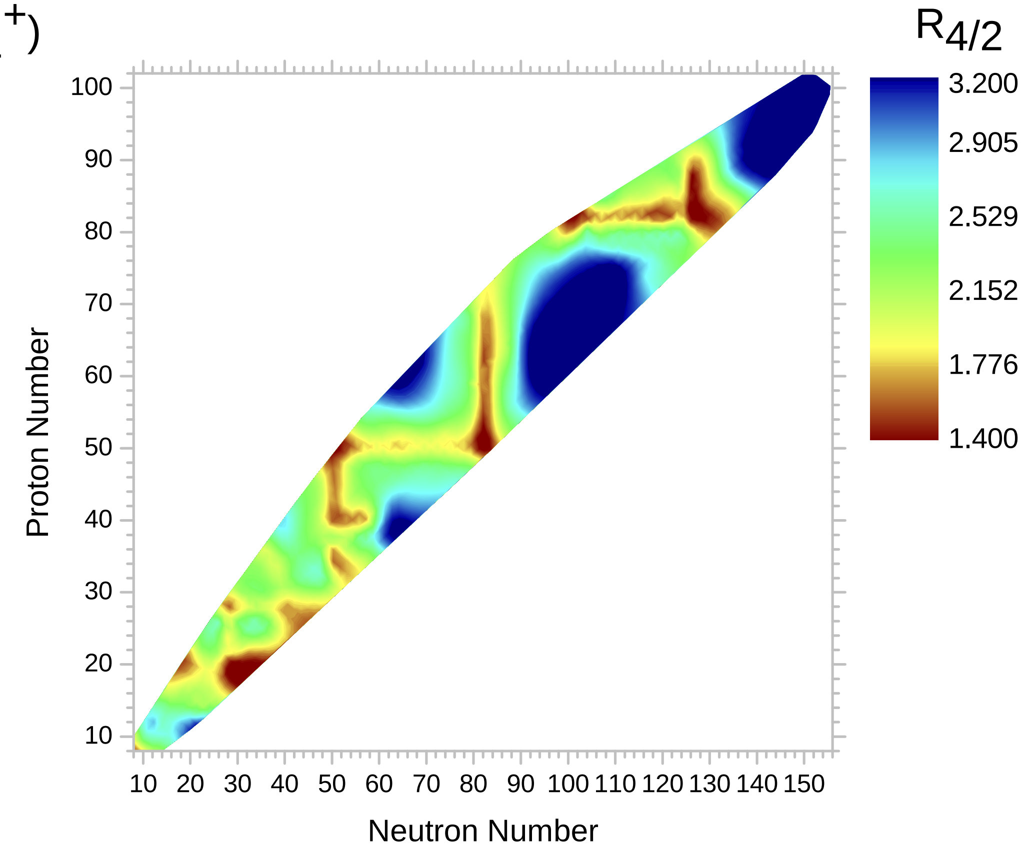
$R_{4/2}$ and $E(2^+_1)$

$R_{4/2}$ across a typical region



Broad perspective on structural evolution
Z=50-82, N=82-126





The remarkable regularity of these patterns is one of the beauties of nuclear systematics and one of the challenges to nuclear theory.

Whether they persist far off stability is one of the fascinating questions for the future

Cakirli

Think about the striking regularities in these data.

Take a nucleus with A ~100-200. The summed volume of all the nucleons is ~ 60 % the volume of the nucleus, and they orbit the nucleus ~ 10^{21} times per second!

Instead of utter chaos, the result is very regular behavior, reflecting ordered, coherent, motions of these nucleons.

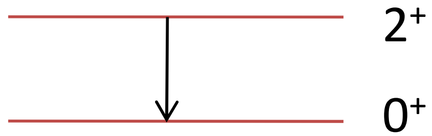
This should astonish you.

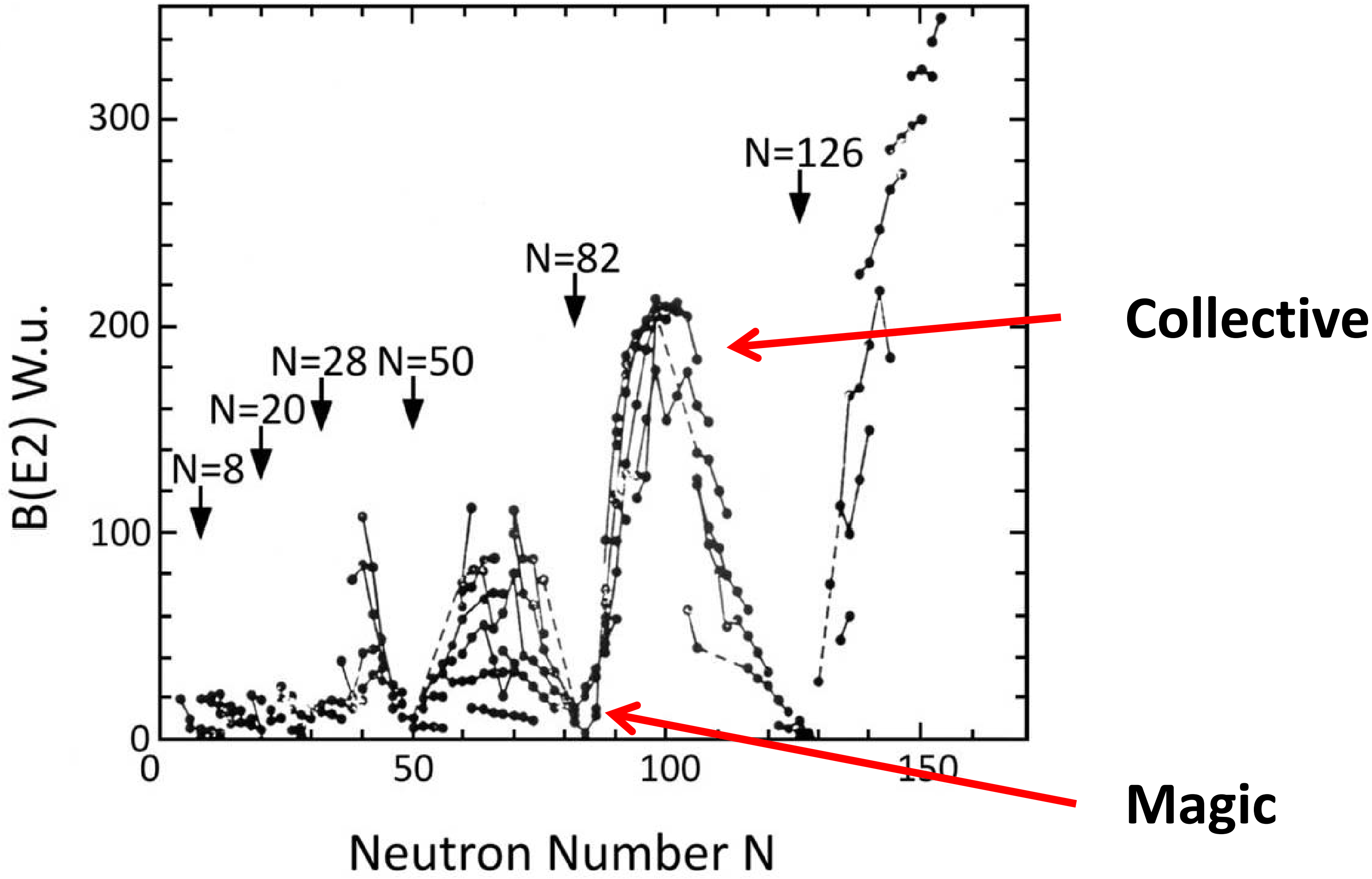
How can this happen??!!!!

Much of understanding nuclei is understanding the relation between nucleonic motions and collective behavior

Transition rates (half lives of excited levels) also tell us a lot about structure

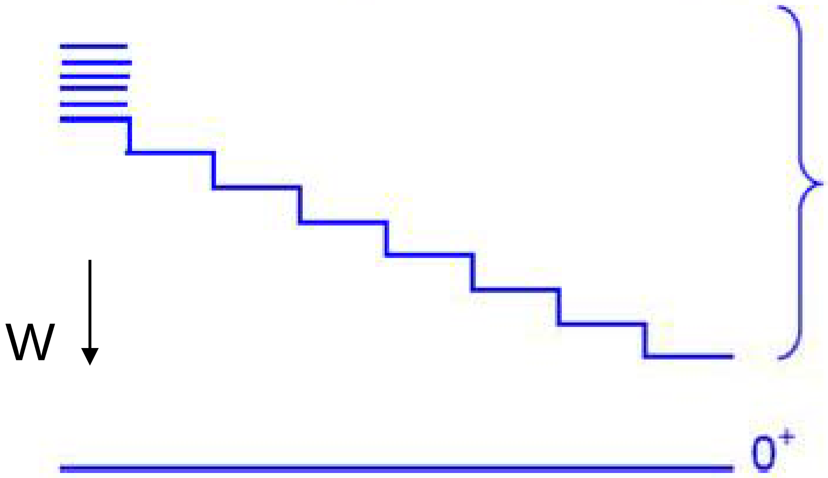
$$B(E2: 0^+_1 \rightarrow 2^+_1) \propto \langle 2^+_1 || E2 || 0^+_1 \rangle^2$$





Coherence and Transition Rates

Consider simple case of N degenerate levels: 2^+



$$\Delta E = (N-1)V$$

$$\Psi = a\varphi_1 + a\varphi_2 + \dots\, a\varphi_N$$

where $a = \frac{1}{\sqrt{N}}$

$$\left(\sum_{i=1}^{N} a^2 = \frac{N}{N} = 1 \right)$$

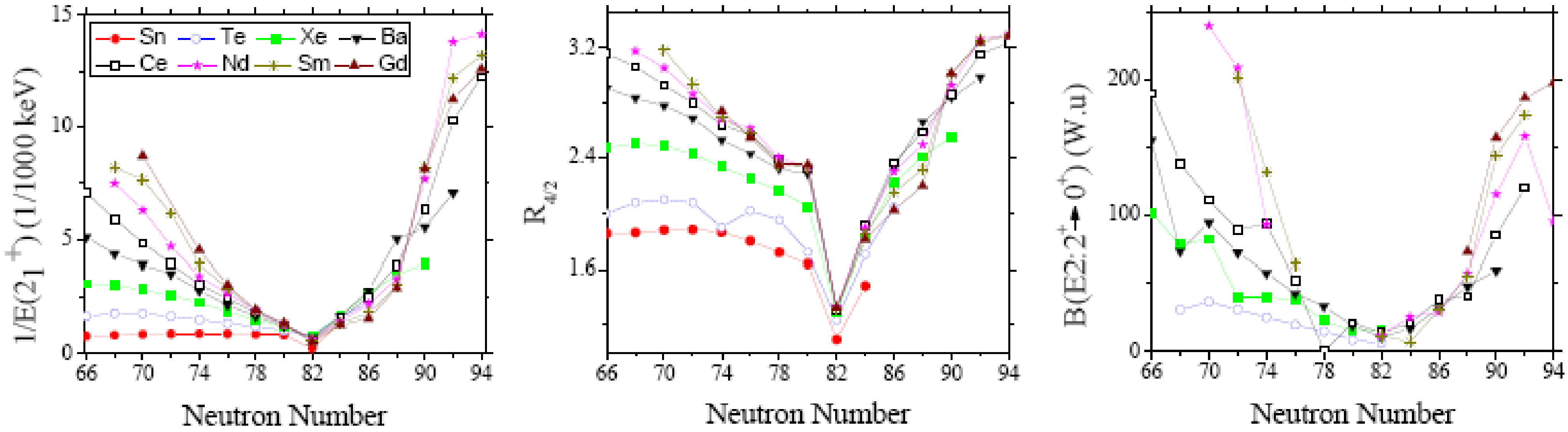
Consider transition rate from $2_1^+ \rightarrow 0_1^+$

$$B(E2;\, 2_1^+ \rightarrow 0_1^+) = \frac{1}{2J_i+1} \left\langle 0_1^+ \| E2 \| 2_1^+ \right\rangle^2$$

$$\left\langle 0_1^+ \| E2 \| 2_1^+ \right\rangle = \left\langle 0_1^+ \| E2 \| \Psi \right\rangle = a \sum_{i=1}^{N} \left\langle 0_1^+ \| E2 \| \varphi_i \right\rangle$$

The more configurations that mix, the stronger the B(E2) value and the lower the energy of the collective state. Fundamental property of collective states.

Alternate look:
Behavior of key observables centered on a shell closure



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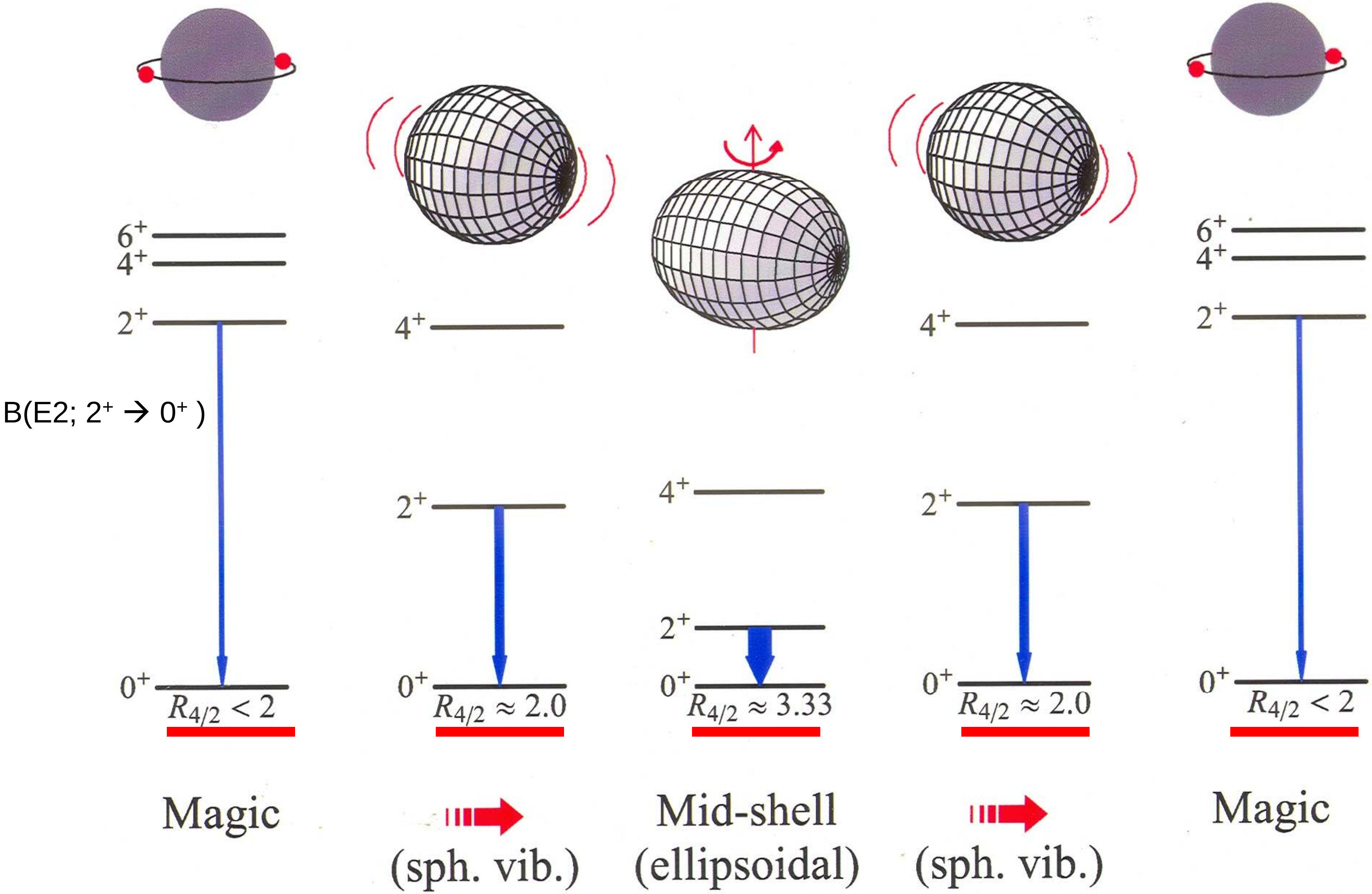
Evolution of nuclear structure
(as a function of nucleon number)

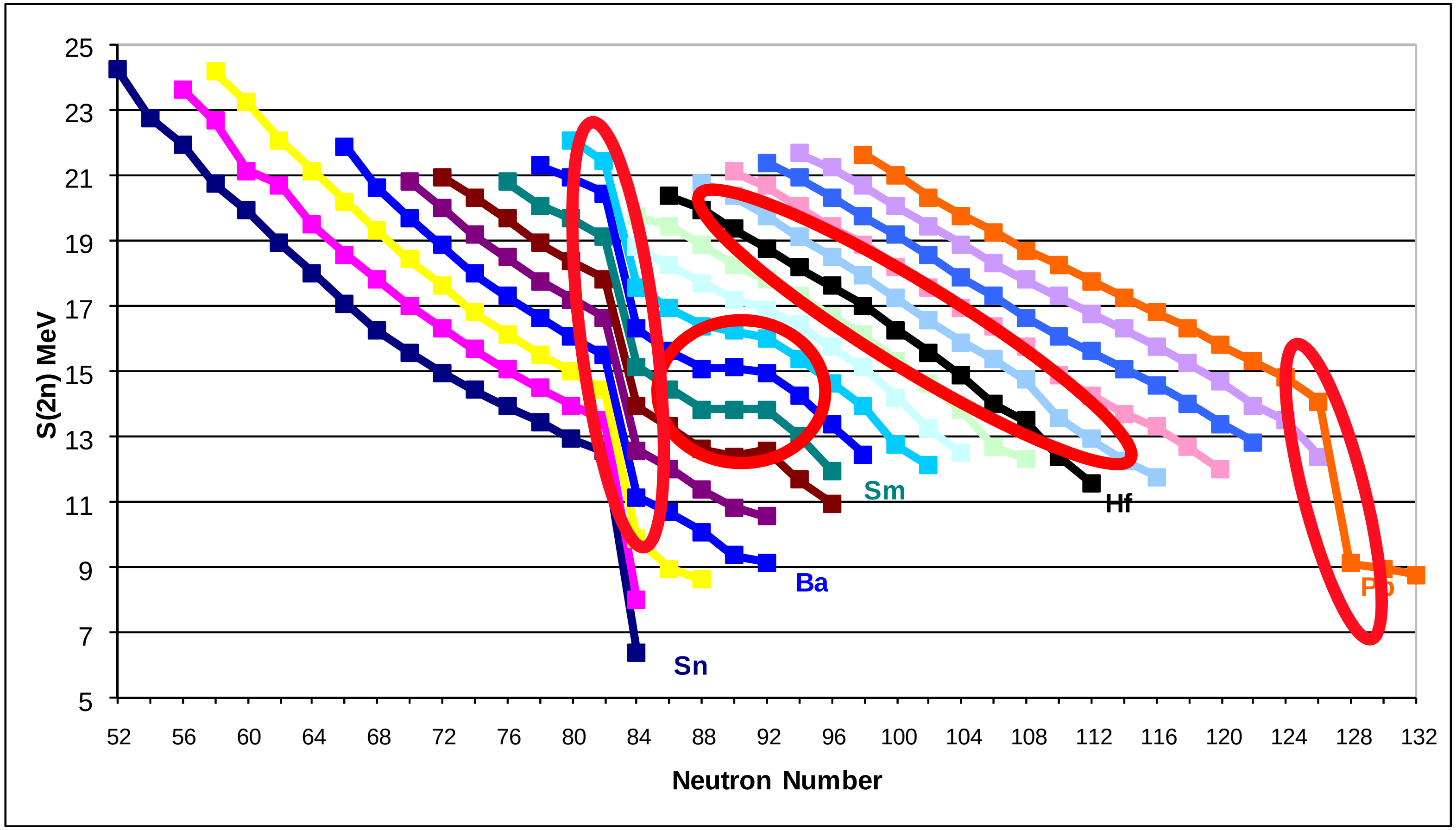


Two-neutron separation energies

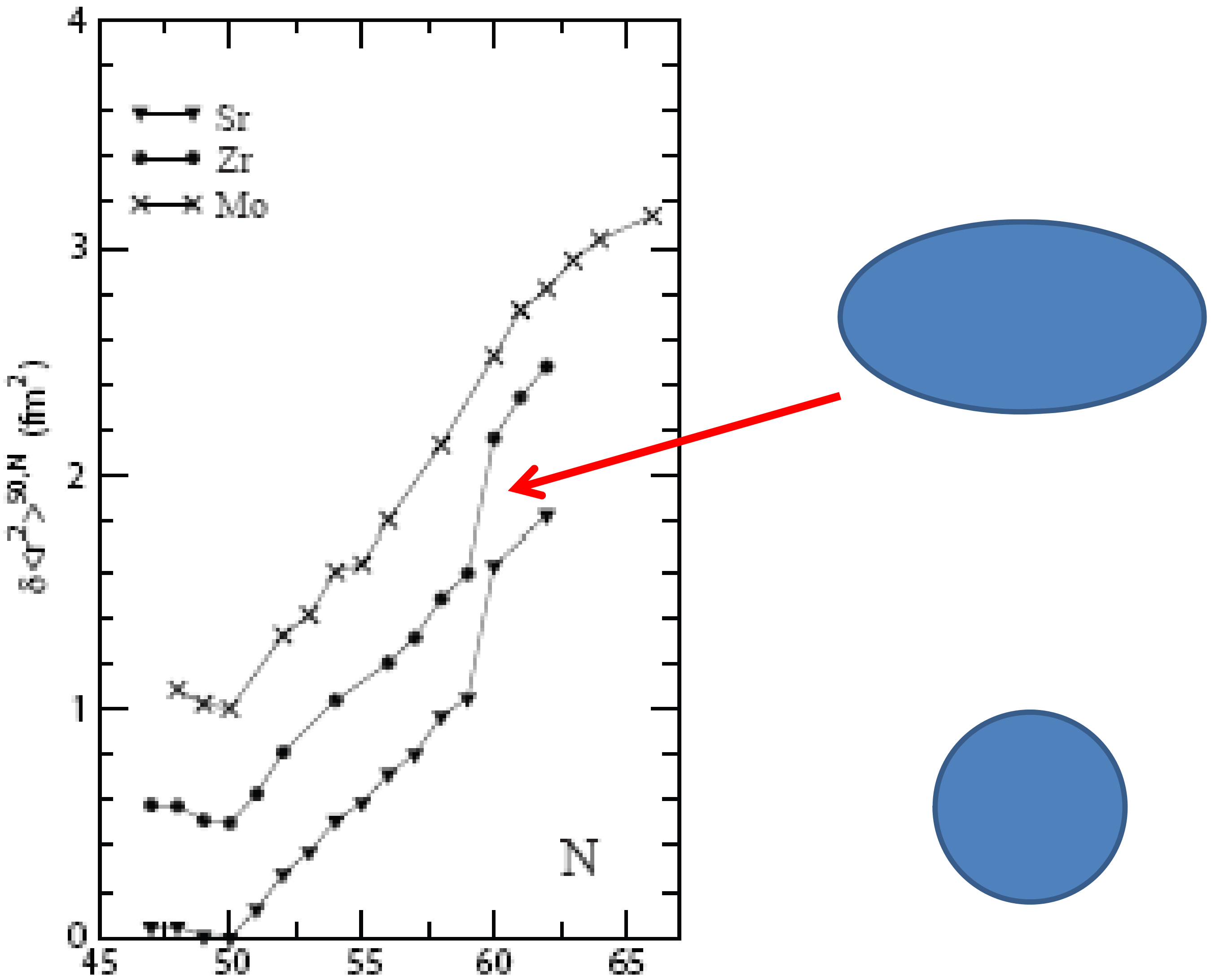
Normal behavior: ~ linear segments with drops after closed shells

Discontinuities at first order phase transitions

$$S_{2n} = A + BN + S_{2n}(\mathrm{Coll.})$$



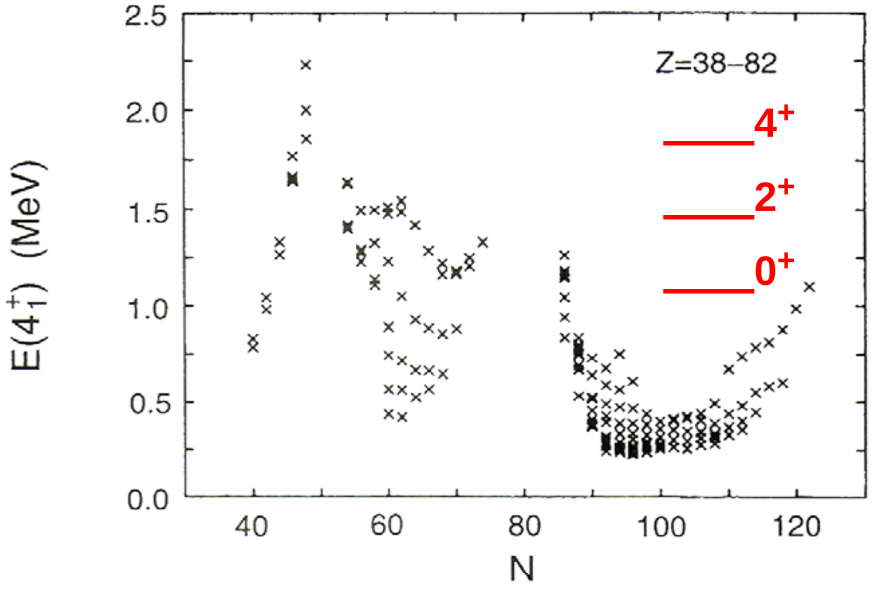
Isotope Shifts – sensitive to structural changes, especially deformation



So far, everything we have plotted has been an individual observable against N or Z (or A)

Now we introduce the idea of correlations of different observables with each other.

Correlations of Collective Observables



There is only one appropriate reaction to this result ….

Wow !!!!!!!!!

There is only one worry, however …. accidental or false correlations. Beware of lobsters !!!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LOBSTERS
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How can we understand collective behavior

- Do microscopic calculations, in the Shell Model or its modern versions, such as with density functional theory or Monte Carlo methods. These approaches are making amazing progress in the last few years. Nevertheless, they often do not give an intuitive feeling for the structure calculated**.**

- Collective models, which focus on the structure and symmetries of the many-body, macroscopic system itself. Two classes: Geometric and Algebraic

Geometrical models introduce a potential which depends on the shape of the nucleus. One can then have rotations and vibrations of that shape.

Algebraic models invoke symmetries of the nucleus and use group theoretical approaches to solve as much as possible analytically.

Nuclear Shapes

- Need to specify the shape. Need two parameters, β and γ. The concept of "intrinsic frame".
  - β specifies the ellipsoidal deformation of the shape. (We consider quadrupole shapes only – American football or frisbee shapes.)
  - γ specifies the amount of axial asymmetry
- $H = T + V(\beta,\gamma)$ Models are primarily a question of choosing $V(\beta,\gamma)$
- Kinetic energy contains rotation if the nucleus is not spherical. So we must specify orientation of the nucleus in space (the lab frame). Introduces three more coordinates, Euler angles.

The Geometric Collective Model

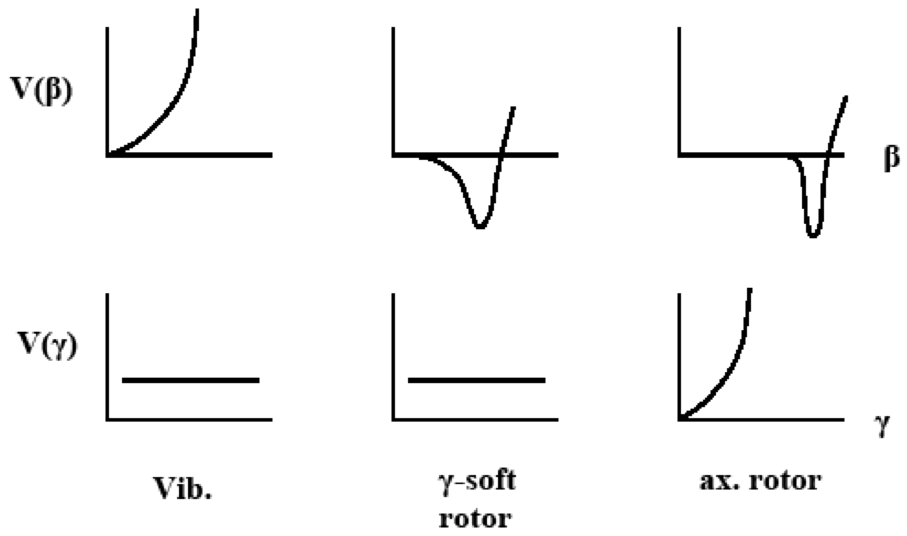
$$H = T + T_{rot} + V(\beta,\gamma)$$

$$V \sim C_2\beta^2 + C_3\,\beta^3 \cos 3\gamma + C_4\beta^4 + \ldots.$$

Six terms in all for the potential. These three are normally the only ones used as they allow a rich variety of collective structures without an explosion of parameters. In addition, there is a kinetic energy term.

This is a phenomenological model which cannot predict anything without being "fed". One selects simple data to help pinpoint the parameters, then uses the model to calculate other observables.

Geometric Collective Model



Vibrator:

$$V = C_2 \frac{1}{\sqrt{5}} \beta^2, \quad C_2 > 0$$

γ-soft:

$$V = C_2 \frac{1}{\sqrt{5}} \beta^2 + C_4 \frac{1}{5} \beta^4, \quad C_2 < 0, \; C_4 > 0$$

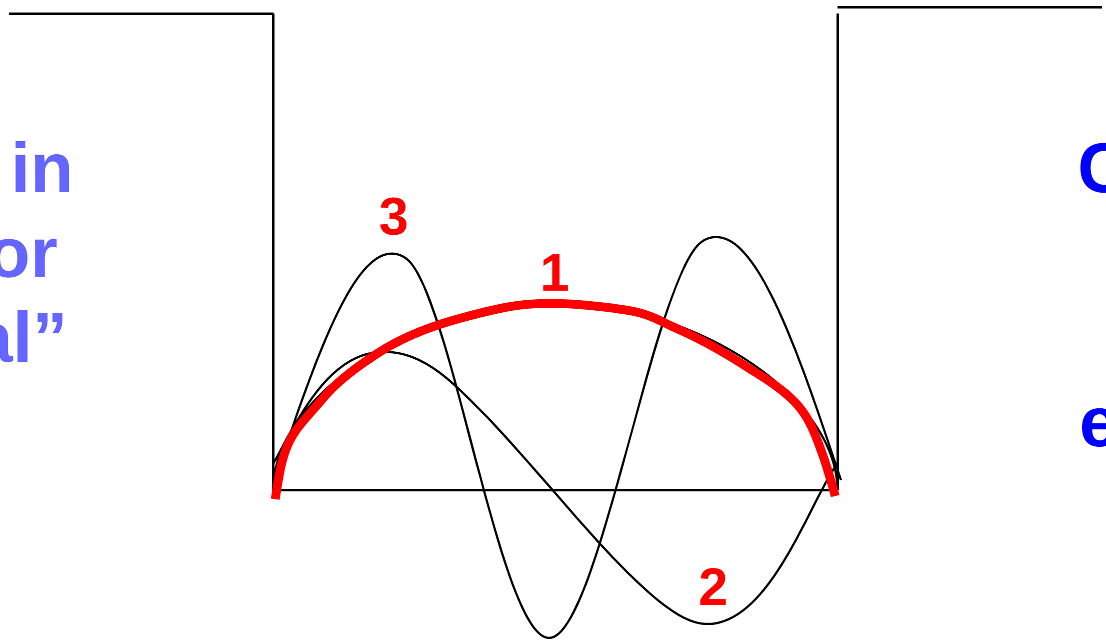
Rotor:

$$V = C_2 \frac{1}{\sqrt{5}} \beta^2 - C_3 \sqrt{\frac{1}{35}} \beta^3 \cos 3\gamma + C_4 \frac{1}{5} \beta^4$$

$$C_2 < 0, \quad C_3 > 0, \; C_4 > 0$$

Key ingredient: Quantum mechanics -- confinement

Particles in a "box" or "potential" well

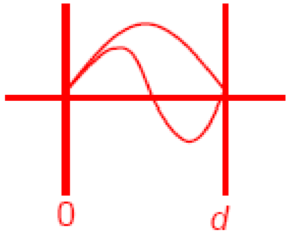


Confinement is origin of quantized energies levels

Energies in an Infinite Square Well

(box)

Simple Derivation



$\Psi(0) = \Psi(d) = 0$

for containment

$$\therefore \frac{n\lambda}{2} = d \qquad n = 1, 2, \cdots$$

Now, use de Broglie relation

$$P = \frac{h}{\lambda} \qquad \text{and} \qquad E = \tfrac{1}{2} m v^2 = p^2/2m$$

$$\text{or} \quad p = \sqrt{2mE}$$

$$\therefore \frac{nh}{2p} = \frac{nh}{2\sqrt{2mE}} = d$$

$$\therefore \frac{n^2 h^2}{8mE} = d^2$$

or $$\boxed{E = \frac{n^2 h^2}{8 m d^2}} \qquad n = 1, 2, \cdots$$

Zero point motion

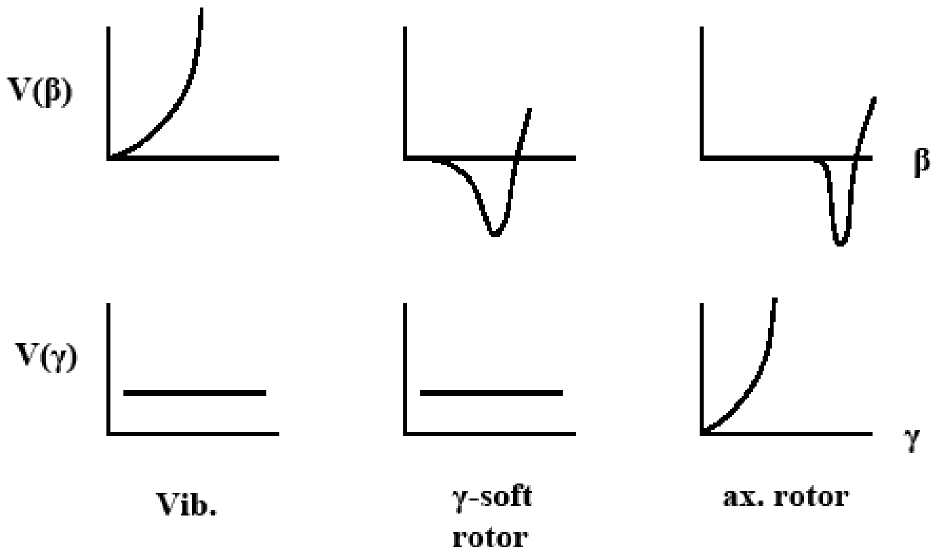
a) confinement

b) wave/particle relation

} → quantization

Geometric Collective Model

$$H = T + T_{rot} + V(\beta,\gamma)$$



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$$C_2 < 0, \quad C_3 > 0, \quad C_4 > 0$$

Next time …

- Geometric models
- Types of collective nuclei
- The microscopic drivers of collectivity – the valence p-n interaction
- Simple ways of estimating the structure of any nucleus
- Introduction to the Interacting Boson Approximation (IBA) Model