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PHYSICS OF NEUTRON STARS[1]

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1 INTRODUCTION

Since the first identification of neutron stars, in pulsars, a decade ago, theoretical and observational knowledge of these unusual objects has grown at a rapid rate. In this article we describe developments that have taken place since our 1975 review article on neutron stars (Baym & Pethick 1975, referred to hereafter as BP), as well as review several of their more astrophysical aspects not discussed there.

The most striking observational fact about neutron stars is their existence: at present 321 pulsars, which are generally accepted to be rapidly rotating neutron stars, have been observed in our galaxy (Manchester et al. 1978, Taylor & Manchester 1977, Manchester & Taylor 1977, Smith 1976). In addition, most of the 16 pulsating compact X-ray sources so far discovered are likely to be accreting neutron stars in close binaries (for a review see Lamb 1977). The association of the Crab and Vela pulsars with supernova remnants provides evidence for the formation of neutron stars in supernovae, a picture supported to a limited extent by comparison of pulsar populations and lifetimes with estimated supernova rates (reviewed in Manchester & Taylor 1977). Optical and X-ray observations of binary X-ray sources provide the possibility of determining the masses of the neutron stars in these objects

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(reviewed in Bahcall 1978); the results are consistent with present theories of neutron star structure and formation in supernovae. Quoted results, with statistical errors, include, for example, $M_{\text{Her X-1}} = 1.33 \pm 0.2\ M_\odot$ (Middleditch & Nelson 1976) and $M_{\text{Vela X-1}} = 1.5 \pm 0.2\ M_\odot$ (van Paradijs et al. 1976, Rappaport et al. 1976); however as Bahcall (1978) emphasizes, systematic errors in determination of these masses could lead to significantly greater true uncertainties.

Measurements of the surface thermal luminosity of a neutron star (in the soft X-ray, for expected surface temperatures) can allow one to deduce its surface temperature, T_e. By this method Wolff et al. (1975) have placed an upper bound, $T_e \leqq 4.7 \times 10^6$ K, for the neutron star in the Crab Nebula. Additional bounds have been reported by Greenstein et al. (1977).

Surface magnetic fields of neutron stars in active pulsars and binary X-ray sources are inferred, from models of these systems, to be $\sim 10^{12}$ G. The principal observational inputs are, for pulsars, the rates of energy loss (reviewed in Ruderman 1972), and, for X-ray sources, the structure of the radiation and spinup rates (Lamb 1977, Ghosh & Lamb 1979). Trümper et al. (1978) observed a feature in the hard X-ray spectrum of Her X-1, which if correctly interpreted as electron cyclotron absorption at ~ 42 keV, would imply a neutron star field of 4×10^{12} G (or if emission at 58 keV, a field $\sim 6 \times 10^{12}$ G).

Information on moments of inertia of neutron stars may be obtained from observations of the secular rates of change of their spin periods. Comparison of the slowdown rate of the Crab pulsar with the luminosity of the Crab Nebula provides a lower bound on its moment of inertia ($> 1.5 \times 10^{44}$ g cm^2) (Ruderman 1972) while observed speedups, on time scales $\sim 10^2$–10^5 yr, of pulsating X-ray sources, combined with model descriptions of accretion torques, indicate moments of inertia and radii consistent with the compact objects being neutron stars (Elsner & Lamb 1976, Rappaport & Joss 1977, Ghosh & Lamb 1979). The wealth of detailed observations of short term variations of pulse arrival times, for both pulsars and pulsating X-ray sources, offers the prospect of enabling one to deduce, within the framework of theoretical models, information about the internal structure of neutron stars. For descriptions of such work see, for example, Lamb (1977), Lamb et al. (1978), and Pines et al. (1974), as well as Section 4.3.

We remind the reader of the general structure of neutron stars. Typical radii are ~ 10 km, masses $\sim M_\odot$, and central densities exceed that of nuclear matter, $\rho_0 \equiv 2.8 \times 10^{14}$ g cm^{-3}. Neutron stars have a solid crust, ~ 1 km thick, beneath which is a liquid interior, likely superfluid in part, beginning at density $\sim \rho_0$. A number of possible phases of matter at densities $\rho \gtrsim 2\rho_0$ have been investigated, but which actually occur remains somewhat uncertain. We now review recent developments on the equation of state. In subsequent sections, we discuss models of neutron stars and dynamical properties.

2 RECENT DEVELOPMENTS ON THE EQUATION OF STATE

The structure of neutron star matter is reasonably well understood (see BP) up to about nuclear matter density ρ_0 where the crust dissolves. In the very low density regime principal progress has been a better description of the properties of matter near the surface in strong magnetic fields. Recent work in the theory of nuclear matter has called into question our understanding of the properties of the liquid regime in the neighborhood of ρ_0, and brought out the sensitivity of the stellar radius and crust thickness to microscopic details of the equation of state in this regime. States of higher density matter that have received considerable recent attention are pion condensation, quark matter, and "abnormal matter" in which the nucleons become essentially massless entities. For detailed reviews of the physics of higher density matter see Baym (1977a,b, 1978).

2.1 *The Liquid Regime*

As in ordinary nuclear matter theory, calculation of the properties of neutron rich matter in the liquid regime is presently beset by a number of uncertainties: the choice of the two-body interaction and how to calculate with it, the role of the internally excited state of the nucleon—the isobar Δ(1236 MeV), or N*—in intermediate states in nucleon-nucleon scattering, and the proper inclusion of tensor correlation effects. Until recently, the phenomenological Reid soft-core nucleon-nucleon potential, fit to phase shifts, was generally felt to be satisfactory for use in calculation of nuclear and neutron star matter. The calculational methods used have been Brueckner-Bethe-Goldstone "nuclear matter theory", which in lowest order sums contributions from two-body scattering processes, and variational techniques based on trial wave functions. See Bethe (1971) and BP for reviews of such calculations.

In the past several years the inadequacies of the conventional calculations have become apparent (Pandharipande et al. 1975, Negele 1976, Bäckman et al. 1972), and more accurate calculational techniques are currently being developed (see reviews by Clark 1978 and Day 1978). With these improved calculations, the indications are that when one uses common phenomenological nucleon-nucleon interactions, such as the Reid, the calculated binding energy and saturation density (i.e. equilibrium zero-pressure density) of symmetric nuclear matter (equal number of neutrons and protons) are too large; the conclusion, given the correctness of these calculations, is that the Reid soft-core potential is too soft. Whether better two-body interactions, derived theoretically from dispersion theory, will give more accurate results for symmetric nuclear matter than the Reid potential remains to be seen. Present indications are that for Reid type interaction potentials the Brueckner method is adequate for neutron matter up to $\rho \sim 2\rho_0$; however, improved potentials can be expected to modify previous results, e.g. the Baym-Bethe-Pethick-Pandharipande and Bethe-Johnson equations of state (surveyed by Canuto 1974).

A second important question, which has great consequences for the structure of neutron stars, is the effect of the medium on the interaction potential itself. The important attractive components of the nucleon-nucleon interaction are believed to arise from processes (analogous to the atomic van der Waals interaction) in which the two nucleons scatter, via pion exchange, to virtual intermediate states in which, for example, one or both nucleons are excited to a Δ state. Since such intermediate states generally have higher energy than the initial states, these processes produce (through the usual second order perturbation formula) a net attraction. In the two-nucleon scattering problem the energies of the intermediate states in these processes have their free-space values, and any potential that fits nucleon-nucleon phase shifts implicitly takes the intermediate states to be in free space. However, as Green and Haapakoski (see Green 1976) pointed out, the nuclear medium will have two important effects on the intermediate range attraction. First, the Pauli exclusion principle forbids processes in which one of the intermediate nucleon states is already occupied; this effect eliminates some of the attraction. Second, because the particles are not in free space the intermediate state energy denominators in such processes will also be modified; this "dispersion correction" also tends to reduce the attraction. The net result is a decrease in the intermediate range attraction in the medium, which becomes more important with increasing density. This effective repulsion is not taken into account in calculations that use a phenomenological two-body interaction.

The process most strongly affected is that in which just one of the nucleons is excited to a Δ state, a process particularly important in neutron matter. [The reason is that two neutrons, or two protons, must have total isospin $T = 1$, while a neutron and proton can have total isospin $T = 1$ or 0. In the isobar process the nucleon plus Δ in the intermediate state can only have $T = 1$ or 2 (the isospin of the Δ is $\frac{3}{2}$) and thus this process occurs only in $T = 1$ states.] To compute the effects of the medium one must solve a coupled channel problem, treating the Δ as an elementary particle that can be present in the medium. Detailed calculations (Holinde & Machleidt 1977, Green 1976, and references therein) show that with Δ's explicitly included, one finds that at higher densities (above ρ_0) the $T = 1$ interactions can change from attractive to repulsive. The effect in symmetric nuclear matter is to lower the saturation density (since in the conventional calculations only interactions in $T = 0$ states became repulsive at higher density; the $T = 1$ interactions remained attractive).

In neutron stars, this effect implies a stiffening, for ρ around ρ_0, of the equation of state, i.e. an increase of the pressure for given density (Smith & Pandharipande 1976), and thus it tends to make a neutron star of a given mass larger in size and lower in density, as well as making the crust increase in mass and volume (Pandharipande et al. 1976). Effects on models are reviewed in Section 3 below.

Another effect on neutron stars is an increase, compared with the predictions of the Reid potential, in the proton fraction in the matter at higher densities. Because in the Reid calculations the $T = 0$ interactions, which are effective between protons and neutrons, become repulsive at high density, while the $T = 1$ do not, it is expensive to have protons at higher densities. However if the $T = 1$ interactions also become repulsive, it is then more favorable to have a fraction of the nucleons be protons.

Calculations for symmetric nuclear matter that include Δ's have been carried out in Brueckner theory and by variational methods only in low order, and do not produce the correct saturation density and binding energy. Also, the matrix elements describing the transitions from the initial neutron-neutron state to the intermediate nucleon-Δ state are uncertain, especially at higher momentum transfer. Thus, the implications for neutron stars should be regarded as tentative. In particular, tensor correlation effects (see, for example, Friman & Nyman 1978), which tend to soften the matter (as well as lead to pion condensation), have not yet been adequately included in either nuclear matter or neutron star matter calculations. Until one has a satisfactory theory of symmetric nuclear matter, the equation of state of neutron star matter must be regarded as uncertain.

The neutron liquid both in the crust and interior as well as the proton liquid in the interior are believed to be superfluid. Pairing calculations, reviewed in BP, indicate that at lower densities the neutrons are paired in $^1S_0$ states, as are the protons in the interior, while at densities $\gtrsim \rho_0$, the neutrons are instead paired in $^3P_2$ states. The calculated energy gaps, a measure of the strength of the superfluid pairing, depend sensitively, however, on detailed assumptions about the interactions between nucleons. Clark et al. (1976) have examined the sensitivity of the neutron $^1S_0$ gaps to interactions between neutrons induced by particle and spin density fluctuation effects, and conclude that at low densities ($\lesssim 0.1\,\rho_0$) such effects reduce the effective interactions between neutrons that are responsible for pairing by $\sim 30\%$, and reduce the gap and corresponding transition temperature by a factor of ~ 3. Further explorations of effects on both neutron and proton gaps of polarization-induced interactions would be useful. Variations in nucleon effective masses assumed in the calculation of pairing strengths can also lead to modifications of the gaps of similar magnitude.

Sauls & Serene (1978) have estimated, via the Ginzburg-Landau approach, corrections to the weak coupling BCS calculations of $^3P_2$ neutron pairing, and conclude that such corrections should not produce a qualitative change in the properties of the superfluid. Effects of superfluidity on cooling and dynamics are discussed in Section 4.

2.2 *Pion Condensation*

A pion-condensed state of matter is one in which the pi meson field, which normally fluctuates about nucleons, develops a nonzero expectation value. In general, pion condensation in matter tends to soften the equation of state, countering the stiffening effect of Δ isobars. It is also of astrophysical interest for its important enhancement of neutrino cooling of neutron stars (described in Section 4.2). Furthermore, pion condensation might lead to possible solidification of high density matter.

Present calculations (reviewed in Brown & Weise 1976, Migdal 1978) indicate the onset of condensation of the charged-pion field in neutron matter at a density $\sim 2\rho_0$. In this charged pion-condensed state the neutrons become rotated in isospin space into coherent superpositions of neutrons and protons, with the microscopic pion field carrying a compensating negative charge density. Methods for describing the properties of the condensed state based on the chiral symmetry of low energy pion-nucleon physics, including effects of isobars and nuclear correlations, have been developed by Campbell, Dashen, and Manassah, and Baym, Au, and Flowers (see Baym & Campbell 1979 for a detailed review and list of references to earlier work). Brown & Weise (1976) have calculated equations of state for spatially uniform charged pion-condensed neutron matter, and Au (1976) has extended this work to include effects of beta equilibrium. The calculations of Migdal and collaborators on the properties of the condensed state are reviewed in Migdal (1978). Calculations of a spatially uniform neutral π^0-condensed state of neutron matter are given by Dautry & Nyman (1979).

As an example of the effect on the equation of state that can be produced by pion condensation we note that Au (1976) finds at $3\rho_0$ a reduction $\sim 75\%$ in the pressure from its value in the noncondensed state. The detailed modification of the equation of state by pion condensation is quite sensitive, however, to the magnitude of the effective nucleon-nucleon interactions (the Landau Fermi-liquid parameter g') assumed, a quantity somewhat uncertain, both theoretically and experimentally, for high density neutron matter; extrapolation of pion-nucleon scattering amplitudes to the "off-shell" regime of pion condensation results in further uncertainties. Thus, present estimates of the modification of the equation of state by pion condensation should be regarded as preliminary but illustrative. A full reliable calculation of the equation of state including the stiffening effects of isobars described in Section 2.1 together with the softening effects of pion condensation has yet to be carried out.

A question of substantial interest is whether neutron matter in the deep interior of a neutron star can solidify. Calculations based on conventional two-body forces acting between neutrons (reviewed in BP) indicate that solidification of neutron matter would not take place. However, as Pandharipande & Smith (1975a) described, π^0 condensation offers a possible mechanism for producing a solid state. Takatsuka et al. (1978) have shown similarly that π^0 condensation might lead to a one-dimensional "solidification" of the matter, analogous to a liquid crystal. The answer to whether such states can actually occur in neutron star matter must await a fuller understanding of the nuclear matter problem.

2.3 *Field Theoretic Models of High Density Matter and the Abnormal State*

At densities much greater than ρ_0 the meson clouds surrounding the nucleons in matter become strongly overlapping, and one does not expect a description in terms of distinct particles—neutrons, protons, etc.—interacting via two-body forces to remain valid. One approach that has been explored is to describe high density matter in terms of relativistic "bare" nucleons interacting via explicit meson fields. The basic type of model is that given by Walecka (1974) in which the nucleons interact attractively via coupling to a scalar meson field σ, and repulsively through coupling to a more massive vector field ω. The meson fields are assumed to be linear, i.e. not coupled to themselves. Chin & Walecka (1974) fitted the coupling constants and meson masses in the theory to reproduce, in the mean field approximation, the properties of symmetric nuclear matter and then derived a neutron matter equation of state. Similar calculations on this model were carried out by Bowers et al. (1975), and by Pandharipande & Smith

(1975b) who include pion exchanges as well. The physics of the model has been explored in further detail by Chin (1977).

Walecka's model predicts that at high densities the energy density $\varepsilon \equiv \rho c^2$ equals $\frac{1}{2}(g_\omega^2/m_\omega^2)n^2$, where n is the baryon density, g_ω is the ω meson-nucleon coupling constant, and m_ω is the ω mass. The behavior $\rho \sim n^2$, which is also characteristic of calculations of matter interacting via finite range two-body potentials, predicts a limiting pressure $P = \rho c^2$. However, in similar models in which the mass of the ω meson is generated dynamically, $\rho \sim n^{4/3}$ and thus $P = \frac{1}{3}\rho c^2$ at high densities (see, for example, Harrington & Yildiz 1974, Krive & Chudnovskii 1976, Källman 1978). In another variation of the mean field theory model, Canuto et al. (1978) include attractive nucleon-nucleon interactions via exchange of massive spin-2 f° mesons (Bodmer 1971, 1973); in this model, however, matter is unstable under collapse to arbitrarily high density.

Lee & Wick (1974) have proposed a possible high density "abnormal state" of matter in which the nucleons become nearly massless. Such a state can arise in a field-theoretic model of matter as follows. The scalar field σ couples to the nucleons as an addition to the nucleon rest mass, i.e. it appears in the energy in the form $[m_n + g\sigma(x)]\bar{\psi}(x)\psi(x)$, where m_n is the usual nucleon mass, g is the coupling constant, and $\psi(x)$ is the nucleon field. Thus the σ field acts as a dynamical modification of the nucleon mass. The energy also contains a potential energy density $V(\sigma(x))$, which is minimum (and $=0$) when $\sigma(x) \equiv 0$, and guarantees that in the vacuum the mean value $\langle\sigma\rangle$ of the σ field vanishes and the effective nucleon mass equals m_n. When the density is finite $\langle\sigma\rangle$ will be nonzero in the ground state and the effective nucleon mass will be $m^* = m_n + g\langle\sigma\rangle$.

Now at high densities it becomes favorable for $\langle\sigma\rangle$ to become $\simeq -m_n/g$, and thus $m^* \simeq 0$, since then the nucleon rest energy is lowered by $\sim m_n n$, while the cost of having $\langle\sigma\rangle \simeq -m_n/g$ is essentially the field energy $V(\langle\sigma\rangle = -m_n/g)$, a term independent of the density. Thus, for $n \gtrsim V(-m_n/g)/m_n$ one might expect the system to undergo a transition to an abnormal state in which $m^* \simeq 0$. In Walecka's model the σ field is linear, $V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2c^5/\hbar^3$, where m_σ is the mass of the quantum of the σ field, and m^* decreases smoothly with increasing density. On the other hand, in more complicated models such as the σ-model (see Baym 1977a, 1978 for a description in this context) in which $V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2(1+g\sigma/2m_n)^2c^5/\hbar^3$ the abnormal state can appear via a sharp phase transition.

The possibility of an abnormal state in pure neutron matter was first considered by Källman (1975) and Källman & Moszkowski (1975).

They included a mean ω field in the σ-model, and predicted that pure neutron matter would have an abnormal state; however, since their model was not fit to normal symmetric nuclear matter, their prediction of an abnormal state was inconclusive. Pandharipande & Smith (1975b) have given a detailed analysis of the problem of fitting the σ-model to the binding energy, saturation density, and symmetry energy of symmetric nuclear matter, and conclude that if the model is made to fit these properties it will not have an abnormal state. Moszkowski & Källman (1977) have also fit mean field models to symmetric nuclear matter and the symmetry energy, through inclusion of a mean ρ field in Walecka's model, or through adjustment in the σ-model of the terms in $V(\sigma)$ cubic and quartic in σ; they no longer find an abnormal state in neutron star matter.

Given the difficulties of fitting the properties of the normal state within the σ-model, one can adopt the point of view that even though the details of the normal symmetric nuclear matter state, which, being on a scale of tens of MeV, are too subtle to be fit by simple models, such models might still give a reasonable description of the larger scale energy changes in the abnormal state. In this spirit Nyman & Rho (1977) have given a phenomenological calculation in the σ-model of a (first order) transition in neutron matter to an abnormal state, but conclude that while the energy of the abnormal phase lies below that of the normal phase at sufficiently high density (the abnormal phase is not self-bound there), the transition to the abnormal phase always occurs at too high a density for abnormal matter to be present in neutron stars. See also Migdal (1978).

Model calculations illustrating the relation of pion-condensed and abnormal states in matter have been given by Chanowitz & Siemens (1977) and by Akhiezer et al. (1979).

2.4 *Quark Matter*

The picture that quarks are the basic constituents of strongly interacting elementary particles (such as nucleons, Δ's, hyperons, and π, ρ and ω mesons) has by now gained wide acceptance, and suggests that a more fundamental description of matter at very high densities is in terms of quarks. In particular, one expects that when matter is sufficiently compressed, the nucleons will merge together and undergo a phase transition to quark matter, a degenerate Fermi liquid, in which the basic constituents are the quarks of which the nucleons were composed. In addition to its possible occurrence in neutron stars, quark matter is of interest in the description of the early universe when the baryon density greatly exceeded that of nuclear matter (Chapline 1976). One can very roughly estimate the density at which the transition to quark matter might occur by noting that nucleons begin to touch at a particle density $\sim(4\pi r_N^3/3)^{-1}$, where $r_N \lesssim 1\,\mathrm{fm} \equiv 10^{-13}\,\mathrm{cm}$ is an effective nucleon radius; this density is of the order of a few times ρ_0.

In the basic quark model, the quarks are spin-$\frac{1}{2}$ fermions, of baryon number $\frac{1}{3}$, which come in at least four "flavors," u, d, s, and c (up, down, strange and charmed). The electrical charges of these four flavors are $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$, and $\frac{2}{3}$ respectively; all have strangeness zero, except s which has strangeness -1. For each quark q there exists a corresponding anti-quark $\bar{q}$ with opposite quantum numbers. Mesons are composed of a quark and anti-quark, and baryons (of unit baryon number) of three quarks. For example, a proton is a *uud* bound state, while a neutron is *udd*. In addition quarks have an internal degree of freedom, color, originally introduced to enable quarks to obey the Pauli principle. In the fundamental model of quark-quark interactions—quantum chromodynamics (or non-Abelian Yang-Mills SU(3) gauge theory), in which colored quarks interact via exchange of eight massless vector gluons (analogues of photons in ordinary electrodynamics)—color functions effectively as a charge for gluon interactions. Loosely speaking, two quarks of the same color "repel," while two quarks of different color "attract" with half the strength. Thus a combination of three quarks each of different color (more correctly, a color singlet) acts as a neutral object, producing no long-range gluon "Coulomb" field.

The u and d quarks are believed to have a fairly small mass, m_u, $m_d \sim 10\,\mathrm{MeV}$; the strange quark is heavier, with m_s perhaps ~ 100–$300\,\mathrm{MeV}$, while the charmed quark is much heavier ($m_c > 1\,\mathrm{GeV}$). Because of its high mass the charmed quark (as well as newer high mass quarks) is not expected to be present in quark matter that could occur in neutron stars.

The quark-gluon theory has the remarkable property that quark interactions at sufficiently short distances become arbitrarily weak. Furthermore in quark matter that is in a color singlet (or color neutral) state, the interactions between quarks at distances large compared to the interparticle spacing will be screened out, analogous to screening of long range Coulomb fields in a plasma in equilibrium. Thus, as Collins & Perry (1975) pointed out, at high densities the net quark interactions in quark matter in an overall color singlet state should be sufficiently weak that the matter can to a first approximation be taken as a non-interacting relativistic Fermi gas. Quark matter formed from compression of pure neutrons will have twice as many down quarks as up quarks (and will have 12 Fermi seas, two for u and d, times 2 for spin, times 3 for color). Similarly high density quark matter consisting of u, d, and s quarks in beta equilibrium can be shown to have equal densities of these three flavors, and no electrons or muons present (Collins & Perry 1975). In both cases the energy density is $\propto n^{4/3}$ at high baryon density n. Because quark-quark interactions become weak at high density (equivalently, the effective quark-gluon coupling constant g decreases with increasing density), one can calculate the high density equation of state as a perturbation expansion in the effective fine-structure constant $\alpha_c = g^2/4\pi$. Terms up to order α_c^2 have been calculated by Baluni (1978a,b) and Freedman & McLerran (1977, 1978). As the density decreases the interactions become more and more important, leading eventually to confinement of the quarks in hadrons, a phenomenon not described by such a perturbation expansion.

One simple phenomenological picture of quark confinement is the MIT bag model (Chodos et al. 1974), in which the quarks in a nucleon are assumed confined to a finite region of space, the "bag," whose volume is limited by the introduction of a term in the nucleon energy equal to the volume of the bag times a constant $B > 0$. With $m_u = m_d = 0$, the parameters $B \simeq 55\,\text{MeV fm}^{-3}$ and a constant $\alpha_c \simeq 2.2$ give a reasonable fit to observed masses of strongly interacting baryons and mesons. To calculate the energy of quark matter in this model one simply adds a term B to the free particle plus interaction energies $\propto n^{4/3}$. The result for the energy density is qualitatively similar to that found from an exact perturbation expansion, and both may be used to produce a first estimate of the transition to quark matter by seeing, at a given baryon density, which phase, quark or nucleon, has a lower energy density.

Such calculations within the framework of the bag model of the phase transition from pure neutron matter to quark matter have been given by Baym & Chin (1976), Chapline & Nauenberg (1976), and Keister & Kisslinger (1976). The conclusion of these calculations is that, for all neutron matter equations of state examined, the phase transition to quark matter takes place at too high a density for quark matter to be found in neutron stars. For example, with the Reid pure neutron equation of state, the density jumps from 14–40 $\times 10^{15}\,\text{g cm}^{-3}$ at the transition, while the maximum central density found in neutron stars described by the Reid equation of state is $4.1 \times 10^{15}\,\text{g cm}^{-3}$. On the other hand the bag model calculations are quite phenomenological and neither they nor the nuclear matter calculations ought to be regarded as conclusive for densities well above that within nucleons, $\sim 1.4\,\rho_0$. When one compares perturbation expansions of the quark matter energy, using a density-dependent coupling constant, with pure neutron equations of state (Chapline & Nauenberg 1977a, Baluni 1978a,b, Freedman & McLerran 1978, Kislinger & Morley 1978, Baym 1977b) one finds that, depending on the assumed coupling strength, the transition to quark matter could well occur at densities sufficiently low that neutron stars could have quark matter cores. At present, the strength of quark-gluon interactions is not well enough established experimentally, nor is the quark confinement problem at low densities adequately understood for one to say more precisely whether quark matter is present in neutron stars. (Possible existence of a class of dense "quark stars" is discussed in Section 3.) The quark matter calculations described here are reviewed in detail in Baym (1977a, 1978).

2.5 *Finite Temperature Equations of State*

The equation of state of dense matter at finite temperature is crucial to understanding formation of neutron stars in supernovae, and also the early moments of their life. In the final stages of stellar collapse in supernovae, matter passes through a range of densities from $\sim 10^9$ g cm^{-3} to about several times ρ_0; temperatures in the collapse may reach a few times 10^{11} K. During the collapse, whose time scale is $\lesssim 1$ sec, substantial electron capture takes place; because of neutral-current weak interactions, the neutrinos produced can be trapped in the matter for time scales up to $\sim$ seconds, sufficiently long to reach approximate thermal equilibrium. The primary constituents of the matter undergoing collapse are nuclei (undergoing various transformations), "free" neutrons and protons, electrons and neutrinos, both of which behave as free Fermi gases, and photons. The initial collapse is reversed in a bounce at densities ($\gtrsim \rho_0$) at which the adiabatic index of the matter, $\Gamma = \partial \ln P/\partial \ln n$ (where P is the pressure), rises well above $\frac{4}{3}$.

The simplest approximation to the equation of state at subnuclear densities treats the matter as a mixture of free neutron, proton, electron, and photon gases (see, for example, Van Riper & Bludman 1977). While this approach is valid at temperatures high compared with that required to dissociate nuclei, ~ 20 MeV, it is necessary in collapse to take the nuclei present into account. In most collapse calculations to date (e.g. Arnett 1977, Mazurek 1979, see also Nadyozhin 1977) this is done by use of a semiempirical mass formula extrapolated from laboratory nuclei. As in the case of zero-temperature matter at subnuclear densities in neutron stars, a more accurate description requires that one go beyond the semiempirical mass formula and include the following important physical effects: (*a*) the effect of nuclear excited states, (*b*) interactions between nucleons outside nuclei, (*c*) the reduction of the nuclear surface energy due to finite temperature, and (*d*) the Coulomb interaction between nuclei. Sato (1975) took (*a*) and (*b*) into account by employing the $T = 0$ result of Baym et al. (1971a), which includes nucleon-nucleon interactions, for the bulk energies of matter both inside and outside nuclei, plus a finite temperature correction calculated assuming that the kinetic energy of the nucleons is that of a free gas (see also Neatrour 1979). One of the important conclusions of this work was that at high densities nuclei survive to relatively high temperatures. Mackie (1976) took (*d*) into account and used an improved nuclear mass formula (Mackie & Baym 1977), which allowed for the reduction of the nuclear surface energy due to the rather large neutron excess in the nuclei. Mazurek et al. (1979) also estimated the effects of (*a*), (*b*), and (*d*).

Bethe et al. (1979) have recently suggested a possible restriction on the range of parameters over which one needs to know the equation of state in collapse. They argue that as the entropy of matter in collapse is likely small, effects of nuclear excited states cause most nucleons to be confined to nuclei, with the pressure of matter being provided chiefly by the degenerate electrons. Consequently, under these circumstances the adiabatic index, Γ, of the matter below $\rho \lesssim \rho_0$ is close to $\frac{4}{3}$, and bounce will occur at $\rho > \rho_0$.

The presently most detailed calculation of the finite-temperature equation of state at subnuclear densities is that of D. Q. Lamb et al. (1978), who include effects (*a*)–(*d*) through a finite-temperature generalization of the work of Baym et al. (1971a). The bulk free energies of matter both inside and outside nuclei are those calculated by Lattimer & Ravenhall (1978) using a Skyrme effective nucleon-nucleon interaction, which was also used to calculate the surface free energy (Ravenhall & Lattimer 1979). The internucleus Coulomb energy is taken into account using the Coulomb liquid calculations of Hansen (1973). An important conclusion of this work is that Γ at subnuclear densities remains close to $\frac{4}{3}$ for a wide range of entropies and lepton fractions, implying that bounce will occur at densities $> \rho_0$ over a large range of initial entropies during the collapse. Calculation of the finite-temperature equation of state in the neighborhood of nuclear matter density, a quantity needed for understanding details of the bounce and subsequent shock formation, contains at least as many difficulties as the zero-temperature calculations (Section 2.1).

Buchler & Coon (1977) and Buchler & Datta (1979) have calculated the equation of state of a hot neutron gas from a more microscopic approach employing various two-body scattering approximations (in the sense of Brueckner theory) for the free energy. One can conclude from these papers that neglect of the temperature dependence of the effective nucleon-nucleon interaction, as when using the Skyrme interaction, is a reasonable approximation in the temperature—density regimes where the interactions produce a significant contribution to the free energy.

Further calculations of hot neutron star matter, using the finite-temperature Thomas-Fermi method, are given by El Eid & Hilf (1977).

An equation of state of finite-temperature neutron matter has been computed by Walecka (1975) from his relativistic mean field theory model (Walecka 1974). (See also Bowers et al. 1977b,c, and Freedman 1977.) While such a calculation omits too much of the physics at low densities to be relevant to the supernova problem, it is possibly applicable at densities well above ρ_0. The "liquid-gas" phase transition at subnuclear densities present in this calculation (for which there is no evidence in potential theory models, expected to be more reliable at such densities) is most likely an artifact of the model and the choice of parameters.

An initial calculation of finite-temperature quark matter within the framework of the MIT bag model has been carried out by Chin (1978), who concludes that the density of the transition to uniform quark matter falls with increasing temperature. See also Shuryak (1978).

2.6 *Matter in High Magnetic Fields*

In the strong magnetic fields expected in neutron stars, the properties of matter at relatively low densities are very different from those in the absence of the field. Individual atoms become compressed, and relatively elongated in the direction of the magnetic field, and can bind together covalently to form polymerized chains in which the electrons are free to move along the length of the chain. The chains may then be bound together by electrostatic forces to form a solid, which at zero pressure has a density much greater than that of terrestrial solids. During the past few years there has been a considerable amount of work on the properties of matter in strong magnetic fields, and while the basic picture remains the same as that described in BP, a number of the quantitive results have changed.

An astrophysically important quantity in descriptions of neutron star surfaces and hence in theories of pulsar emission (Ruderman & Sutherland 1975) is the cohesive energy, i.e. the energy of the condensed polymerized matter in a strong field, compared to that of isolated atoms in the same field. Hillebrandt & Müller (1976) pointed out an error in the earlier calculations of Chen et al. (1974) that led to an overestimate of the cohesive energy. The most reliable calculation to date is that of Flowers et al. (1977), who used a variational approach including electron exchange, the importance of which was stressed by Glasser & Kaplan (1975), and allowed for buildup of the electronic charge in the vicinity of nuclei. They find cohesive energies ranging from 2.6 keV/atom at field $B = 10^{12}$ G to 8.0 keV/atom at $B = 5 \times 10^{12}$ G, compared with Chen et al's (1974) results of 10 keV/atom and 20 keV/atom. The zero-pressure densities are not significantly modified from previous results.

The new calculated cohesive energies are small compared with the binding energy of an individual iron atom, ~ 50 keV. Given the fact that the cohesive energy is the difference of two large energies, each calculated variationally, one cannot conclude with certainty that at zero pressure the condensed state formed of polymerized chains is energetically favorable compared with a more ordinary state of elongated atoms with weaker metallic binding. (Because of the compression of atoms in strong fields, this latter state would at zero pressure still be at density much higher than terrestrial solids, but perhaps an order of magnitude below that of the magnetically polymerized solid.) To improve upon the estimates of Flowers et al. will be difficult, and the extent to which models of pulsar radio emission will be affected by a reduced cohesive energy is an open question.

The spectra of atoms in strong fields have been reviewed by Garstang (1977), with particular emphasis on H and He. Also, Sarazin & Bahcall (1977) have calculated Zeeman splitting of X-ray emission lines from Fe XXV and XXVI and other ions, and suggest that observations of these line profiles may enable one to determine both the direction and magnitude of neutron star fields.

3 NEUTRON STAR MODELS

Construction of neutron star models is of interest not only for learning their physical properties—radius, moment of inertia, density profile—for a given mass (and equation of state), but also for determining the allowed range of neutron star masses. In particular, knowledge of the maximum allowable neutron star mass is an important ingredient in attempts to identify black holes from measurement of masses of compact objects.

Neutron star models are constructed from a given equation of state $P(\rho)$ by integration of the equation of hydrostatic balance, which for general relativity is the Tolman-Oppenheimer-Volkoff equation. See Baym et al. (1971b) for a discussion of numerical integration methods, and Arnett & Bowers (1977) for an extensive comparison of models.

Figure 1 shows (for general relativistic stars) the resultant gravitational masses M as a function of the central mass density ρ_c for a representative sample of equations of state: MF is the Pandharipande-Smith (1975b) mean field theory calculation, similar to that of Chin & Walecka (1974); TI is the Pandharipande-Smith (1975a) tensor-interaction model, which incorporates approximately stiffening effects on the equation of state due to isobars (Section 2.1); BJ is for the Bethe-Johnson (1974) equation of state I, which includes small effects of hyperons (see also Malone et al. 1975); R is for the pure neutron equation of state with the Reid potential (Pandharipande 1971); π is for the Reid equation of state as modified by charged-pion condensation of a strength that may reasonably be expected, while π' shows the results of somewhat stronger pion condensation, in which there is a large first order phase transition at the onset of condensation (Maxwell & Weise 1976). The estimates of Hartle et al. (1975) on effects of pion condensation (their case n′) are quite similar to the case π'. The maximum possible neutron star mass, given by the maximum of $M(\rho_c)$, is fairly sensitive to the equation of state, tending to increase as the equation of state becomes stiffer, but remaining, as we see, $\lesssim 3\,M_\odot$, and above the masses $\sim 1.4\,M_\odot$ observed in Her X-1 and Vela X-1. The central density for a given mass decreases as the equation of state becomes stiffer.

The corresponding radii of the stars, shown in Figure 2, are also sensitive to the stiffness of the equation of state, and in particular to the detailed behavior in the neighborhood of nuclear matter density. (The moments of inertia of these models are discussed in the respective references.) One sees from the TI curve how inclusion of explicit effects

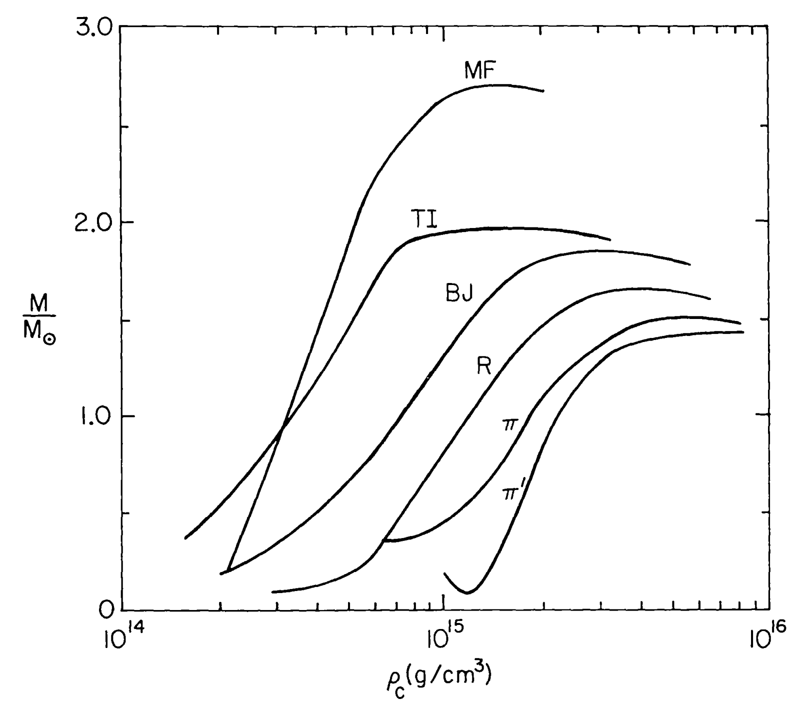

Figure 1 Gravitational mass (in solar units) versus central density for a variety of equations of state. The rising portions of the curves represent stable neutron stars. See text for identification of curves.

of isobars can increase the radius of a neutron star of a given mass. Stars with stiffer equations of state will also have thicker crusts since, for a given mass liquid core, the stiffer the equation of state is around ρ_0, the weaker is gravity at the core-crust interface. For example the crust of a 1.33 $M_\odot$ TI star reaches from a radius of 11.3 to 16.1 km, while in the same mass R star the crust exists between 9.1 and 9.9 km. (See Pandharipande et al. 1976 for more detailed discussions.) On the other hand, as we described earlier, pion condensation can significantly soften the equation of state, an effect not included in the stiffer MF or TI models; it tends to contract neutron stars of a given mass, as well as decrease $M_{\max}$. Because there do not yet exist equations of state reliably including effects of both isobars and pion condensation, one must regard the models shown in Figures 1 and 2 as illustrating the range of possibilities.

A further interesting possibility could occur were neutron star matter to develop a self-bound state, due, for example, to pion condensation or an abnormal state. Then, as illustrated by Hartle et al's (1975) case C′, stable neutron stars could exist with arbitrarily small mass and radius ("golf balls"), and surface density equalling that of the self-bound state.

As discussed in Section 2.4, the transition to uniform quark matter may occur at densities above the maximum central density $\rho_{\max}$ in neutron stars. An interesting question then is whether there perhaps

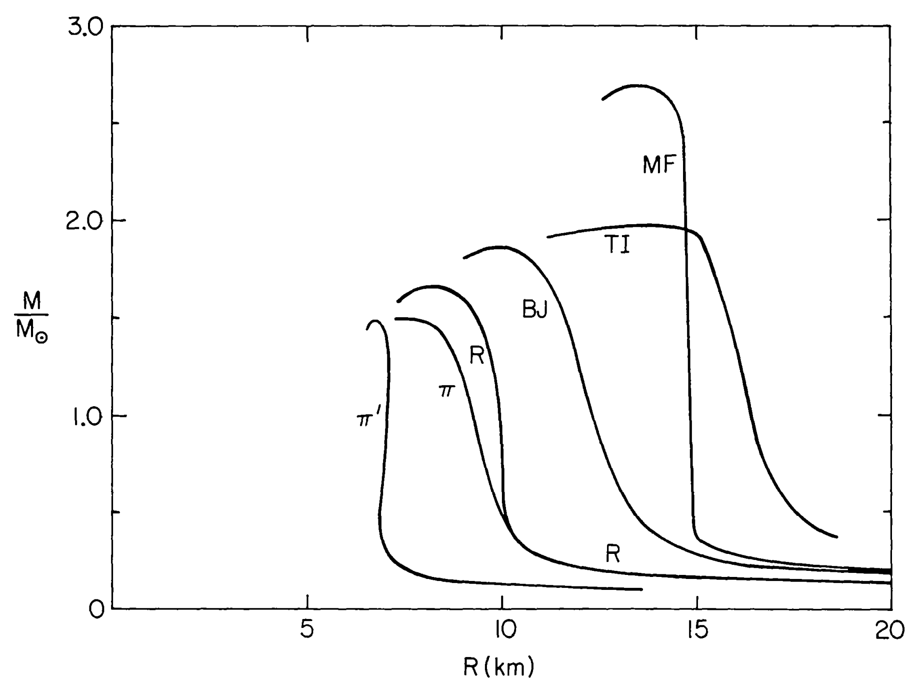

Figure 2 Gravitational mass (in solar units) versus radius for the same equations of state described in Figure 1.

exists a third stable branch of cold stars (after white dwarfs and neutron stars), "quark stars," whose central densities lie beyond ρ_{max}, and which are supported against gravitational collapse primarily by the degeneracy pressure of unconfined quarks (Itoh 1970). The general relativistic stability condition for a star of mass M and radius R—that the mean adiabatic index Γ exceed $\frac{4}{3}(1+KR_s/R)$, where $R_s = 2MG/c^2$ is the Schwarzschild radius and K is of order unity—places a strong constraint on those equations of state of quark matter that can yield a stable third branch (see Gerlach 1968). The stability requirement on Γ tends to grow with increasing ρ_c, and is ~ 2 at M_{max} for neutron stars. Since in the limit of large density, $P \propto n^{4/3}$ in quark matter, and thus $\Gamma \propto \frac{4}{3}$, such a condition is unlikely to be fulfilled for a significant range of densities of quark matter (see Bowers et al. 1977a). We note though that, by appropriate choice of parameters, one can construct model equations of state of quark matter that yield a stable quark star branch (Chapline & Nauenberg 1977b, Fechner & Joss 1978).

Substantial effort has been devoted to determination of exact bounds on masses and moments of inertia of neutron stars. Since this subject is lucidly reviewed by Hartle (1978, see also Sabbadini & Hartle 1977), we quote only a few major results. If the equation of state is regarded as known below a fiducial density ρ_f, assumed to be not far above ρ_0 (so that $P/\rho c^2$ is small at ρ_f), then the bound on the maximum mass is given to a good approximation by

$$M_{bound}/M_\odot = 6.75(\rho_0/\rho_f)^{1/2}, \tag{1}$$

assuming only that the equation of state above ρ_f obeys $\rho > 0$ and $\partial P/\partial\rho > 0$. If one furthermore imposes the "causality condition" $\partial P/\partial\rho < c^2$, then the bound is sharpened to

$$M_{bound}/M_\odot = 4.0(\rho_0/\rho_f)^{1/2}. \tag{2}$$

It should be noted that these bounds lie, for $\rho_f = \rho_0$, significantly above the various M_{max} that one finds from equations of state derived from microscopic theory.

The above bounds were derived assuming the validity of general relativity. Other theories of gravity, however, such as the Rosen-Rosen bimetric theory and some versions of Ni's theory, permit significantly greater maximum neutron star masses than does general relativity (see Hartle 1978).

4 NONEQUILIBRIUM PROCESSES

In this section we review aspects of the physics underlying non-equilibrium processes in neutron stars that determine their evolution and dynamical behavior, concentrating in particular on transport and hydrodynamic properties, cooling processes, and nonequilibrium effects of superfluidity.

4.1 *Transport Properties and Hydrodynamics*

Transport properties of dense matter have an important influence on neutron star magnetic fields and cooling, as well as on other features of their behavior. Detailed calculation of the electrical and thermal conductivities of matter at densities $\lesssim \rho_0$ have been given by Flowers & Itoh (1976), who have also discussed transport properties of the liquid regime, assuming it to be a mixture of normal (i.e. nonsuperconducting and nonsuperfluid) Fermi liquids (Flowers & Itoh, 1979). Ewart et al. (1975) have discussed the electrical conductivity of the crust, and conclude that decay of large-scale magnetic fields in the crust during a characteristic pulsar lifetime will be small unless either the temperature is higher than estimated, or the crust is very impure or composed of microcrystallites.

The high magnetic fields expected in neutron stars can have a drastic effect on transport properties, since electrons can move more easily along field lines than perpendicular to them. Consequently, when $\omega_c \tau > 1$, where ω_c is the cyclotron frequency and τ is the relevant transport relaxation time, the electrical and thermal conductivities, and the viscosity, are much larger along the field than perpendicular to it. At densities such that the electron Larmor radius is large compared with the electron spacing $[\rho \gg 2 \times 10^4 (B/10^{12})^{3/2}\,\mathrm{g\,cm^{-3}}]$ the structure of the matter, and consequently the scattering rates, are little affected by the magnetic field, and transport properties in the presence of the field may be calculated straightforwardly in terms of zero-field quantities (see, for example, Flowers & Itoh 1976, Easson & Pethick 1979). At lower densities, which are of particular interest for determining the properties of neutron star surfaces, the problem of calculating transport coefficients is much more difficult. Itoh (1975) has discussed the electrical conductivity due to electron-phonon scattering, and has also discussed the refractive index. See also the review by Canuto & Ventura (1977). The reflection of X rays by a neutron star surface assumed to be composed of the magnetically condensed matter discussed in Section 2.6 has been considered by Lenzen & Trümper (1978); their results are applicable only if the neutron star surface may be treated as smooth on the scale of $\sim 10\,\mathrm{keV}$ X-ray wavelengths and scattering outside the surface may be neglected.

Sources of photon opacity, especially in the relatively low densities in the outer parts of a neutron star, have an important effect on neutron star surface temperatures. These processes have been studied extensively for zero magnetic field (see, for example, Carson 1976). Lodenquai et al. (1974) have studied free-free, bound-free, and Thomson scattering in the presence of a field; they find that for photons of frequency ω having an electric field vector perpendicular to the magnetic field the opacity is reduced, for $\omega \ll \omega_c$, by a factor $\sim(\omega/\omega_c)^2$.

The rather unusual conditions in neutron star interiors lead to magnetohydrodynamic behavior different from that in more common situations. Hide (1971) has pointed out that, for rapid stellar rotation, the elementary magnetohydrodynamic modes will have frequency spectra modified from the soundlike dispersion relations in the absence of rotation. For a review see Acheson & Hide (1973). Time scales to establish beta equilibrium, allowing for magnetohydrodynamic flows, have been calculated by Baym et al. (1979); they conclude that the possible force-free configuration of a magnetic field in a neutron star in magnetohydrostatic and beta equilibrium suggested by Easson (1976) can never be realized in practice. The basis of the magnetohydrodynamic equations has been examined by Easson & Pethick (1979), who also point out that when the neutrons are superfluid, the strong interactions will couple the motion of the charged particles to that of the neutrons, even though collisional effects are unimportant. Jones (1975) pointed out that proton superconductivity could drastically influence the magnetic contribution to the stress tensor, which is of importance for the static deformation of neutron stars, and for the dynamics. Detailed calculations of the effect were carried out by Easson & Pethick (1977), who showed that the components of the stress tensor are $\sim BH_{c1}/4\pi$ (where H_{c1} is the lower critical field), a result typically a few orders of magnitude greater than the normal result $B^2/8\pi$.

4.2 *Cooling of Neutron Stars*

Measurement of surface temperatures of neutron stars, through observation of thermal black body emission, can in principle yield substantial information about the interior structure of the stars. One may ask, for example, what are the internal states of matter and processes that could have enabled the Crab pulsar, made in 1054 A.D. with an interior temperature $\sim 10^{11}$ K, to cool to its present state with a surface temperature $<4.7 \times 10^6$ K (Wolff et al. 1975). Knowledge of the temporal evolution of the internal temperatures of neutron stars is also important for estimating temperature-dependent properties such as transport coefficients, transitions to superfluid states, and solidification of the crust.

After formation the predominant cooling mechanism is neutrino emission. Photon emission begins to play a role only when the internal temperature falls a thousandfold to $\sim 10^8$ K, with a corresponding surface temperature about two orders of magnitude smaller.

In degenerate matter in neutron star cores cooling by the URCA process

$$n \rightarrow p + e^- + \bar{\nu}_e, \quad e^- + p \rightarrow n + \nu_e \tag{3}$$

becomes ineffective unless momentum can be transferred, in the reactions, to the rest of the system. We can see this by considering the neutron beta decay. In beta equilibrium, $\mu_n = \mu_p + \mu_e$ to within terms of order $(\kappa T)^2/\mu_p$, where μ_n, μ_p and μ_e are the neutron, proton, and electron chemical potentials. The neutrons capable of decaying lie within $\sim \kappa T$ of the neutron Fermi surface, and thus the final proton and electron must lie within κT of their Fermi surfaces; the neutrino energy is also $\sim \kappa T$. Because the electron and proton Fermi momenta are small compared with the neutron Fermi momentum p_n, the final proton, electron, and neutrino, as well, must have small momenta. But the initial neutron must have momentum $\sim p_n$, and thus the decay cannot conserve momentum if it conserves energy. In order for the process to work, a bystander particle must absorb momentum, as in the "modified URCA process" (Chiu & Salpeter 1964), $n + n \rightarrow n + p + e^- + \bar{\nu}_e$, $n + p + e^- \rightarrow n + n + \nu_e$. The neutrino luminosity due to this process has been calculated by Bahcall & Wolf (1965), Itoh & Tsuneto (1972), and most recently by Friman & Maxwell (1979), who use more realistic expressions for the nucleon-nucleon interactions, and find a stellar luminosity

$$L_\nu \sim (6 \times 10^{39}\,\mathrm{erg/s})(M/M_\odot)(\rho_0/\rho)^{1/3} T_9^8 \tag{4}$$

(where T_9 is the interior temperature in units of 10^9 K) an order of magnitude larger than that calculated by Bahcall & Wolf. Due to the available phase space for the bystander neutron this rate is down by a factor $\sim (\kappa T/\mu_n)^2$ from the rate for (3), were that process allowed by energy and momentum conservation. The calculated rate (4) is sensitive to the effective masses assumed for the nucleons.

The experimental discovery of weak neutral currents in 1974 suggested further cooling processes. The most important in the interior of neutron stars are the nucleon pair bremsstrahlung processes $n + n \rightarrow n + n + \nu + \bar{\nu}$ and $n + p \rightarrow n + p + \nu + \bar{\nu}$, first considered by Flowers et al. (1975), and re-examined by Friman & Maxwell (1979). The luminosity from these processes also varies as T^8, but is less than $\frac{1}{30}$ the magnitude of that from the modified URCA process (Friman & Maxwell 1979).

All the above calculations assumed the nucleons to be normal. If instead they are superfluid, the rates are reduced by factors $\sim \exp(-\Delta_{n,p}/\kappa T)$ due to reduction of the number of thermal excitations

(Wolf 1966, Itoh & Tsuneto 1972). (Here $\Delta_{n,p}$ are the neutron and proton superfluid gaps.) Under these circumstances the neutrino pair bremsstrahlung process,

$$e^- + (Z,A) \to e^- + (Z,A) + \nu + \bar{\nu} \tag{5}$$

from nuclei in the crust, can be important. Initially estimated by Festa & Ruderman (1969), who considered only the weak charged-current contribution, it has more recently been discussed by Flowers (1973, 1974), who also allowed for the finite nuclear size, and by Dicus (1972), Dicus et al. (1976), and Soyeur & Brown (1979), who calculated neutral-current contributions. The luminosity from this process varies as T^6, and therefore decreases less rapidly than the modified URCA process with decreasing temperature. Maxwell (1979) estimates the total luminosity due to this process as

$$L_\nu \sim (5 \times 10^{39}\,\mathrm{erg/s})(M_{cr}/M_\odot)T_9^6, \tag{6}$$

where M_{cr} is the mass of the crust. In the inner crust, where free neutrons coexist with nuclei, neutrino pairs can also be produced by bremsstrahlung in the scattering of neutrons from nuclei (Flowers & Sutherland 1977); the rate of this process is of order $\eta(\varepsilon_f/30\,\mathrm{MeV})^{3/2}$ $(A/200)^{1/3}(30/Z)^2$ times that for electron bremsstrahlung, where $\eta \sim \frac{1}{4}$–$\frac{1}{2}$, and ε_f is the neutron Fermi energy; this factor is generally <1.

Flowers et al. (1976) have pointed out that if neutrons are superfluid, two neutron-like excitations can annihilate to produce neutrino pairs. This process is most important just below the transition temperature, and it can dominate the pair bremsstrahlung process under some circumstances.

Pion condensation can significantly enhance the cooling rate of neutron stars in their hot early period, since it permits the analogue of (3) to occur conserving energy and momentum. (The condensed pion field itself does not beta decay.) Essentially the excess momentum in (3) is absorbed by an Umklapp process involving the condensed pion field. In a weak condensate one can think of the neutron decay process as occurring by first $n \to p + e^- + \bar{\nu}_e$, with the proton after the beta decay existing only in a virtual intermediate state before scattering from the condensed pion field, changing into a neutron and absorbing energy $\mu_\pi = \mu_n - \mu_p$ and momentum $\mathbf{k}$, the wave vector of the condensed pion field. Thus as long as the initial and final neutron states are separated by $\sim\mathbf{k}$ across the Fermi surface, the process is allowed by energy and momentum conservation. More precisely, because the nucleon eigenstates are linear combinations of neutrons and protons, nucleons in these states (call them f) can beta decay into themselves: $f \to f + e^- + \bar{\nu}_e$,

$f + e^- \rightarrow f + \nu_e$. In the first process, for example, the neutron component of the initial f decays to a proton state, which has nonzero overlap with the final f. In the beta decay the particle f moves a momentum $\sim \mathbf{k}$ across the f Fermi surface. The net result is a luminosity (Maxwell et al. 1977)

$$L_\nu^\pi \sim (1.5 \times 10^{46}\,\mathrm{erg/s})\theta^2(M/M_\odot)(\rho_0/\rho)T_9^6, \tag{7}$$

where $\theta \sim 0.3$ is an angle measuring the degree of pion condensation. This rate is $\sim 2.5 \times 10^6(\rho_0/\rho)^{2/3}\theta^2/T_9^2$ larger than the modified URCA rate (4), and will, if there is pion condensation, dominate at all temperatures of interest. [The result (7) is quite close to the estimate of Bahcall & Wolf (1965) for cooling via decay of free pions, were they to exist, in the medium.]

The cooling time of a neutron star may be estimated using the fact that the thermal energy U, which resides almost exclusively in the degenerate neutrons, is $U \simeq (10^{47}\ \mathrm{erg})(M/M_\odot)(\rho/\rho_0)^{-2/3}T_9^2$. Equating $\mathrm{d}U/\mathrm{d}t$ to the neutrino luminosity, one finds that the star cools from an initial temperature $T(\mathrm{i})$ to a final temperature $T(\mathrm{f})$ in a time

$$\Delta t \simeq (0.2\,\mathrm{yr})(\rho/\rho_0)^{-1/3}[T_9(\mathrm{f})^{-6} - T_9(\mathrm{i})^{-6}] \tag{8}$$

for the modified URCA process (4), and

$$\Delta t \simeq (3\ \mathrm{sec})\theta^{-2}(\rho/\rho_0)^{1/3}[T_9(\mathrm{f})^{-4} - T_9(\mathrm{i})^{-4}] \tag{9}$$

for the pion condensation process (7).

The temperature that determines the thermal emission from a neutron star is that at the surface T_e, rather than the interior temperature T. Neutron star interiors are to a good approximation isothermal, but near the surface the temperature drops rapidly, and $T_e/T \sim 10^{-2}$–10^{-3}. Detailed calculations of T_e/T have been made by Tsuruta (1974) and by Malone (1974).

The relative importance of various cooling processes on the interior temperature of a neutron star of mass $\sim M_\odot$, as a function of time, may be assessed from the schematic Figure 3; each line in this figure shows $T(t)$, assuming only a single process, that labelling the line, to be operant. Results are shown for the modified URCA process (4), for pion condensation cooling (7), for cooling from crust bremsstrahlung (6), and for emission of photons from the surface, assuming $T_e = (10T)^{2/3}$, an approximate fit to Tsuruta's (1979) calculations. The formulae were evaluated for $\rho = \rho_0$ and $\theta^2 = 0.1$. At any given time the most effective process will be the one with the lowest $T(t)$, and the temperature of the star will be roughly that T.

The curves were plotted assuming the neutrons and protons to be

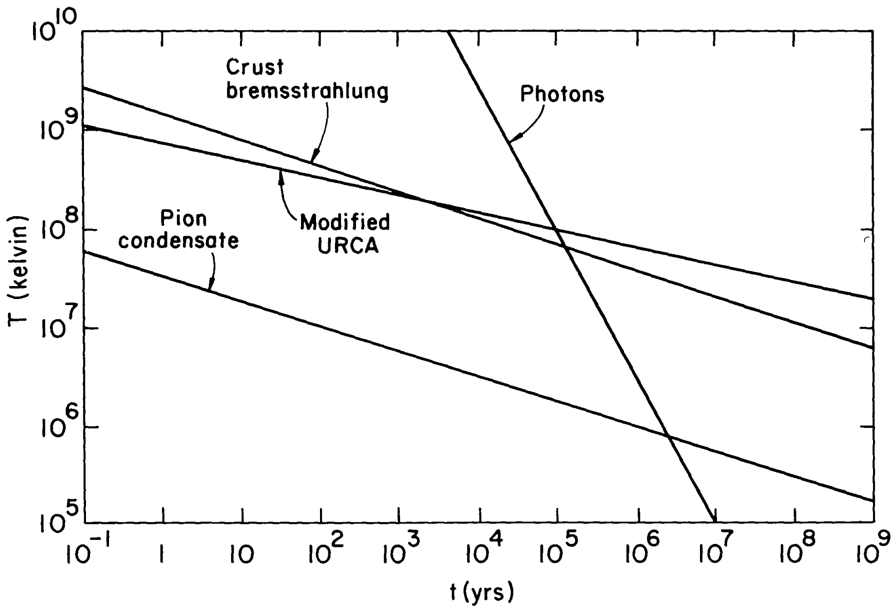

Figure 3 Schematic neutron star cooling curves of interior temperature versus time for various processes, were they to operate alone.

normal. Superfluidity has two effects; first it modifies the specific heat, which at the transition temperature jumps discontinuously to a value greater than that in the normal state, and then falls off exponentially at lower temperatures. This tends to decrease the cooling rate immediately below T_c, and to increase it at lower temperatures. A second effect is to suppress neutrino processes in the interior, and thereby decrease the cooling rate. Which of these two effects dominates depends on how important a role the neutrino processes in the interior play in the cooling.

While magnetic fields do not affect the neutrino processes, they can, as discussed in Section 4.1, reduce the opacity of the surface of a neutron star appreciably, and thus, for a given internal temperature, increase the surface temperature, and hence increase the photon luminosity (Tsuruta et al. 1972).

Detailed studies of neutron star cooling have been made by Tsuruta (1975) and Malone (1974) and reviewed by Tsuruta (1979). See also Brown (1977). More recently Maxwell (1979) has calculated the cooling curve for some simplified models, and concludes that the present upper bound on the Crab pulsar temperature is compatible with any cooling scenario involving neutrinos; however, a bound as low as 2×10^6 K would be more difficult to understand without pion condensation.

The neutrino emissivity from a neutron star immediately after its formation may be detectable on earth. At formation the neutrinos in the star are degenerate and the neutrino mean free paths are short, due to scattering via neutral current interaction with nucleons, as well as absorption of neutrinos by neutrons (Sawyer & Soni 1979). As Sawyer & Soni (1977) pointed out, even when the neutrinos are no longer degenerate, but the neutron star is still very hot, neutrino mean free paths are small compared with the stellar radius, and neutrino luminosities and stellar cooling rates will be reduced from those given above. These effects will not significantly affect the temperature of neutron stars more than a few hours old. Considerable work is in progress on neutrino processes in very young neutron stars, and more detailed calculations of the early life of a neutron star should be available in the near future.

4.3 *Dynamical Effects of Superfluidity*

The irregular fluctuations in neutron star rotational periods observed in both pulsars and pulsating X-ray sources, as well as the behavior of pulsars after sudden speedup (Manchester & Taylor 1977, Lamb 1977), may be governed in part by superfluidity of the internal regions of the stars. The rotational dynamics of the two-component model of a neutron star with a superfluid interior weakly coupled to a normal crust, introduced by Baym et al. (1969), provides a plausible explanation of the long relaxation times associated with speedups of the Crab and Vela pulsars (see Pines et al. 1974); however, relation of the observed relaxation times to possible microscopic coupling processes requires detailed study. Aspects of the dynamics were discussed by Ruderman & Sutherland (1974), and by Greenstein (1975, 1976, 1977), who pointed out possible internal heating of neutron stars due to the superfluid-crust coupling; Harding et al. (1978) have recently calculated relaxation processes between superfluid neutron vortex lines and the crust. Effects of pinning of the vortices to nuclei in the crust were considered by Anderson & Itoh (1975), by Ruderman (1976), who suggested possible breaking of the crust due to pinning forces, by Alpar (1977), and by Shaham (1977).

An interesting exploration of possible relations to relaxation after a sudden pulsar speedup has been the experimental simulation of such behavior by Tsakadze & Tsakadze (1975), using rotating superfluid helium. Theoretical interpretation of these experiments for neutron stars has been given by Anderson et al. (1978) and Alpar (1978).

5 CONCLUSION

As can be judged from this review, knowledge of the properties of neutron stars has advanced remarkably over the past decade. One is now much more aware of the gaps and uncertainties in the picture, and the important problems for further research. Particularly crucial is an improved description of nuclear matter and forces, from which we will gain a better understanding of the equation of state, of the effects of pion condensation and superfluidity, and of neutron star models. A second outstanding problem, which we have only touched on peripherally, is the formation of neutron stars in supernovae, a problem on which the discovery of weak neutral currents has had a strong recent impact. For an overview of problems in this area see, for example, Arnett (1977) and Freedman et al. (1977).

Much work has been done on many areas of neutron star behavior that we unfortunately have not had adequate space to discuss. Among these are: internal dynamical processes, including models of short term variability of neutron star rotation periods and neutron star wobble (Lamb 1975, 1977, F. K. Lamb et al. 1978); processes outside neutron stars, including pulsar emission mechanisms (Manchester & Taylor 1977), accretion and X-ray emission (Sunyayev 1978, Lightman et al. 1978), surface nuclear burning (Woosley & Taam 1976, Joss 1978, Lamb & Lamb 1978, Taam & Picklum 1978), and neutron star models of X-ray (Lamb et al. 1977) and γ-ray burst sources (Lamb et al. 1973, Ruderman 1975); interactions of neutron stars with other stars, including binary pulsars (Manchester & Taylor 1977), interactions with black holes (Lattimer & Schramm 1976, Lattimer et al. 1977), and neutron stars as cores of red giants and supergiants (Thorne & Żytkow 1977).

We are grateful to our colleagues in Urbana for numerous helpful discussions during the preparation of this review.

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