Isospin symmetry breaking in mirror nuclei

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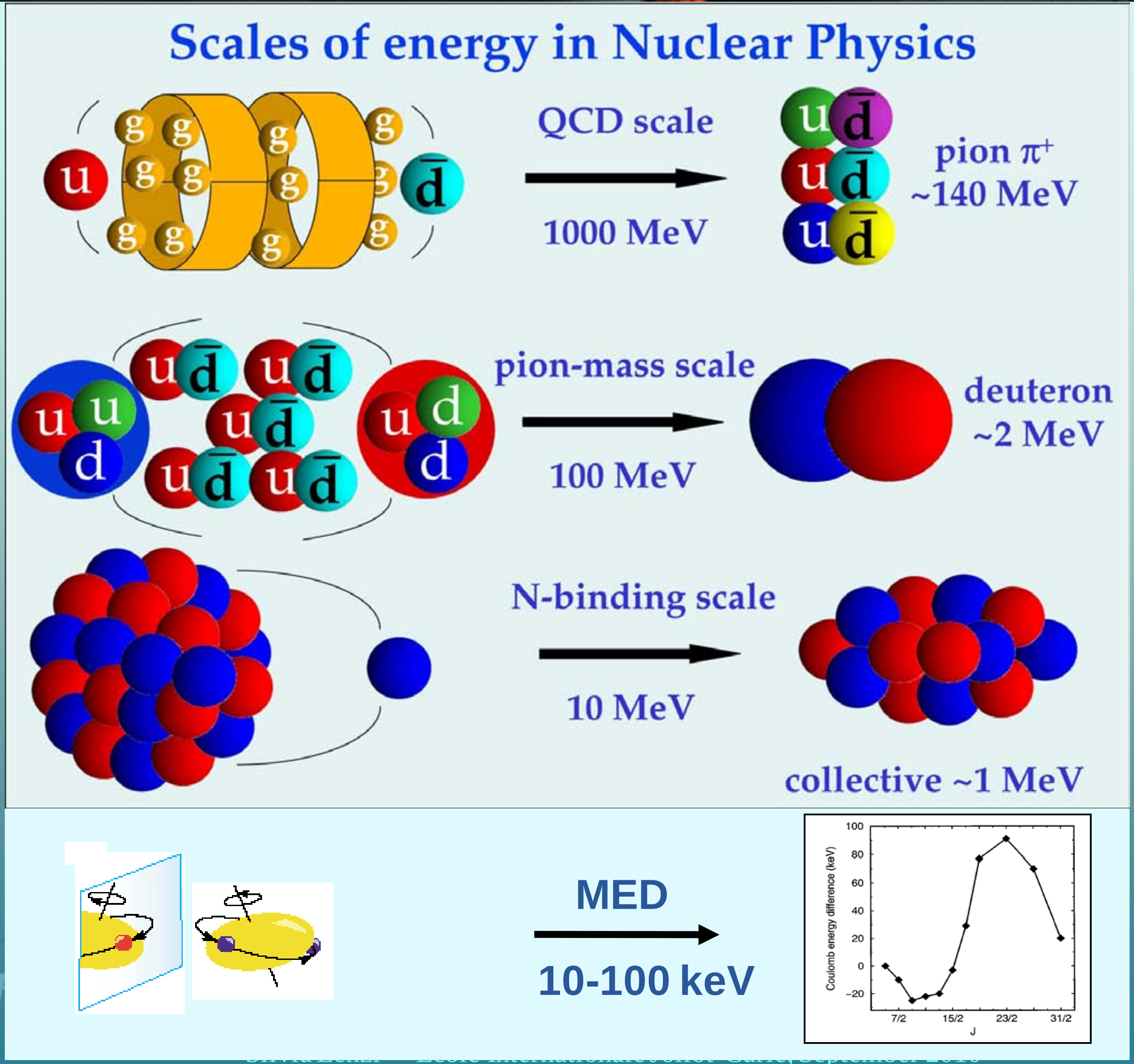
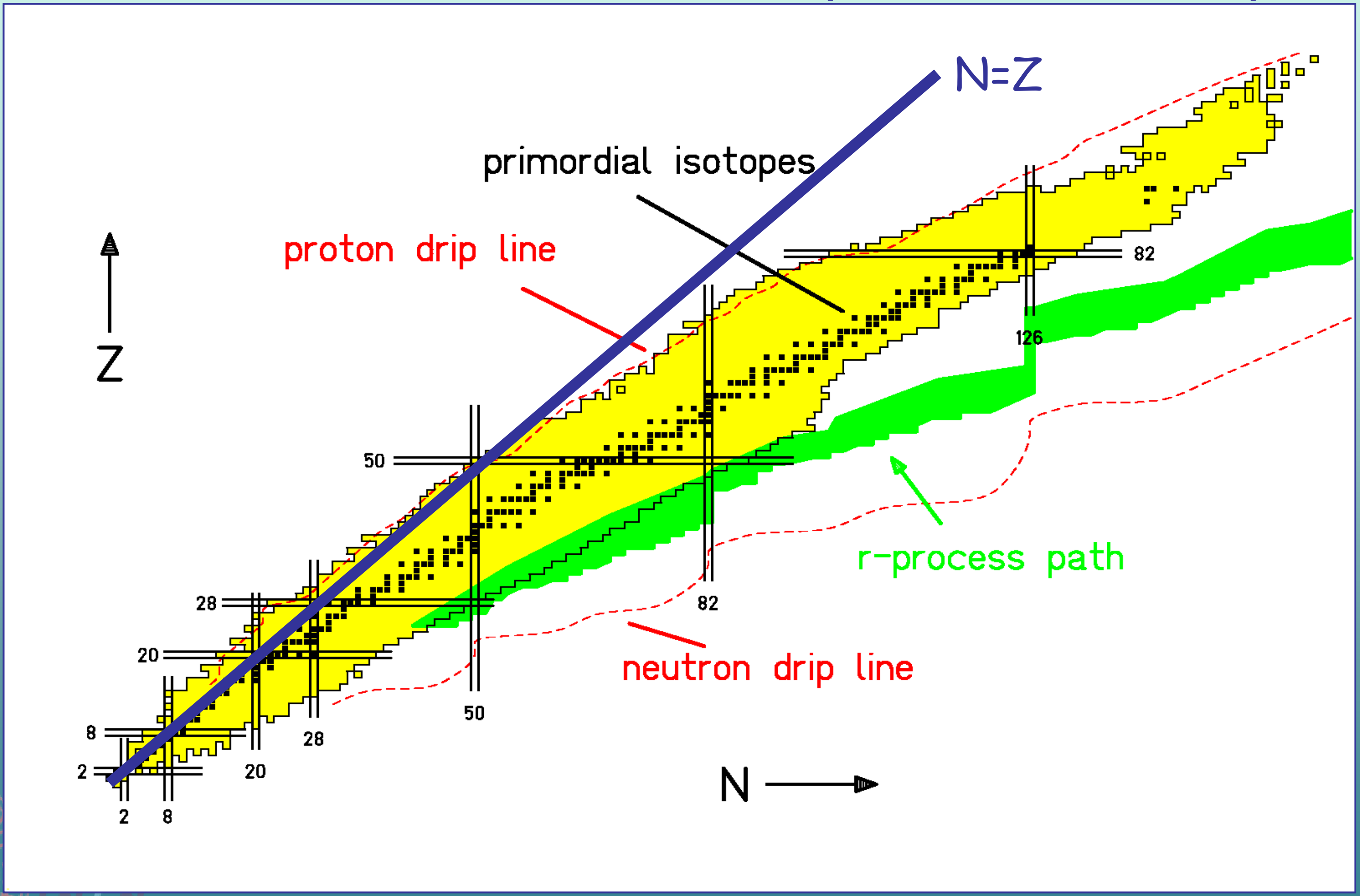
Energy scales

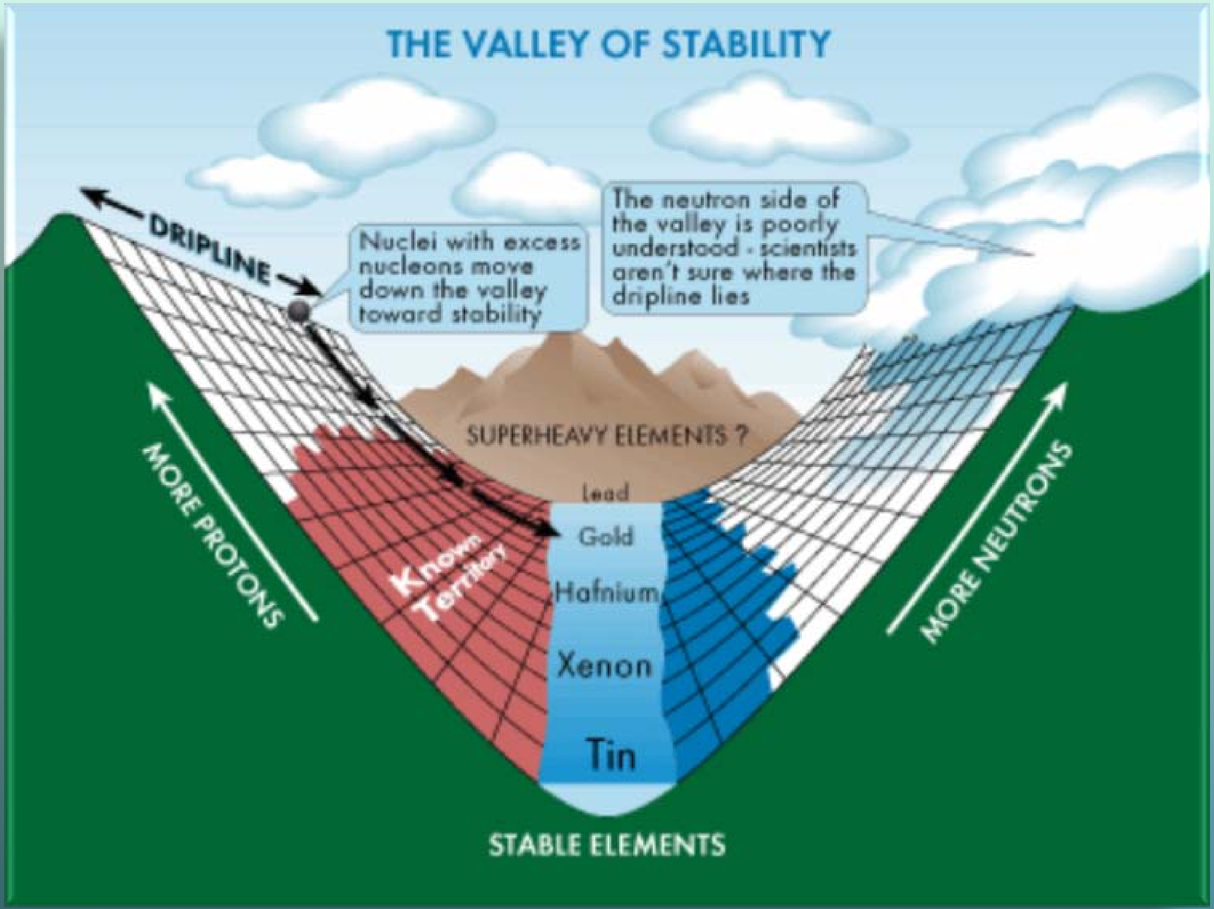


The Nuclear landscape

- 287 primordial isotopes exist in Nature
- ~6000 nuclei are bound (3000 are known)

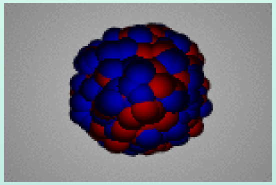


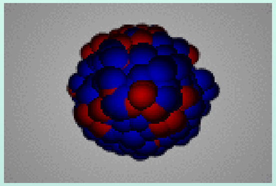
A cut view

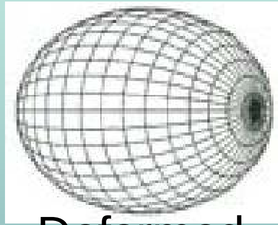

THE VALLEY OF STABILITY
DRIPLINE
Nuclei with excess nucleons move down the valley toward stability
The neutron side of the valley is poorly understood - scientists aren't sure where the dripline lies
SUPERHEAVY ELEMENTS ?
Lead
Gold
Hafnium
Xenon
Tin
MORE PROTONS
MORE NEUTRONS
Known Territory
STABLE ELEMENTS

Nuclear shapes and excitation modes

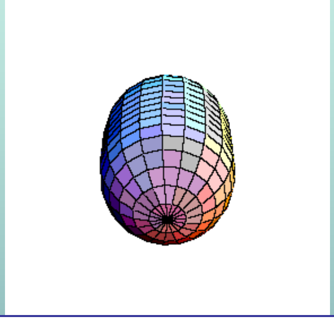
Spherical

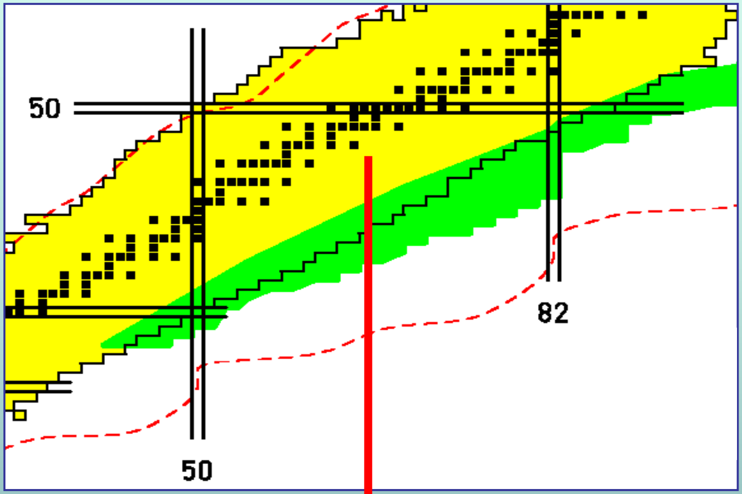






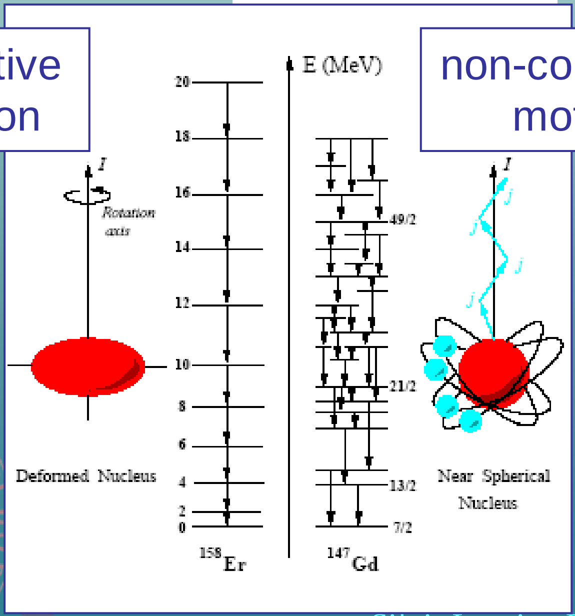
Deformed

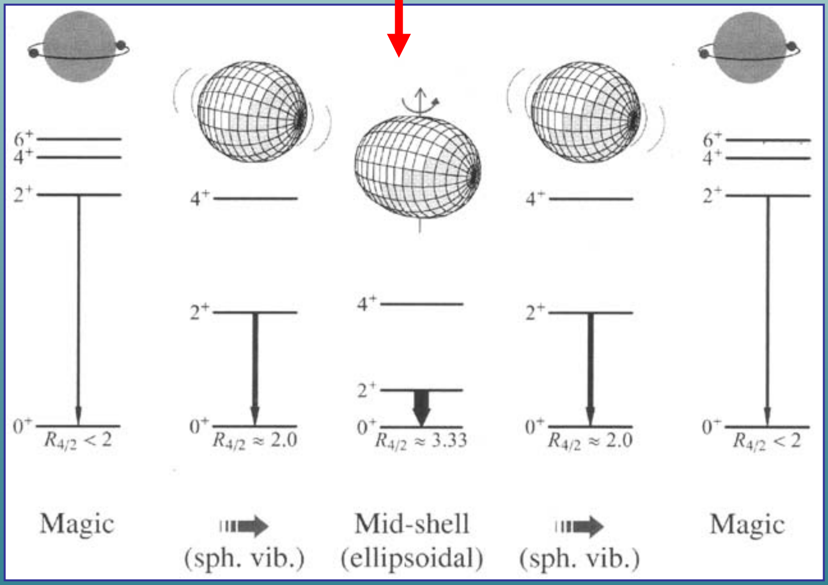




collective motion

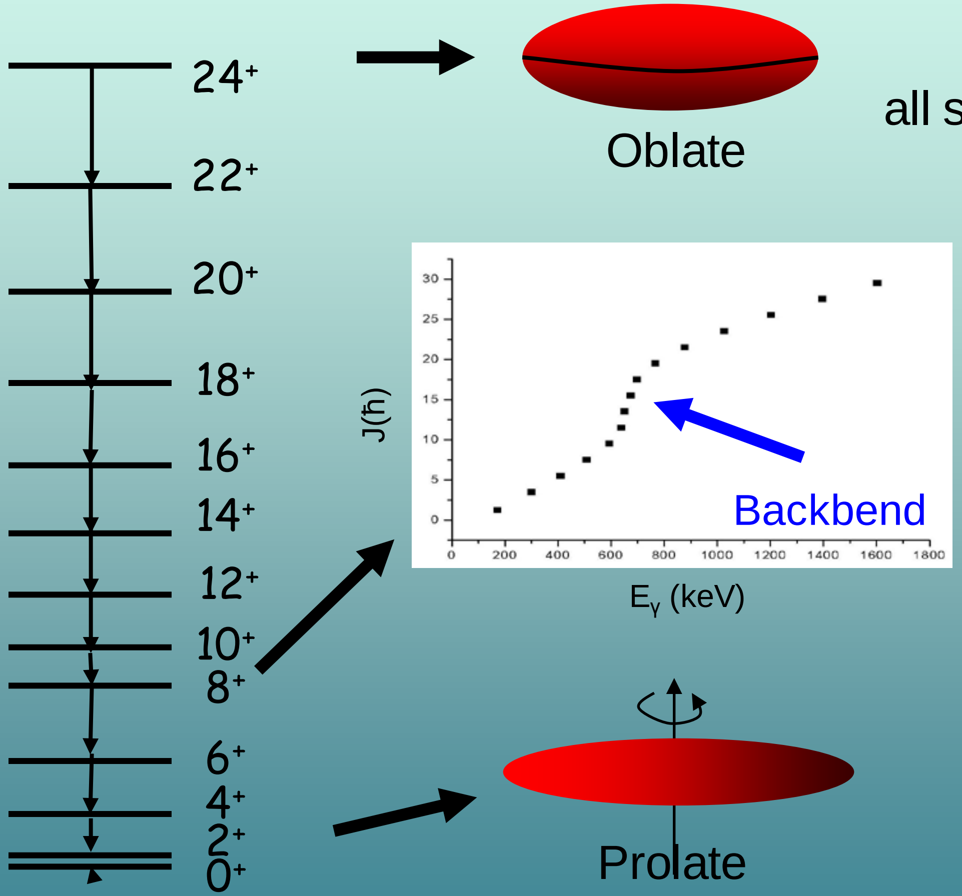
non-collective motion





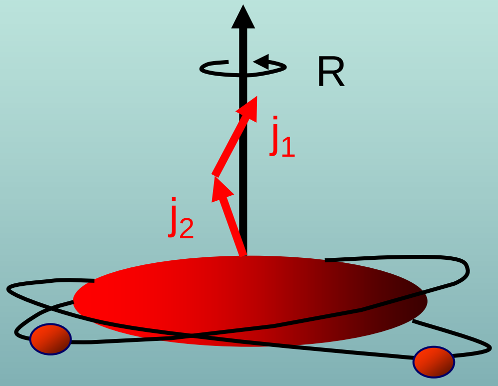
Changing deformation along the band

Rotational band



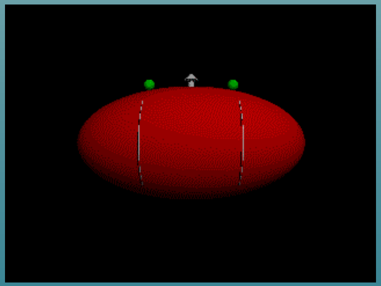
Band termination

all single spins are aligned

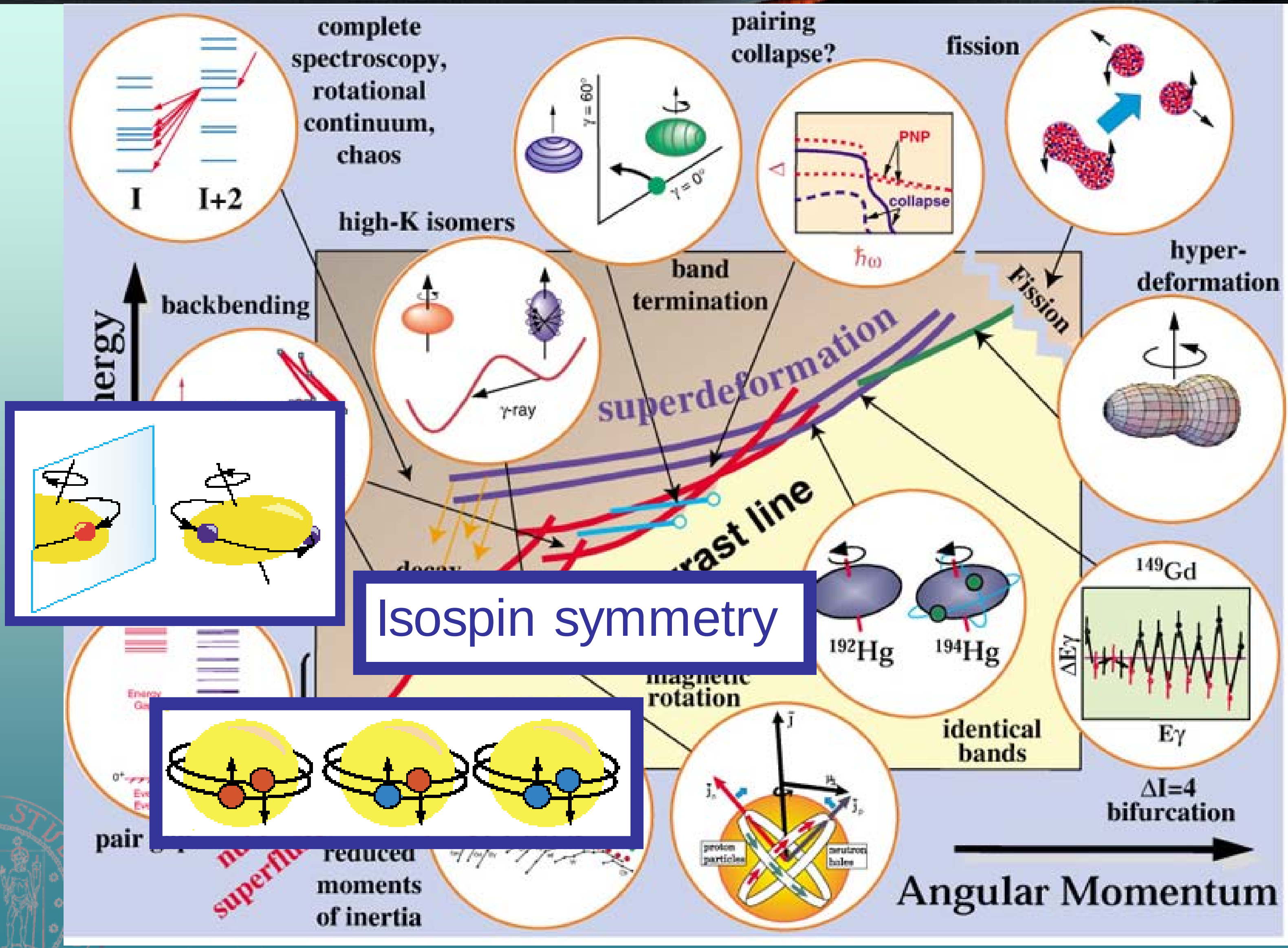


J=**R**+**j₁**+**j₂**

Deformed nucleus



The physics at the yrast line



Outline

What's isospin?

- Why isospin symmetry?

How do we study isospin symmetry?

- Experimental methods
- Theoretical methods

What do we learn from the data?

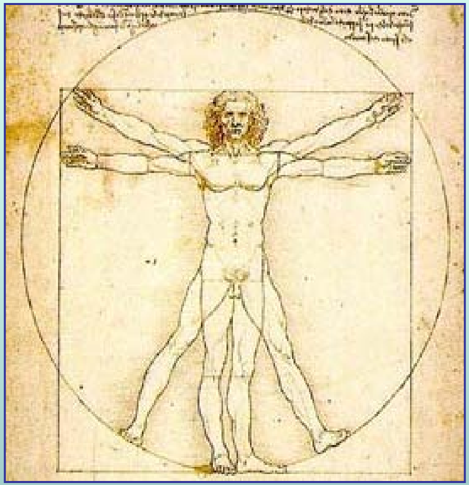
Background

What's isospin?

Why isospin symmetry?

Symmetries

Symmetries help to understand Nature



Examination of fundamental symmetries:
a key questions in Physics

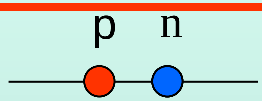
conservation laws

good quantum numbers

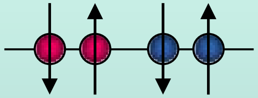
Conserved quantities imply some underlying symmetry enjoyed by the interactions

In Nuclear Physics symmetries help
to understand the behaviour of matter…

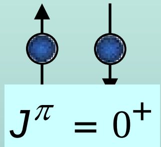
Symmetries in nuclear physics



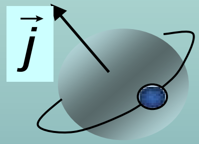
Isospin Symmetry: 1932 Heisenberg SU(2)



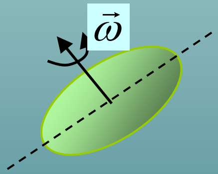
Spin-Isospin Symmetry: 1936 Wigner SU(4)



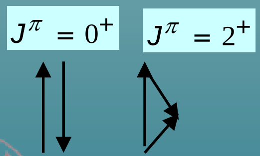
Seniority Pairing: 1943 Racah



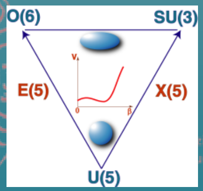
Spherical Symmetry: 1949 Mayer



Nuclear Deformed Field (spontaneous symmetry breaking)
Restore symm. →rotational spectra: 1952 Bohr-Mottelson
SU(3) Dynamical Symmetry: 1958 Elliott



Interacting Boson Model (IBM dynamical symmetry):
1974 Arima and Iachello

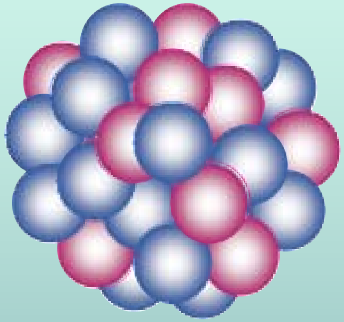


Critical point symm. E(5), X(5) ….
2000… F. Iachello

K. Heyde

The nucleus: a unique laboratory

Composed by two types of fermions differing only on its charge



Strong interaction: largely independent of the charge

Proton – Neutron exchange symmetry

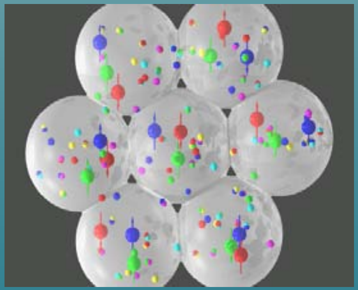
Proton and neutron can be viewed as two alternative states of the same particle: *the nucleon*.

The quantum number that characterizes the two charge states is the *isospin*

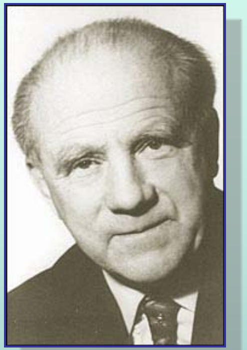
$$t = \frac{1}{2}$$

proton : π
$t_z = -\frac{1}{2}$

neutron : ν
$t_z = +\frac{1}{2}$

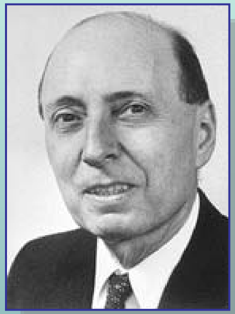


Charge symmetry and Isospin



1932 Heisenberg applies the Pauli matrices to the new problem of labeling the two alternative charge states of the nucleon.

1937 Wigner: isotopic spin is a good quantum number to characterize isobaric multiplets.

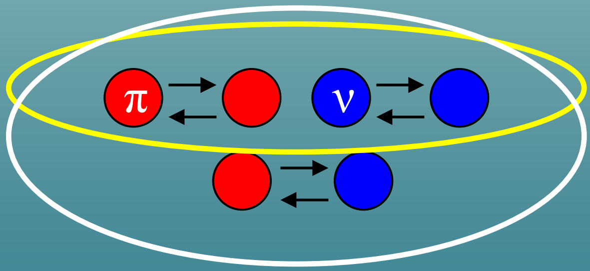


$$T_z = \sum_{i=1}^{A} t_{z,i} = \frac{N-Z}{2}$$

$$\left|\frac{N-Z}{2}\right| \leq T \leq \frac{N+Z}{2}$$

⇨ Isobaric analogue multiplets

- Charge Symmetry: $V_{pp}=V_{nn}$
- Charge Independence: $V_{pp}=V_{nn}=V_{pn}$



Two-nucleon system

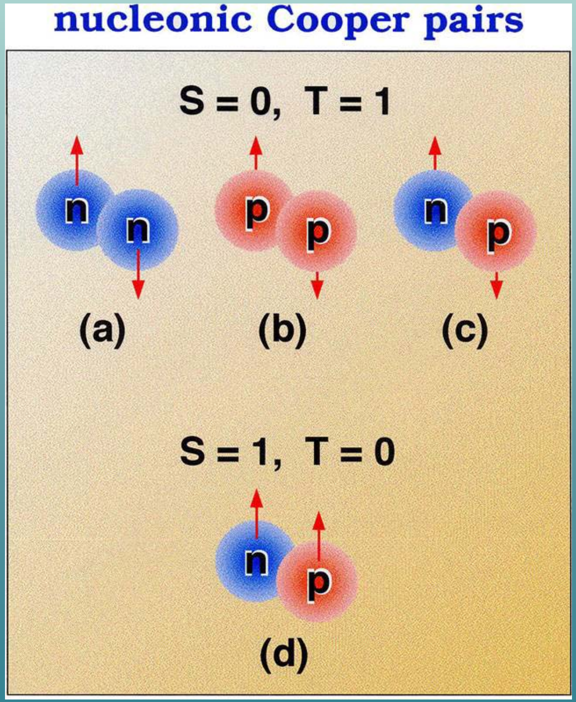
For a two-nucleon system, four different isospin states can exist:

Triplet T=1

$$|T=1, T_z=1\rangle = \uparrow\uparrow \qquad |T=1, T_z=-1\rangle = \downarrow\downarrow \qquad |T=1, T_z=0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$$

Singlet T=0

$$|T=0, T_z=0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$



Total wavefunction must be antisymmetric on exchange of all (spin, space, isospin) co-ordinates…

$$\Psi_{\text{space}} \otimes \Psi_{\text{spin}} \otimes \Psi_{\text{isospin}}$$

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Isobaric multiplets

It is easy to extend the formalism to many-nucleon systems

The isospin quantum number T directly couples together the two effects of charge symmetry/independence and the Pauli principle

$$T_z = \sum_{i=1}^{A} t_{z,i} = \frac{N-Z}{2} \qquad \left|\frac{N-Z}{2}\right| \leq T \leq \frac{N+Z}{2}$$

All nuclear states can be classified by the isospin quantum number, T, which to a good approximation can be treated as a good quantum number.

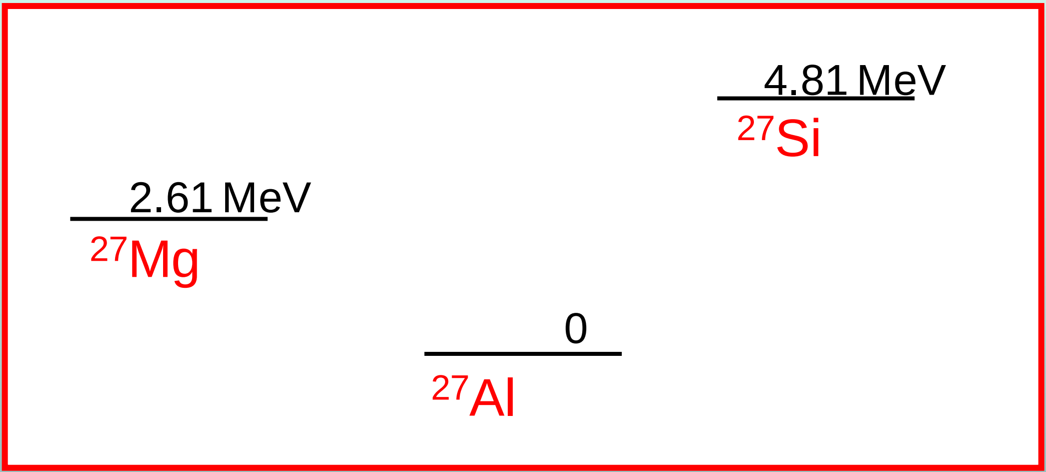
Since $T<T_z$ is forbidden, states of a given T can only occur in a set of nuclei with $T_z=T,T-1,\ldots,-T$ → isospin multiplet or “isobaric multiplet”.

This set of states of the same T in such a multiplet are termed “isobaric analogue states” (IAS).

Energy difference of ground states in A=27

Look at the difference in absolute mass/binding energy

Nuclei of same number of particles A=27



Protons	12	13	14
Neutrons	15	14	13
pp-pairs	66	78	91
nn-pairs	105	91	78
np-pairs	180	182	182
Total	351	351	351

pn-pairs can exist in states NOT allowed for pp or nn pairs - PAULI PRINCIPLE

^{27}Mg is less bound than ^{27}Al even though it has less protons, but it has also less p-n T=0 pairs → T=0 stronger than T=1

R.F. Casten

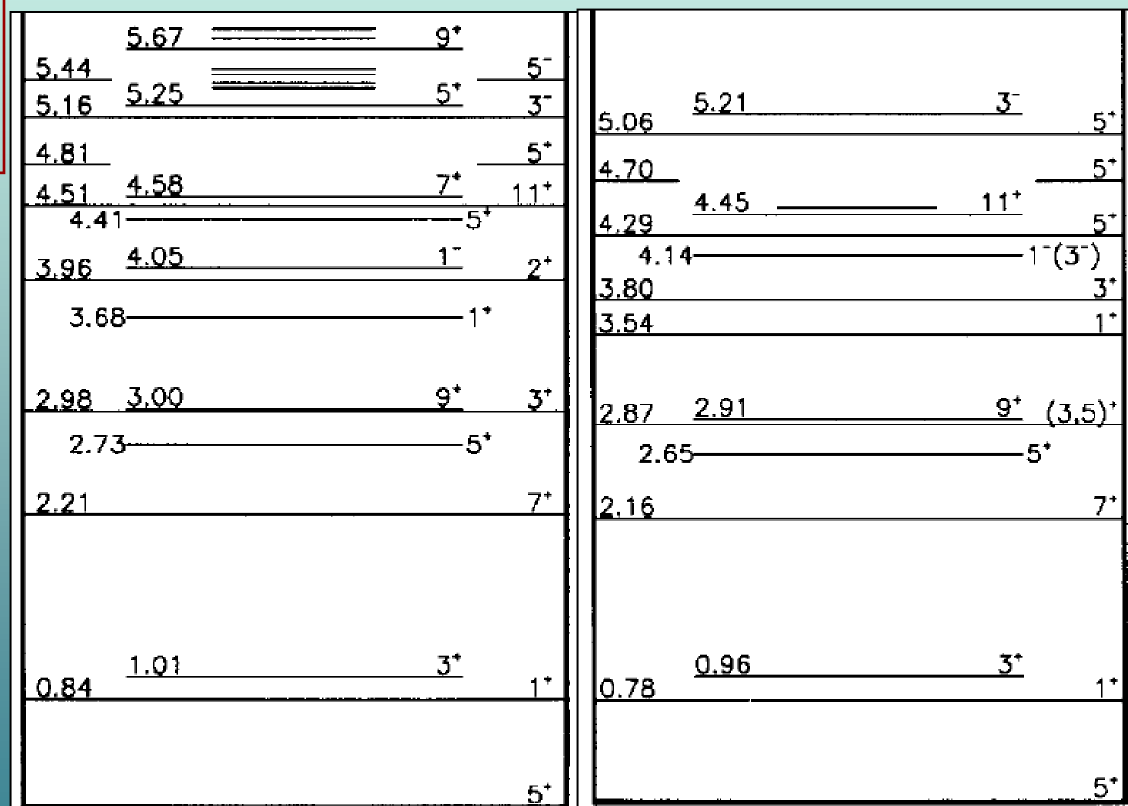
Analog states in isospin doublets

Mirror nuclei

with $T_z = \pm 1/2$

Test of isospin symmetry

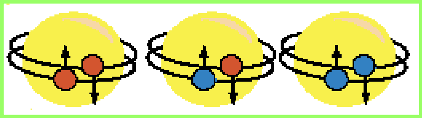
$^{27}_{13}Al_{14}$ $^{27}_{14}Si_{13}$



stable

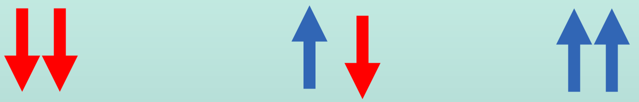
4.16s

T=1 isobaric triplets



The nucleus can be characterized by isospin quantum numbers which restrict the possible states in which the many-nucleon system can exist.

Look at the isobaric triplet: $^{22}_{12}Mg_{10}$ $^{22}_{11}Na_{11}$ $^{22}_{10}Ne_{12}$

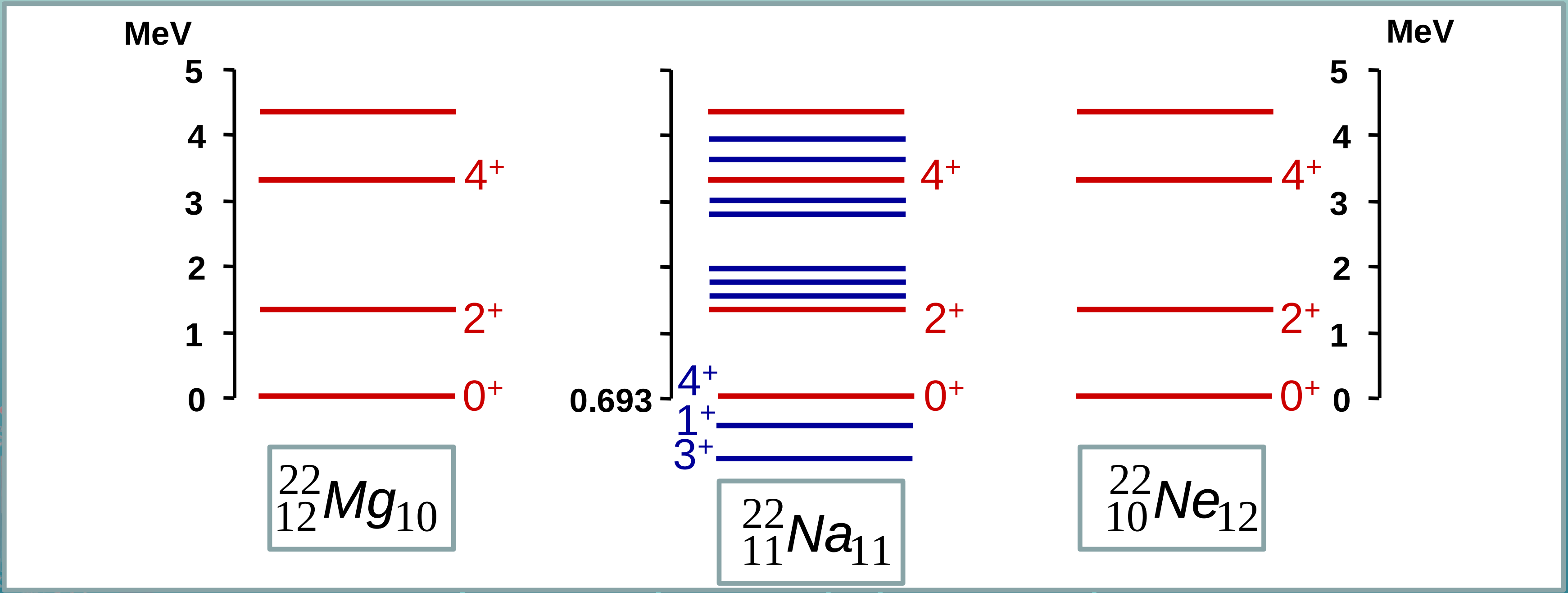


We expect:

T=1 states low in energy in ^{22}Mg and ^{22}Ne

T=0 and *T=1* states in ^{22}Na (N=Z)

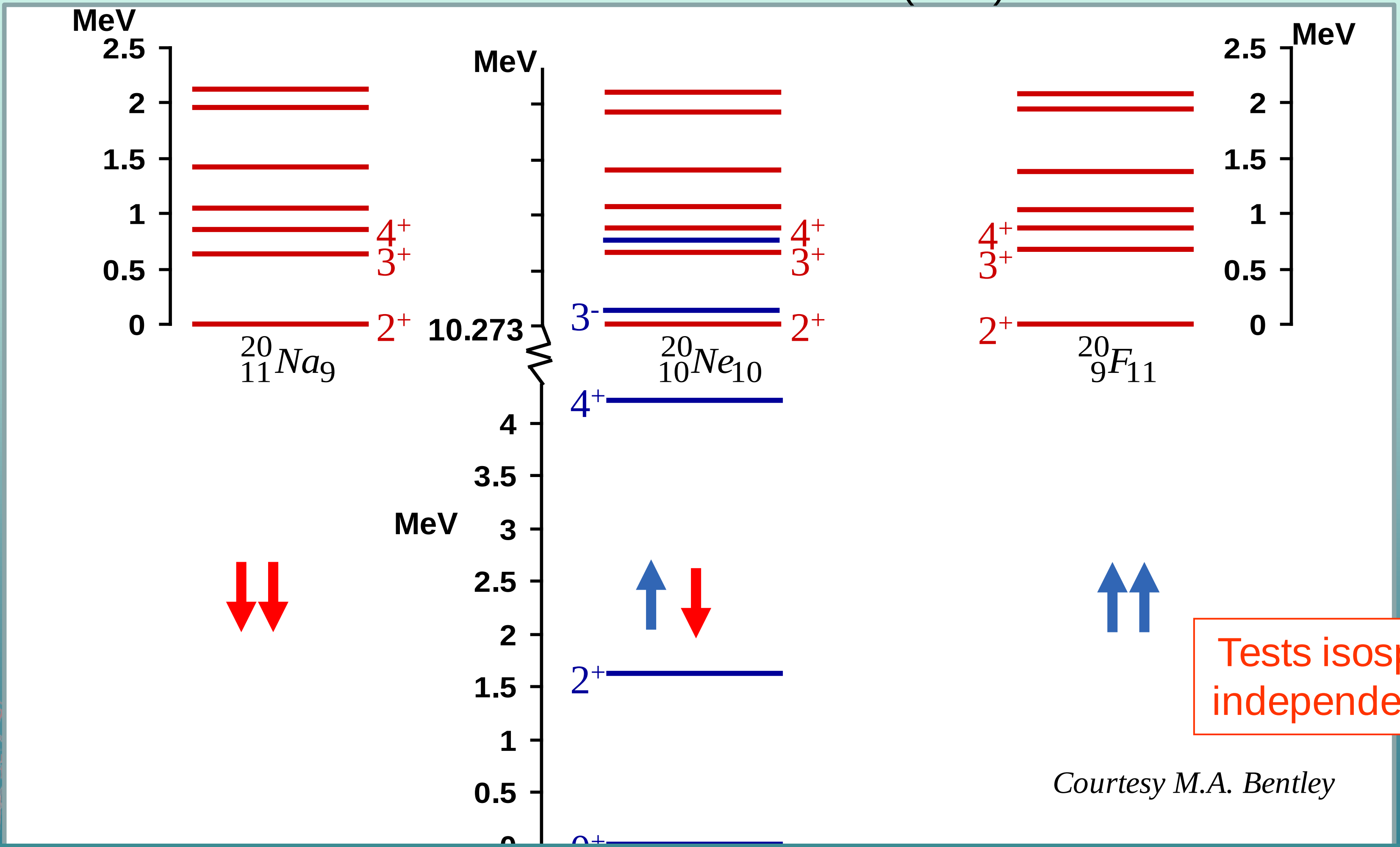
Tests isospin independence



T=1 isobaric triplets

We expect: T=1 states low in energy in ^{20}Na and ^{20}F

T=0 and T=1 states in ^{20}Ne (N=Z)



T ≥ 1 allowed | T ≥ 0 allowed | T ≥ 1 allowed

Three nucleon-system

We can form a doublet T=1/2

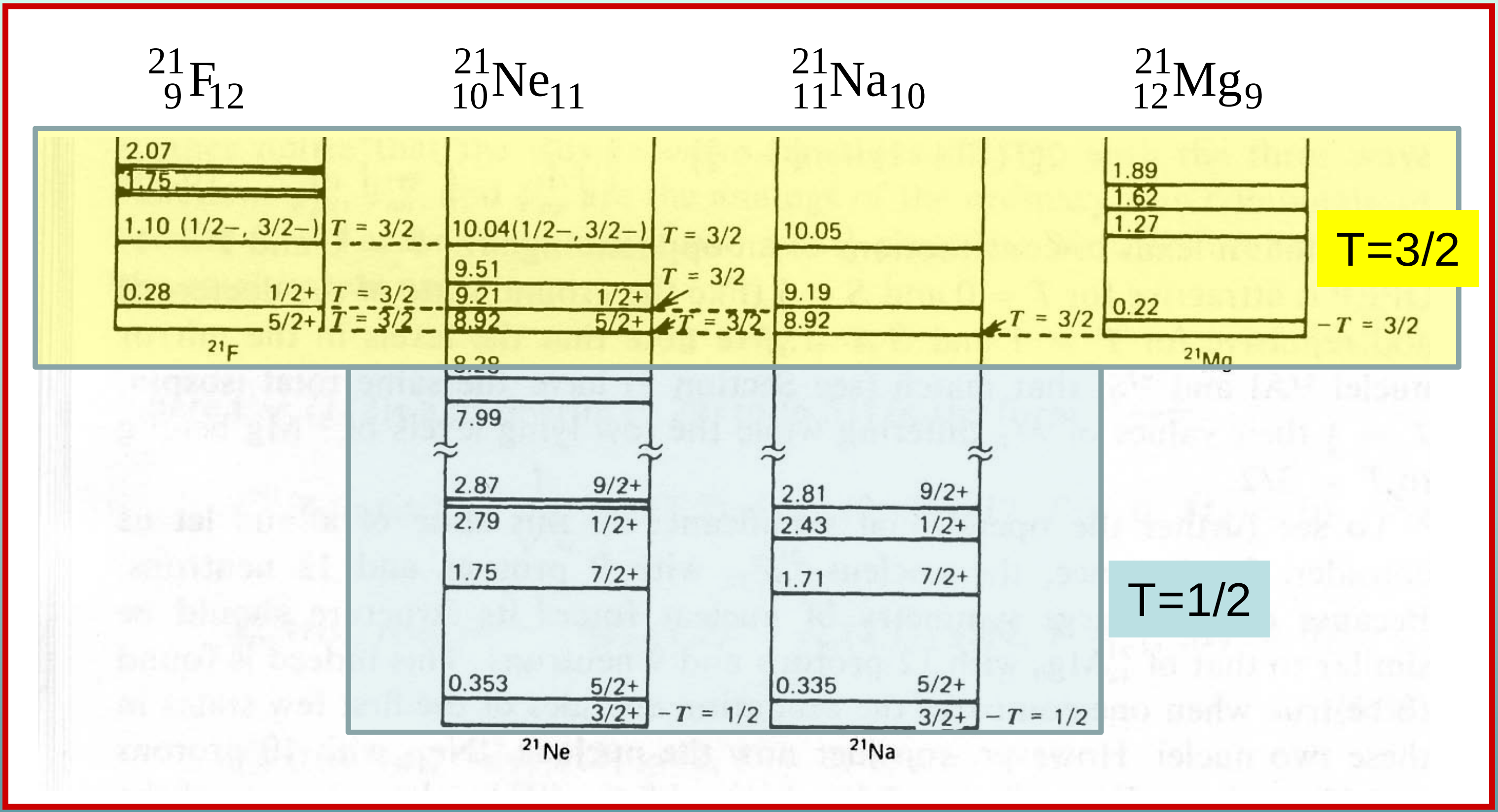
$$\left|T = \tfrac{1}{2}, T_z = +\tfrac{1}{2}\right\rangle = \uparrow\uparrow\downarrow \qquad \left|T = \tfrac{1}{2}, T_z = -\tfrac{1}{2}\right\rangle = \uparrow\downarrow\downarrow$$

or a quadruplet T=3/2

$$\left|T = \tfrac{3}{2}, T_z = +\tfrac{3}{2}\right\rangle = \uparrow\uparrow\uparrow \qquad \left|T = \tfrac{3}{2}, T_z = -\tfrac{3}{2}\right\rangle = \downarrow\downarrow\downarrow$$

$$\left|T = \tfrac{3}{2}, T_z = +\tfrac{1}{2}\right\rangle = \uparrow\uparrow\downarrow \qquad \left|T = \tfrac{3}{2}, T_z = -\tfrac{1}{2}\right\rangle = \uparrow\downarrow\downarrow$$

T=3/2 Isobaric quadruplets: the spectra

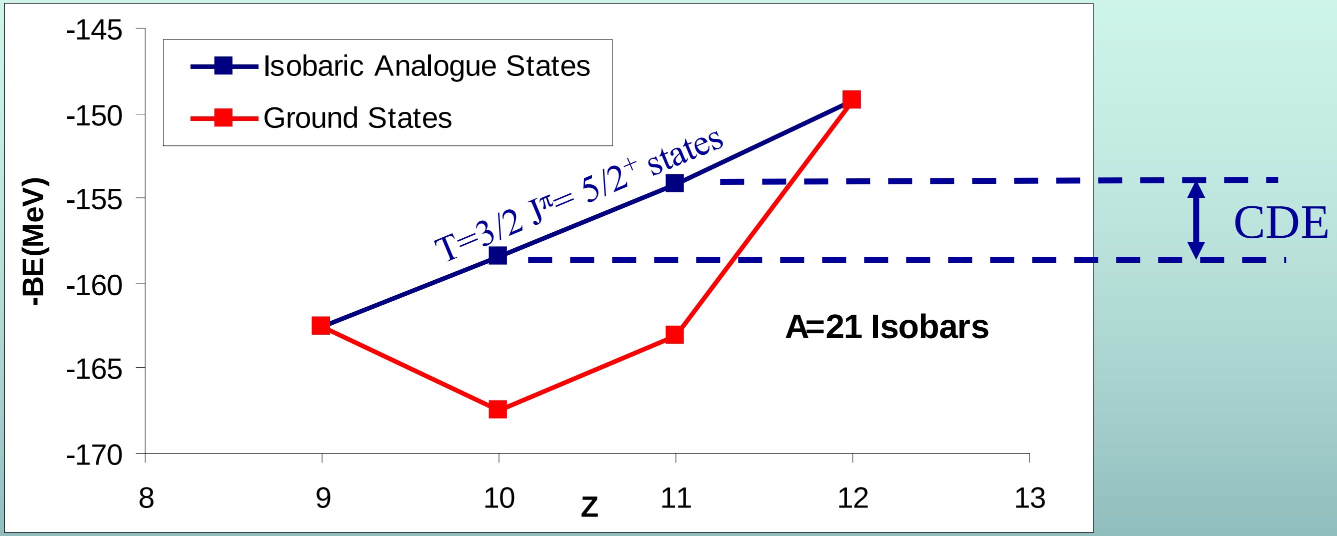


SMALL differences in excitation energy due mainly to Coulomb effects

LARGE differences in mass/binding energy - also due mainly to Coulomb effects….

Courtesy M.A. Bentley

Coulomb displacement energies (CDE) in A=21



CDE : For any two members of a multiplet of isospin T, transformed through exchange of k protons for neutrons is given by

$$(CDE)_{\alpha,T,T_z} = M_{\alpha,T,T_z+k} - M_{\alpha,T,T_z} + k\Delta M_{np}$$

(α = Isobaric analogue state, defined by A, T, J)

Isobaric Multiplet Mass Equation (IMME)

A simple relationship between mass and T_z for a set of analogue states (E.P.Wigner 1957)

To derive it we start with eigenstates of a charge-invariant (CI) Hamiltonian
The eigenvalues do not depend on T_z and analogue states are degenerate

$$H_{CI}\left|\alpha T T_z\right\rangle = E_{\alpha T}\left|\alpha T T_z\right\rangle$$

A charge-violating interaction lifts this degeneracy:

$$H_{CV} = \sum_{k=0}^{2} H_{CV}^{(k)}$$

Any two-body force can be written in the isospin space in terms of 3 components

Isoscalar: $H_{CV}^{(0)} = (V_{nn}+V_{pp}+V_{pn})/3$

Isovector: $H_{CV}^{(1)} = V_{pp} - V_{nn}$

Isotensor: $H_{CV}^{(2)} = V_{pp} + V_{nn} - 2\,V_{pn}$

IMME (2)

The total binding energy can be obtained as

$$BE(\alpha T T_z) = \langle \alpha T T_z | H_{CI} + H_{CV} | \alpha T T_z \rangle$$

The energy splitting in an isobaric multiplet can be obtained as:

$$\Delta BE(\alpha T T_z) = \langle \alpha T T_z | H_{CV} | \alpha T T_z \rangle$$

Let's apply the Wigner-Eckart theorem

$$\langle \alpha T T_z | H_{CV} | \alpha T T_z \rangle = \sum_{k=0}^{2} (-1)^{T\text{-}T_z} \begin{pmatrix} T & k & T \\ -T_z & 0 & T_z \end{pmatrix} \langle \alpha T \| H_{CV}^{(k)} \| \alpha T \rangle$$

Calling

$$M^{(k)} = \langle \alpha T \| H_{CV}^{(k)} \| \alpha T \rangle$$

we obtain …

IMME (3)

$$\Delta BE(\alpha T T_z) = \frac{1}{\sqrt{2T+1}} M^{(0)} + \frac{T_z}{\sqrt{T(2T+1)(T+1)}} M^{(1)} + \frac{3T_z^2 - T(T+1)}{\sqrt{(2T-1)T(2T+1)(T+1)(2T+3)}} M^{(2)}$$

We can now write the total BE into the form (a, b and c independent on T_z):

$$BE(\alpha T T_z) = a + bT_z + cT_z^2$$

Isoscalar, ~ 100's MeV (includes the contribution of V_{Cl})

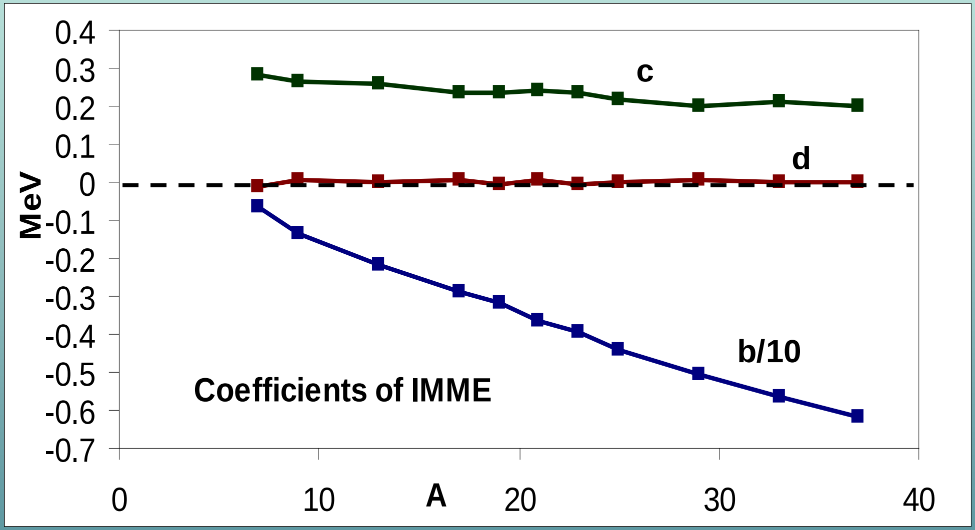
Isovector (Vpp≠ Vnn) ~ 3-15 MeV (~ $A^{2/3}$)

Isotensor ($V_{pp}+V_{nn} \neq 2V_{np}$) ~200-300 keV

The shift of the BE (mass) in an isobaric multiplet depends quadratically on T_z

Tests of IMME

IMME widely tested through fitting $T=3/2$ quadruplets. Coefficient of cubic term (dT_z^3) should be zero.



N. Auerbach Phys. Rep ***98****(1983)273*

*Benenson & Kashy RMP****51****(1979)527*

<u>IMME works beautifully!</u>

The major contribution comes from the Coulomb interaction…

Courtesy M.A. Bentley

The Coulomb contribution to IMME

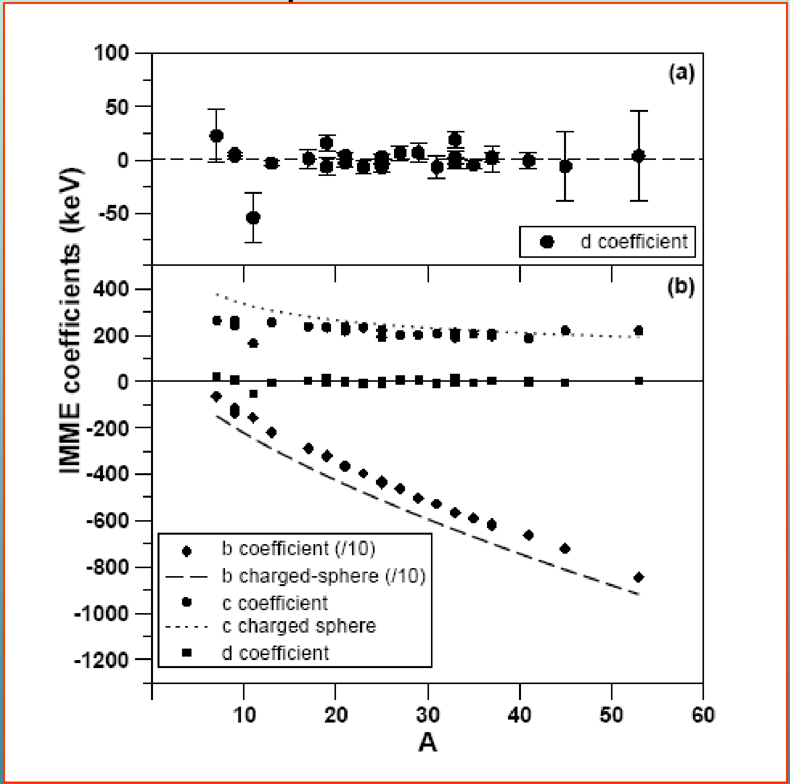
Consider the nucleus a uniform charged sphere, the Coulomb energy is:

$$E_C = \frac{3e^2 Z(Z-1)}{5R_C} = \frac{3e^2}{5r_0 A^{\frac{1}{3}}}\left[\frac{A}{4}(A-2)+(1-A)T_z+T_z^2\right]$$

$$a = \frac{3e^2}{5r_0 A^{\frac{1}{3}}}\frac{A}{4}(A-2)$$

$$b = -\frac{3e^2}{5r_0 A^{\frac{1}{3}}}(A-1)$$

$$c = \frac{3e^2}{5r_0 A^{\frac{1}{3}}}$$



This crude estimate (dashed-line in the figure) has to be corrected by exchange terms (Pauli Principle), etc.

The Nolen-Schiffer Anomaly

IMME works well, but…reproducing magnitude of coefficients → historical problem

For two adjacent nuclei,

$$\mathrm{CDE}_{\alpha,T,T_z} = M_{\alpha,T,T_z} - M_{\alpha,T,T_z+1} + \Delta M_{np}$$
$$= -b - c(2T_z + 1) + \Delta M_{np}$$

Nolen & Schiffer (1969) calculated CDE for wide range of isobaric multiplets…

Used independent-particle models (with exchange term) → Corrected for other phenomena (e.g. electromagnetic spin-orbit (coming soon), magnetic effects, core-polarisation, isospin impurity, etc.).

But…magnitude of predicted CDE ALWAYS ~5-8% lower than experimental values. Underestimated by ~500 keV!

Auerbach (1983) improves the theoretical description but the anomaly remained

Duflo and Zuker (2002) reduce the difference to ~100-200 keV

Coulomb Energy Differences (CED)

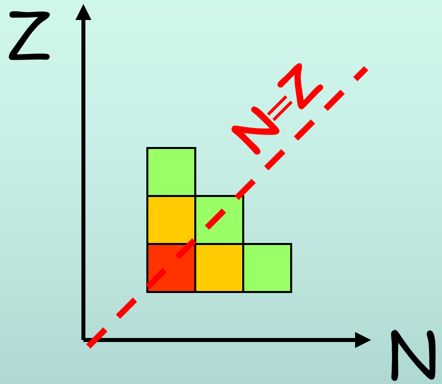
What about excited states - Coulomb Energy Differences…

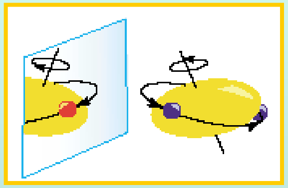
"Normalise" ground state energies and take differences in excitation energy (CED)

How does Coulomb energy (+ other INC) change with E_x, J ?

Does Nolen-Schiffer Anomaly only concern ground states?

CED: MED and TED

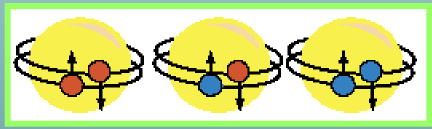




Mirror Energy Differences

$$\text{MED}_J = \text{E}x_{J,T_z=-1/2} - \text{E}x_{J,T_z=+1/2} = -\Delta b_J$$

Test the charge symmetry of the interaction



Triplet Energy Differences

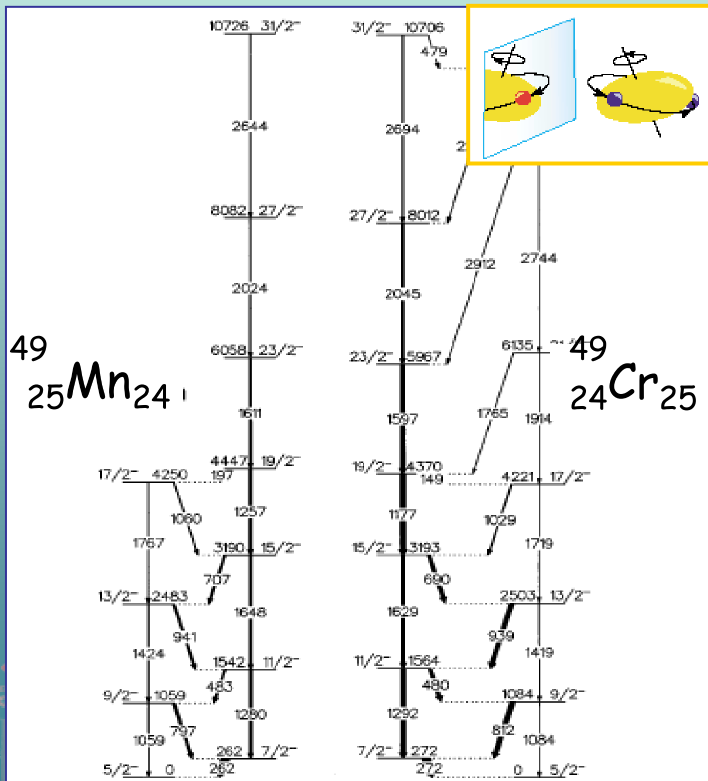
$$\text{TED}_J = \text{E}x_{J,T_z=-1} + \text{E}x_{J,T_z=+1} - 2\text{E}x_{J,T_z=0} = -2\Delta c_J$$

Test the charge independency of the interaction

A classical example: MED in T=1/2 states

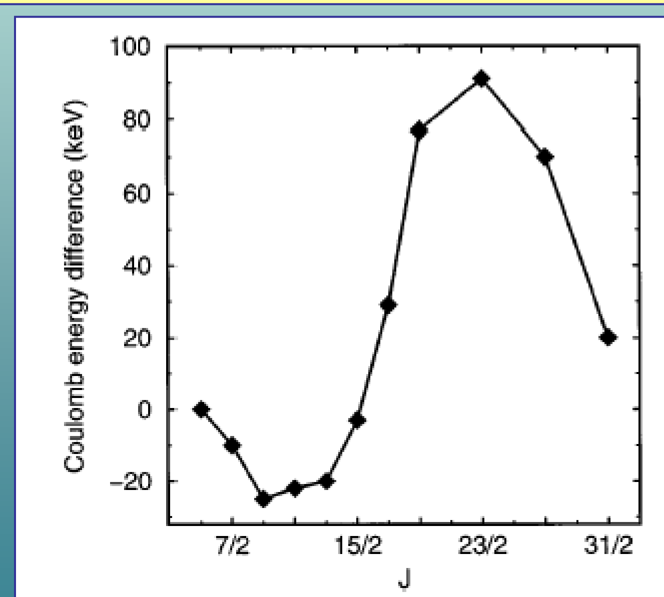
Coulomb effects in isobaric multiplets:

- **bulk energy (100's of MeV)**
- **displacement energy (g.s.) CDE (10's of MeV)**
- **differences between excited states (10's of keV)**

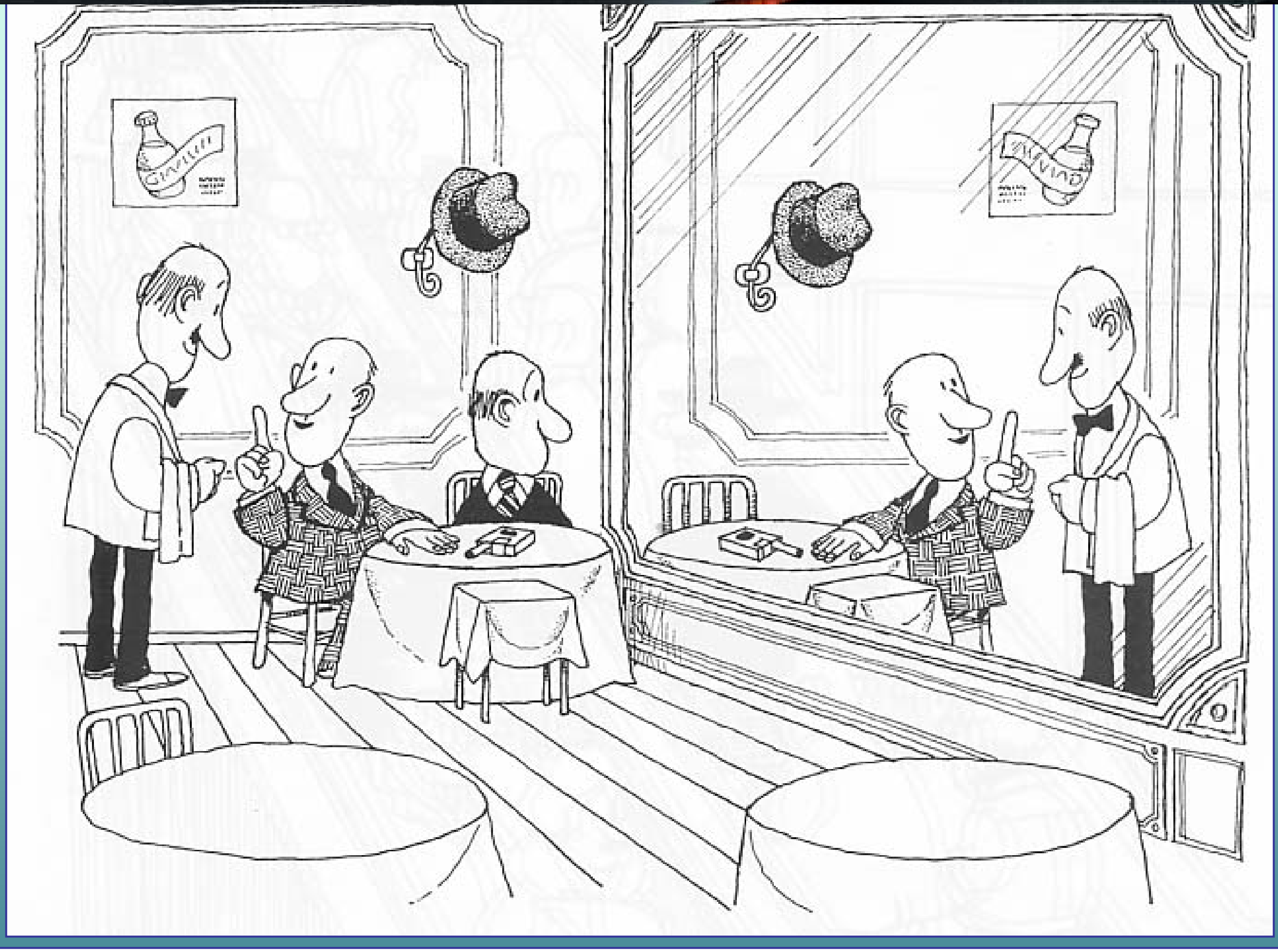


Mirror Energy Differences

$$MED_J = E_J(Z>N) - E_J(Z<N)$$



Symmetry breaking



Isospin symmetry breakdown, mainly due to the Coulomb field, manifests when comparing mirror nuclei. This constitutes an efficient observatory for a direct insight into nuclear structure properties.

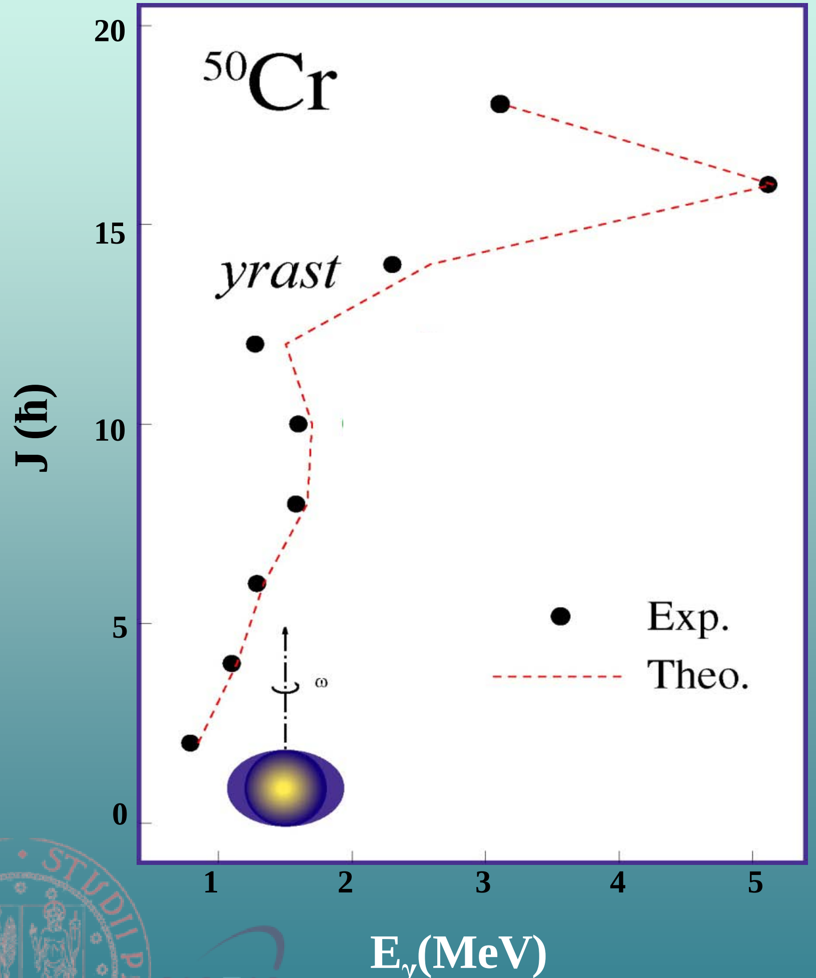
Measuring the Isospin Symmetry Breaking

We will see that CED are extremely sensitive to quite subtle nuclear structure phenomena which, with the aid of shell model calculations, can be interpreted quantitatively at the level of 10's of keV.

We measure ***nuclear*** structure features:

- How the nucleus generates its angular momentum
- Evolution of the radii (deformation) along a rotational band
- Learn about the configuration of the states
- Isospin non-conserving terms in the nuclear interaction

Understanding structure features

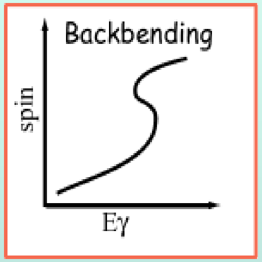


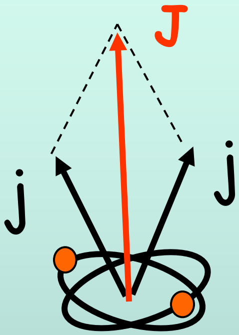
Most of the structure features of nuclei in the $f_{7/2}$ shell are very well described by shell model calculations in the full fp valence space

What happens at the backbending?
- band-crossing?
- alignment?
- which nucleons are aligning?

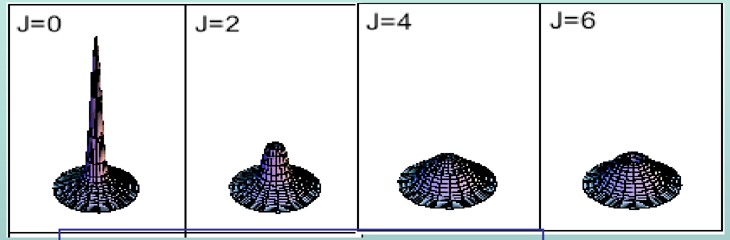
S.M.Lenzi et al., PRC ***56****, 1313 (1997)*

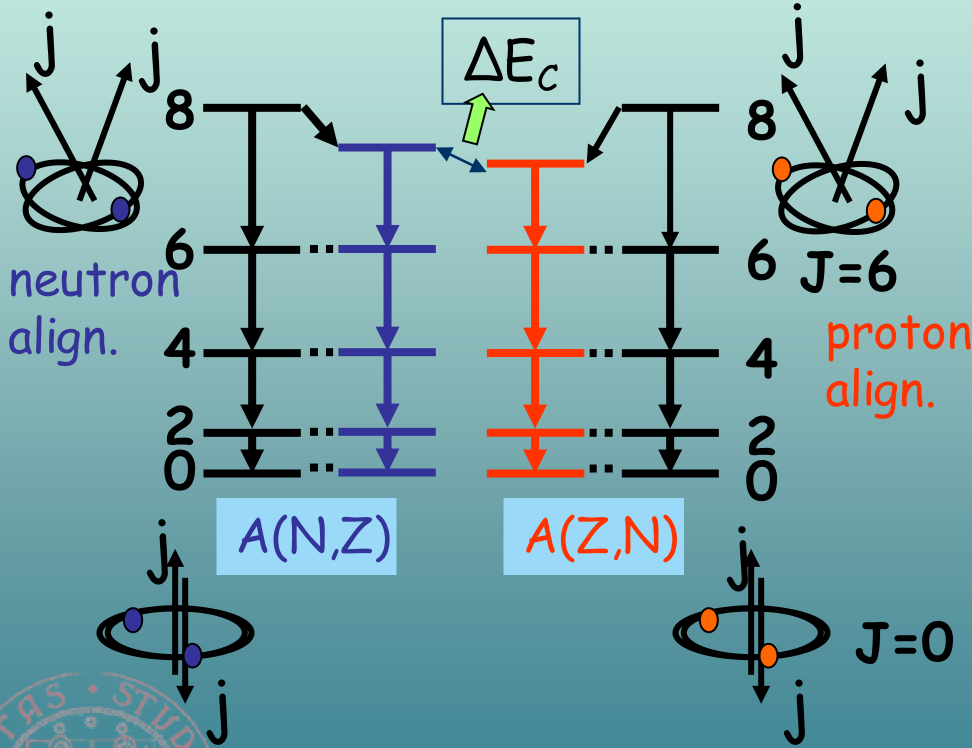
Mirror energy differences and alignment

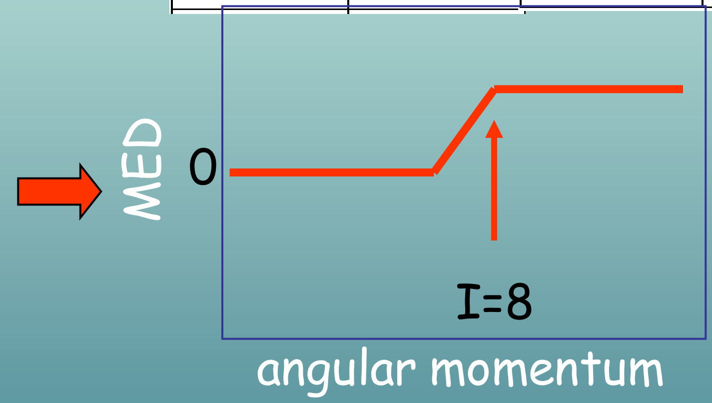




probability distribution for the relative distance of two like particles in the *f*7/2 shell





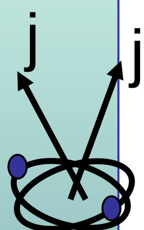


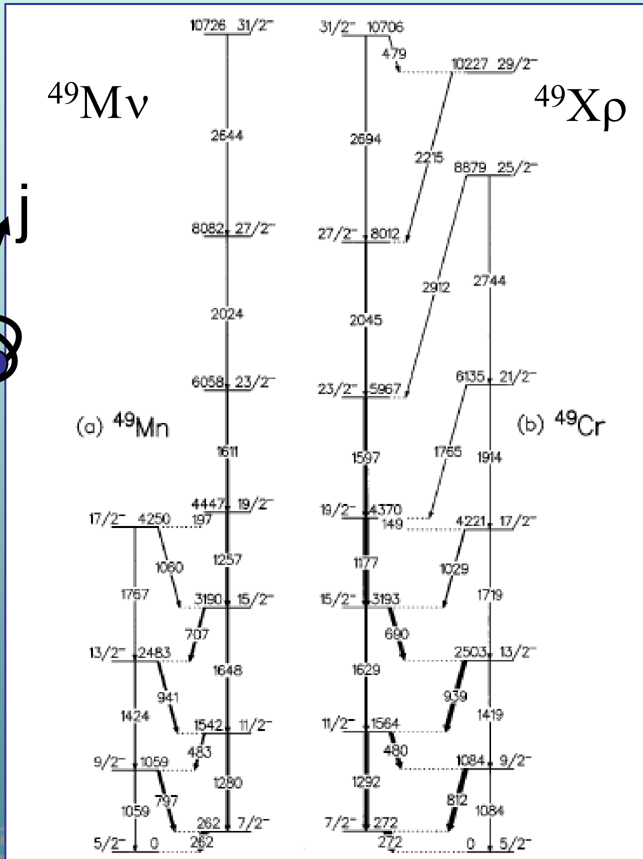
J.A. Cameron *et al.*,
Phys. Lett. B **235**, 239 (1990)

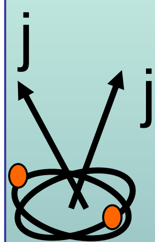
Shifts between the excitation energies of the mirror pair at the back-bend indicate the type of nucleons that are aligning

Nucleon alignment at the backbending

J.A. Cameron *et al.*, Phys. Lett. **B 235**, 239 (1990)
C.D. O'Leary *et al.*, Phys. Rev. Lett. 79, 4349 (1997)

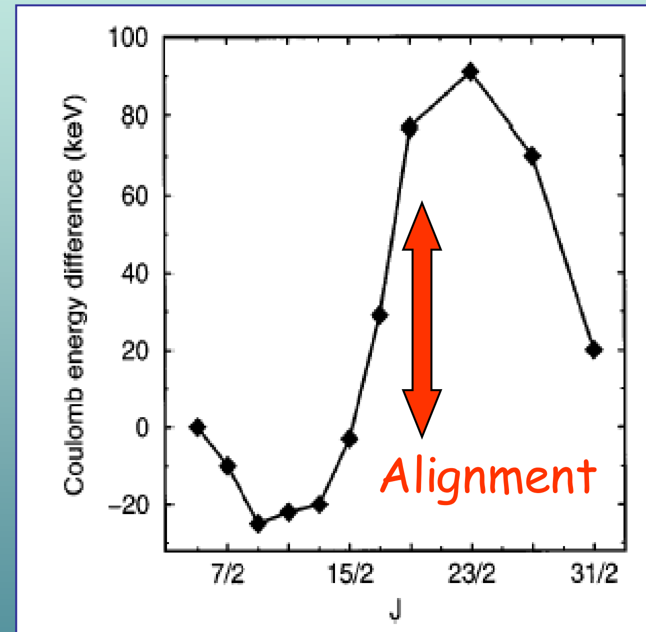






J=6

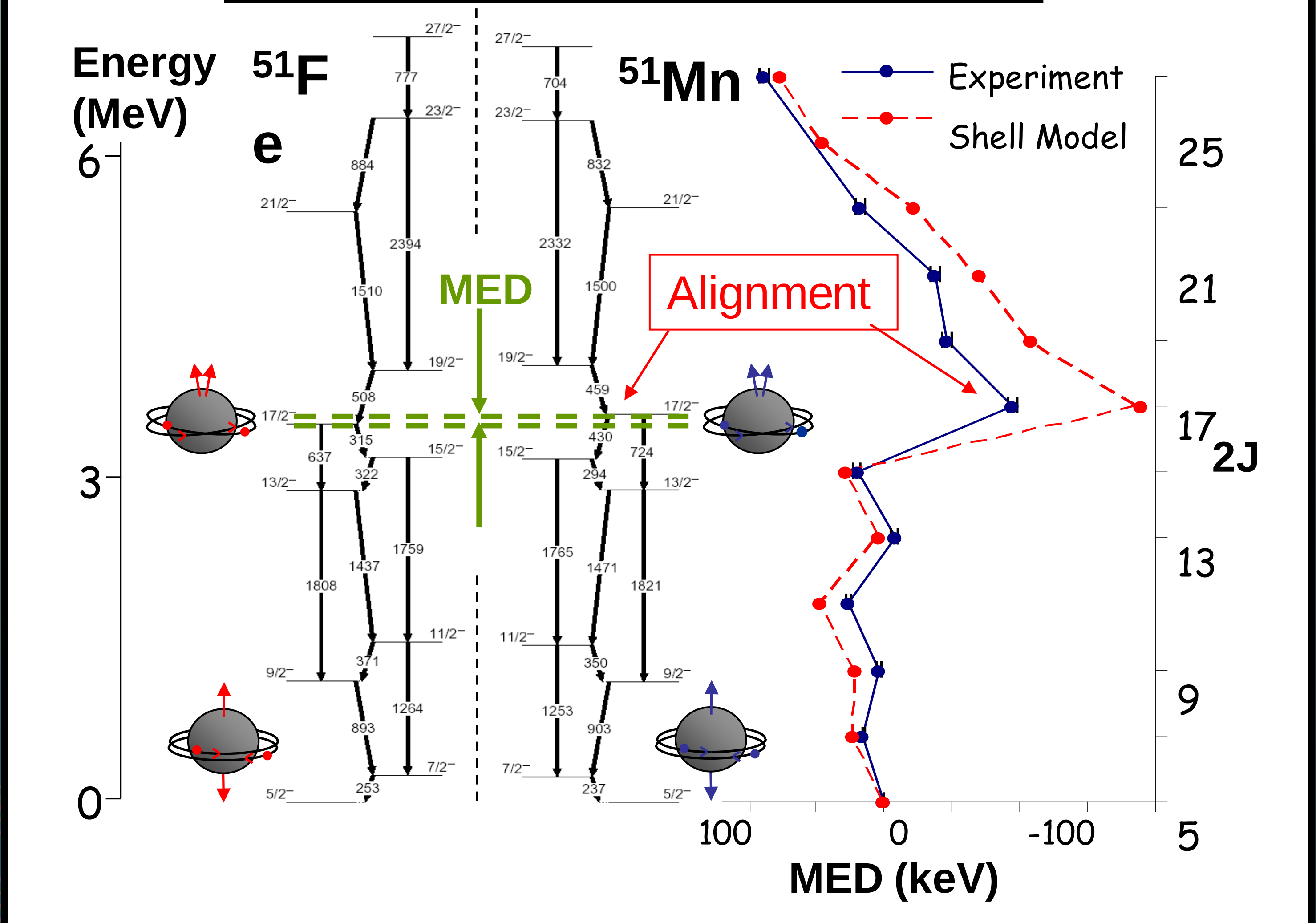
Experimental MED



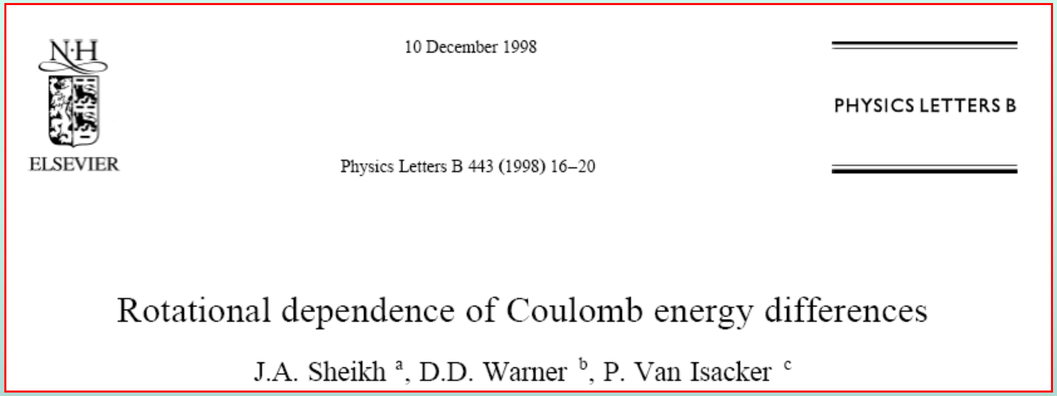
MED are a probe of nuclear structure:
reflect the way the nucleus generates its angular momentum

Nucleon alignment at the backbending

D.D. Warner, M.A. Bentley and P. Van Isacker., Nature Physics 2 (2006) 311



Cranked shell model and alignment


10 December 1998

PHYSICS LETTERS B

Physics Letters B 443 (1998) 16–20

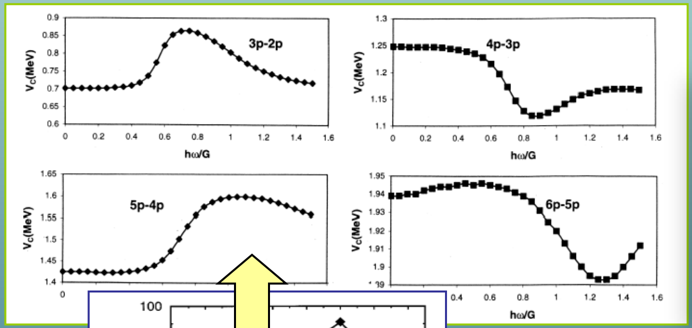
ELSEVIER

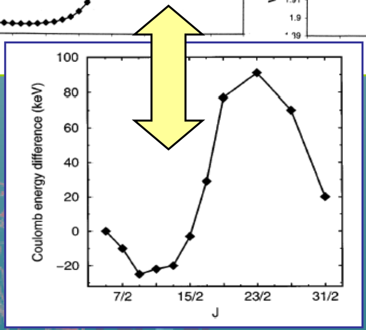
Rotational dependence of Coulomb energy differences

J.A. Sheikh [a], D.D. Warner [b], P. Van Isacker [c]

Cranked shell-model

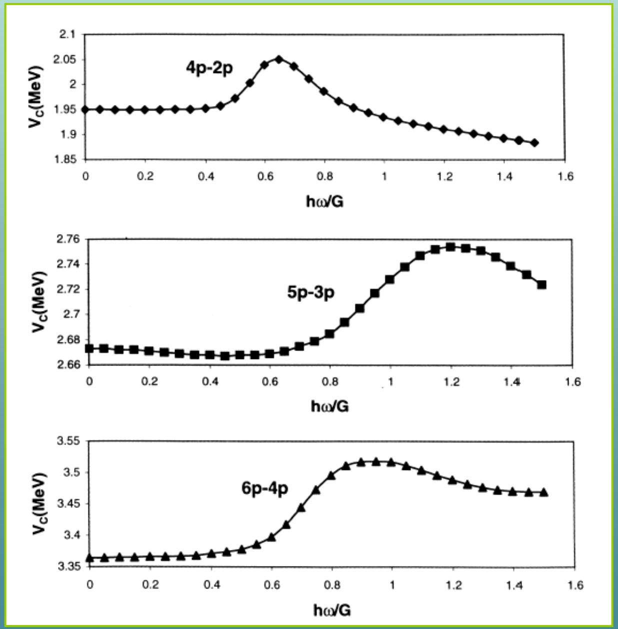
$$H' = h_{def} + V_2 - \omega J_x$$





Approximations:
- one shell only
- fixed deformation
- no p-n pairing

CSM: good qualitative description of the data



J.A. Sheikh, P. Van Isacker, D.D. Warner and J.A. Cameron, Phys. Lett. B 252 (1990) 314

Alignment and shell model

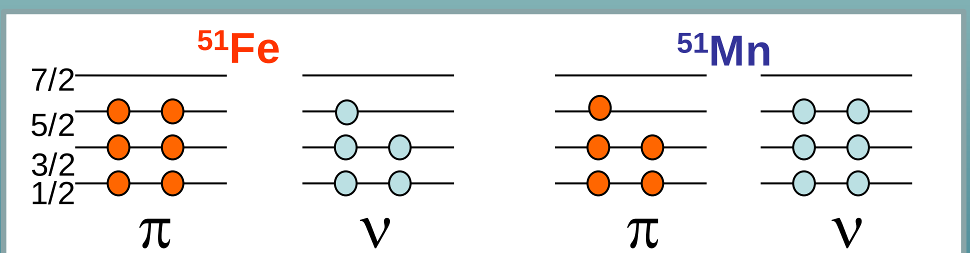
Define the operator

$$\mathbf{A}_\pi = \left[\left(a_\pi^+ a_\pi^+ \right)^{J=6} \left(a_\pi a_\pi \right)^{J=6} \right]^0$$

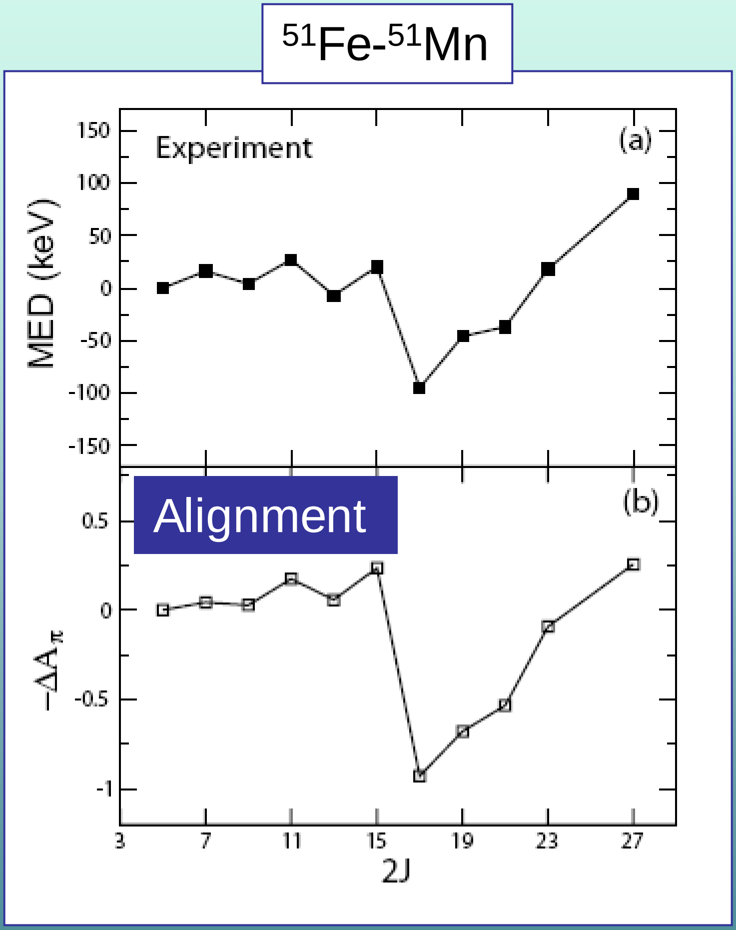
"Counts" the number of protons coupled to J=6

Calculate the difference of the expectation value in both mirror as a function of the angular momentum

$$\Delta\mathbf{A}_{\pi,J} = \langle \Phi_J | \mathbf{A}_\pi(Z_>) | \Phi_J \rangle - \langle \Phi'_J | \mathbf{A}_\pi(Z_<) | \Phi'_J \rangle$$



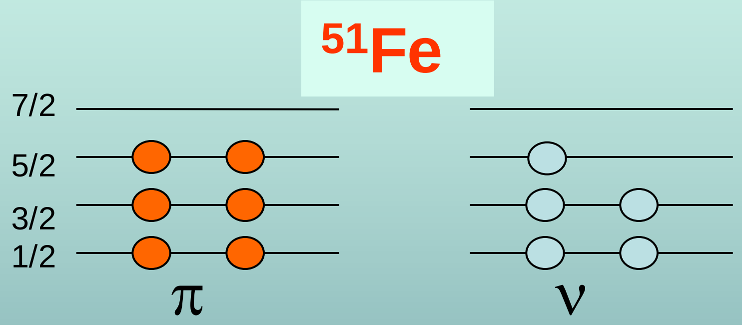
In ^{51}Fe (^{51}Mn) a pair of protons (neutrons) align first and at higher frequency align the neutrons (protons)

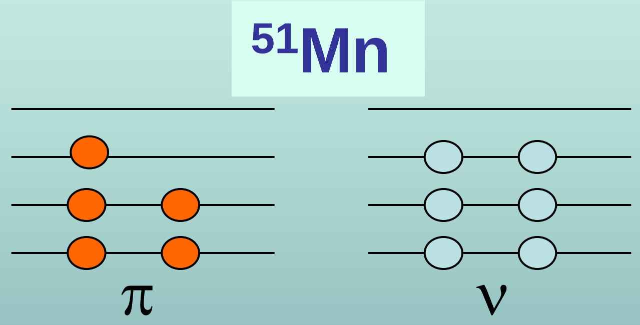


M.A.Bentley et al. Phys Rev. C62 (2000) 051303

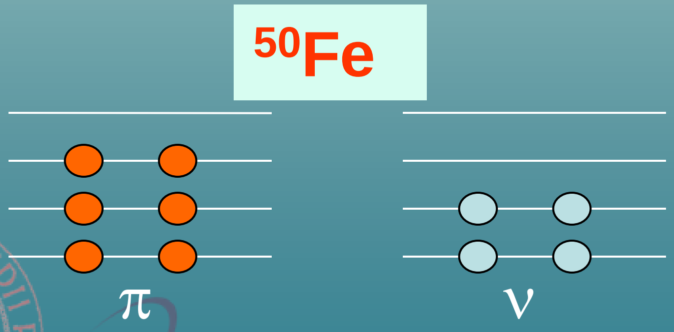
Alignment in odd- and even-mass nuclei

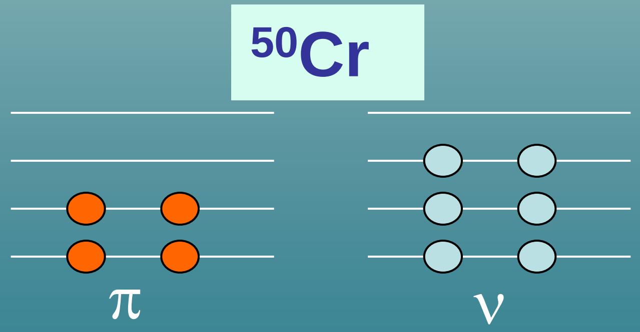
In odd-mass nuclei, the type of nucleons that aligns first is determined by the **blocking effect** → the even fluid will align first



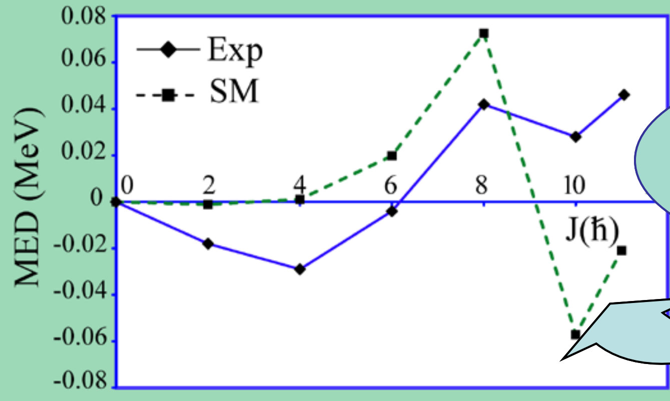


What about even-even rotating nuclei?





Alignment in even-even rotating nuclei

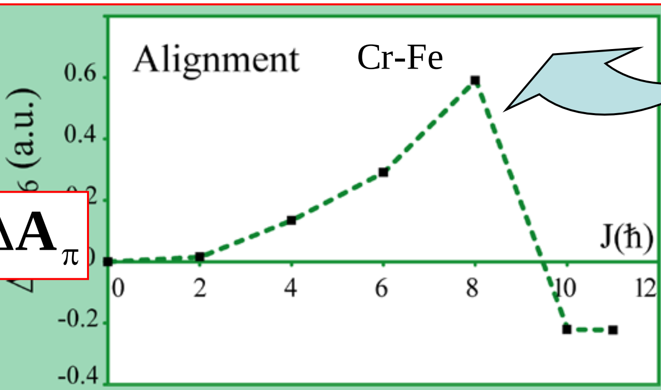


Renormalization of the Coulomb m.e.? Only a Coulomb effect?

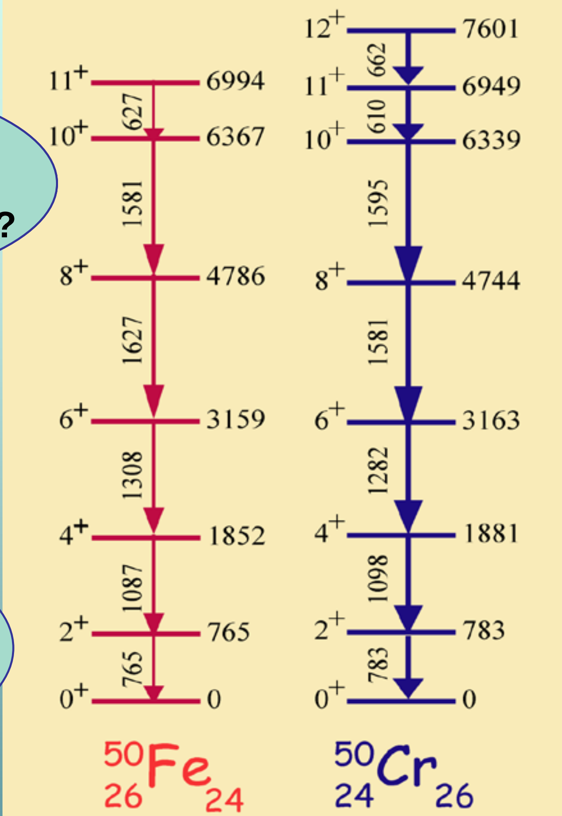
calculate for protons in both mirrors:

$$\Delta \mathbf{A}_{\pi} = \langle \Phi_J | \mathbf{A}_{\pi}(Z_>) | \Phi_J \rangle - \langle \Phi'_J | \mathbf{A}_{\pi}(Z_<) | \Phi'$$

counts the number of aligned protons



$-\Delta \mathbf{A}_{\pi}$



In ^{50}Cr (^{50}Fe) a pair of protons (neutrons) align first and at higher frequency align the neutrons (protons)

can shell model do better?

S.M.Lenzi et al., Phys. Rev. Lett. 87, 122501 (2001)

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Lecture 1

The end