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# Traitement des Réactions de Transfert: Détermination de Facteurs Spectroscopiques

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# Scope of these lectures

We shall give a brief overview of the most commonly used direct reaction models, concentrating on how they are used in practice to extract spectroscopic information

We shall *not* present details of the theory behind these models

Sources of uncertainty in derived spectroscopic quantities will be discussed

The ultimate aim of these lectures is to present you with sufficient information to enable you to assess critically (in the best sense of the term!) analyses of direct reactions in the literature:

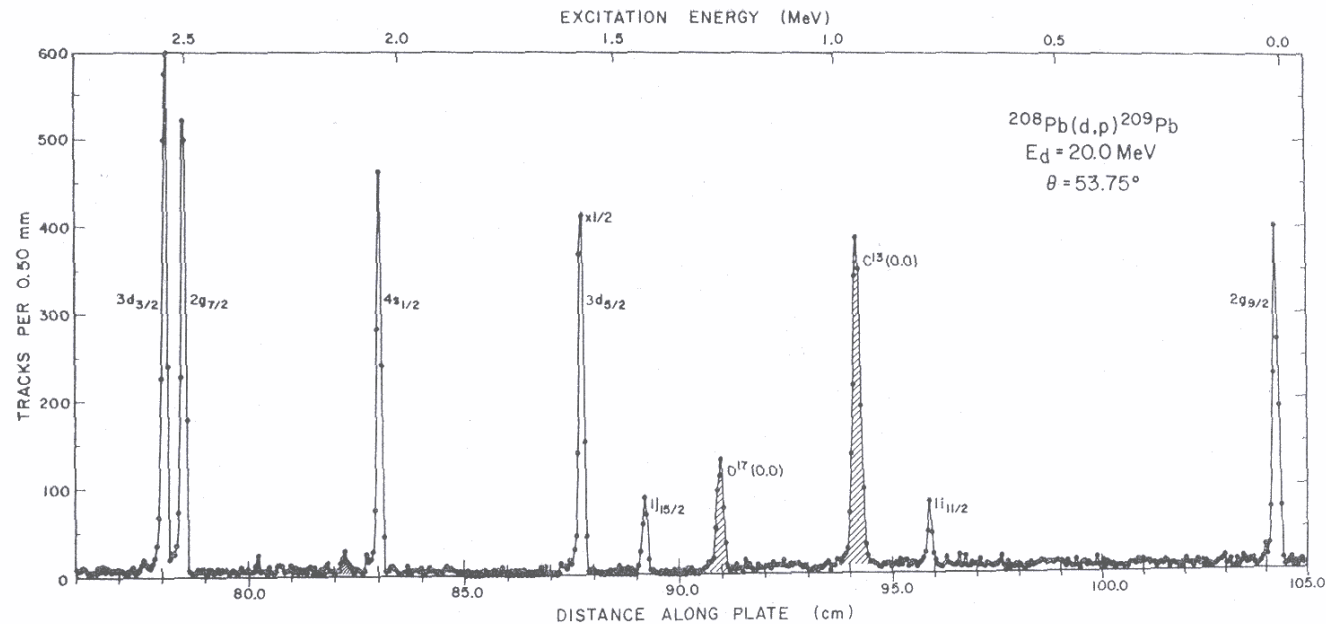
Is the reaction model used appropriate to the circumstances?

What is the likely uncertainty in the derived spectroscopic information?

Are the conclusions drawn fully justified by the analysis?

## Introduction: direct reactions as a spectroscopic tool

Direct reactions are a useful spectroscopic tool due to their selectivity – they favour the population of single particle levels:



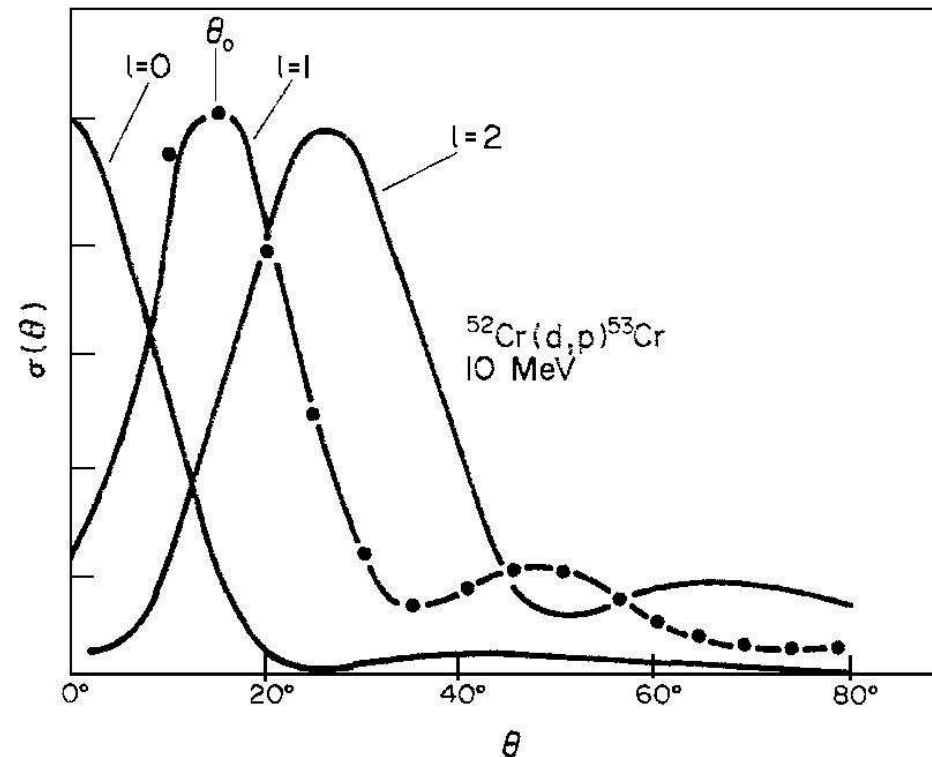
D.G. Kovar et al., Nucl. Phys. **A231** (1974) 266.

Apart from excitation energies (from spectra) how does one obtain the desired information (spin, parity, spectroscopic factor) about these levels?

# How do we obtain the spectroscopic information?

Measure an **angular distribution** of the differential cross section ( $d\sigma/d\Omega$ )

The form of the distribution depends on the angular momentum transferred in the reaction [S.T. Butler, Phys. Rev. **80** (1950) 1095]:



L.D. Knutson and W. Haeberli, Prog. Part. Nucl. Phys. **3** (1980) 127.

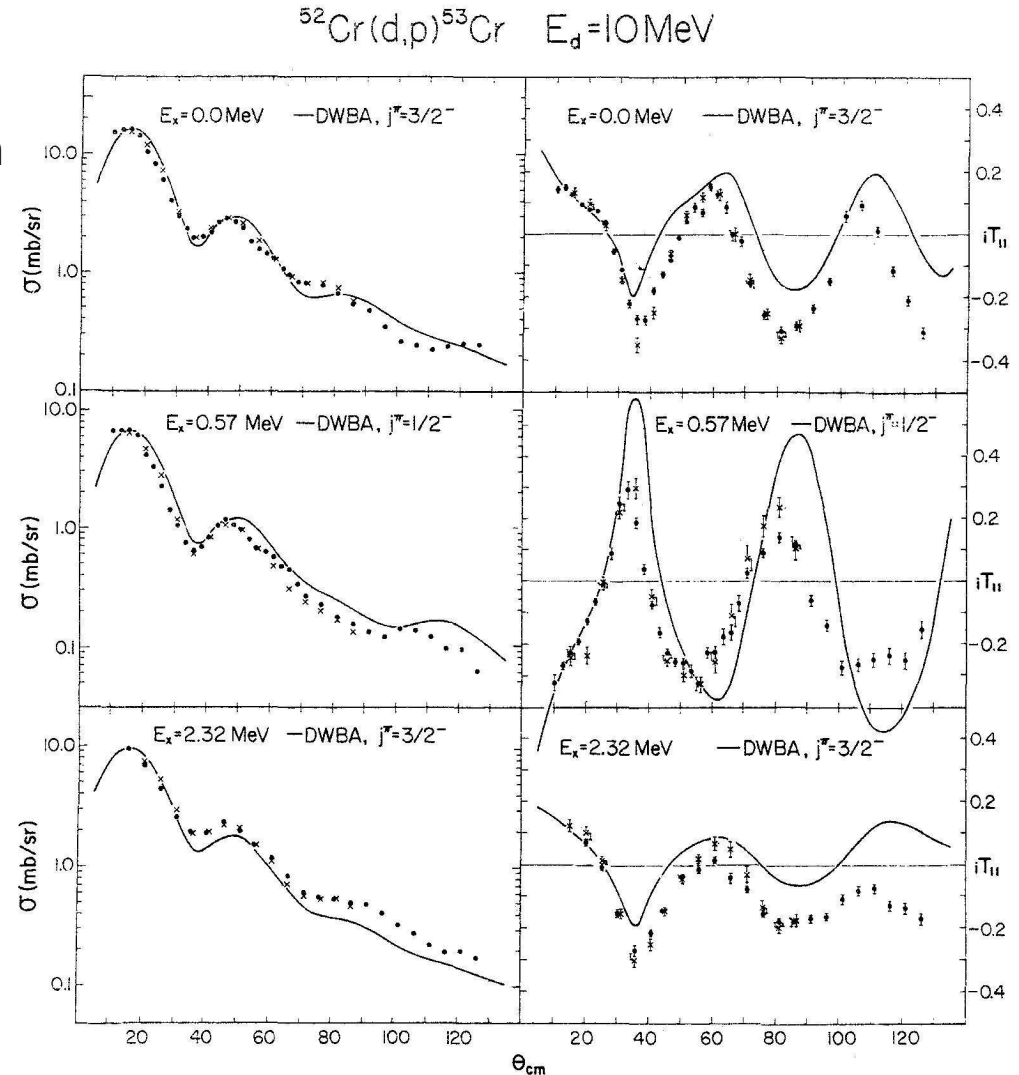
This tells us the relative angular momentum,  $L$ , of the transferred particle with respect to the “core” and the parity of the state;

$$\pi_T \pi_R = (-1)^L$$

However, we wish to determine  $J_R$ , the total spin of the state, where  $\mathbf{J}_R = \mathbf{J}_T + \mathbf{L} + \mathbf{s}$  and  $s$  is the intrinsic spin of the transferred particle (1/2 for nucleons)

To do that, we need a polarised beam (or target) to measure the

**analysing powers** →



L.D. Knutson and W. Haeberli, Prog. Part. Nucl. Phys. **3** (1980) 127.

# How do we actually extract L from the measured angular distributions?

It is useful to know that different L transfers give angular distributions of different forms, but how do we go about determining *which* L for a specific case?

We need a *model* of the reaction process, which is also essential if we wish to determine the *spectroscopic factors* (we shall define these shortly)

There are four reaction models with which we shall concern ourselves here:

- 1) The **D**istorted **W**ave **B**orn **A**pproximation (**DWBA**) – the simplest useful reaction model, assumes a direct, one-step process that is weak and may be treated by perturbation theory
- 2) The adiabatic model – a modification of DWBA for (d,p) and (p,d) reactions that takes deuteron breakup effects into account in an approximate way
- 3) The **C**oupled **C**hannels **B**orn **A**pproximation (**CCBA**) – used when the assumption of a one-step transfer process breaks down; strong inelastic excitations modelled with coupled channels theory, transfers still with DWBA
- 4) **C**oupled **R**eaction **C**hannels (**CRC**) – does not assume one-step or weak transfer process. All processes on equal footing; (complex) rearrangements of flux possible

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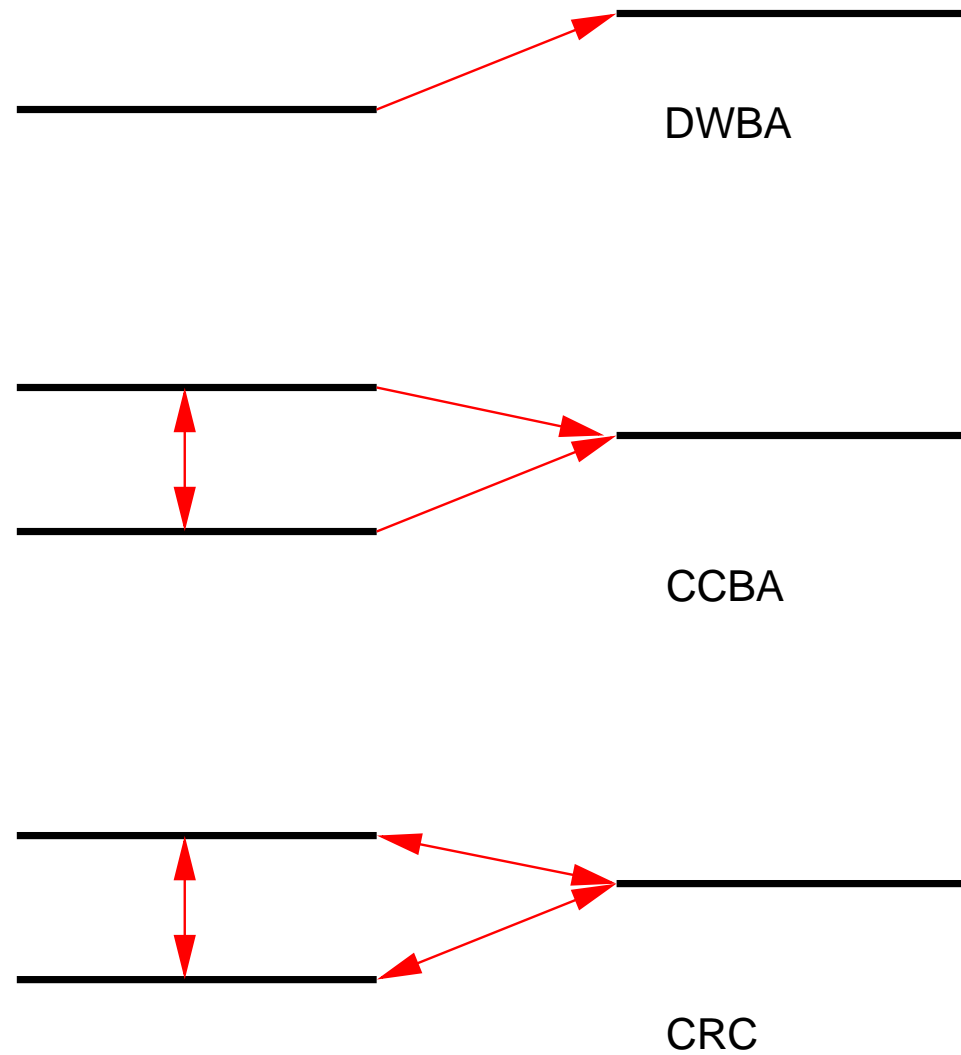
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# Simplified visualisation of DWBA:CCBA:CR

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## (Simple) definition of the spectroscopic factor

The differential cross section,  $d\sigma(\theta,E)/d\Omega$ , for a transfer reaction may be written (for the simplest case) in the following form:

$$d\sigma(\theta,E)/d\Omega = S_{JL} F_{JL}(\theta,E)$$

$S_{JL}$  is a number depending on the initial and final states and the quantum numbers  $J,L$  of the transferred particle, and  $F_{JL}(\theta,E)$  is a factor that depends on the reaction mechanism, containing all the angular and energy dependence

$S_{JL}$  is the spectroscopic factor – it often includes the isospin Clebsch-Gordan coefficient,  $C$ , and is sometimes then written as  $C^2S$

If we have a code that can calculate  $F_{JL}(\theta,E)$  and a measured angular distribution of  $d\sigma(\theta,E)/d\Omega$ , then we may obtain  $S_{JL}$  (in practice,  $S_{JL}$  is the product of two spectroscopic factors, one of which is determined by other means)



# The Distorted Wave Born Approximation

What are the basic “ingredients” of a DWBA calculation?

- 1) Optical model potentials that describe the elastic scattering in the entrance and exit channels
- 2) Wave functions and potentials that bind the transferred particle in the “donor” and to the “acceptor” nucleus – e.g. for the  $^{12}\text{C}(d,p)^{13}\text{C}$  reaction, the d is the donor and the  $^{12}\text{C}$  the acceptor of the transferred neutron. Thus, we need  $V_{pn}$  to calculate the internal wave function of the deuteron and  $V_{nC}$  (where C represents  $^{12}\text{C}$  here) to calculate the internal wave function of the  $^{13}\text{C}$  state of interest

In practice, it may arise that appropriate elastic scattering data are not available and we are constrained to use global potentials – often far from ideal

The wave functions for 2) are usually calculated by binding the particle in a Woods-Saxon potential well of fixed “geometry” with a depth adjusted to give the known binding energy of the state in question (sometimes referred to as the “well-depth prescription”)

# Ingredients of a DWBA calculation continued

To calculate the internal wave functions we need some further information:

- 1) The spin-parity ( $J_R^\pi$ ) of the state of the “composite” nucleus
- 2) The angular momentum ( $L$ ) of the transferred particle relative to the “core”
- 3) The number of nodes ( $N$ ) in the radial wave function

These quantities are known for the light particle (the  $d$  in a  $(d,p)$  reaction) but for the heavy particle ( $^{13}\text{C}$  in our example) they are part of the information we wish to determine

In practice, we assume different values for  $L$  and compare with experiment. To do this, we must assume a definite  $J_R$ , even though cross section data alone do not determine this quantity. To determine the number of nodes in the radial wave function, for single nucleon transfer we consult a shell model scheme and find a reasonable level with the desired  $L$  and  $J_R$ .  $N$  is then the principal quantum number of that state (there are complications for the transfer of composite particles such as  $d$ ,  $^3\text{He}$ ,  $^4\text{He}$  etc.)

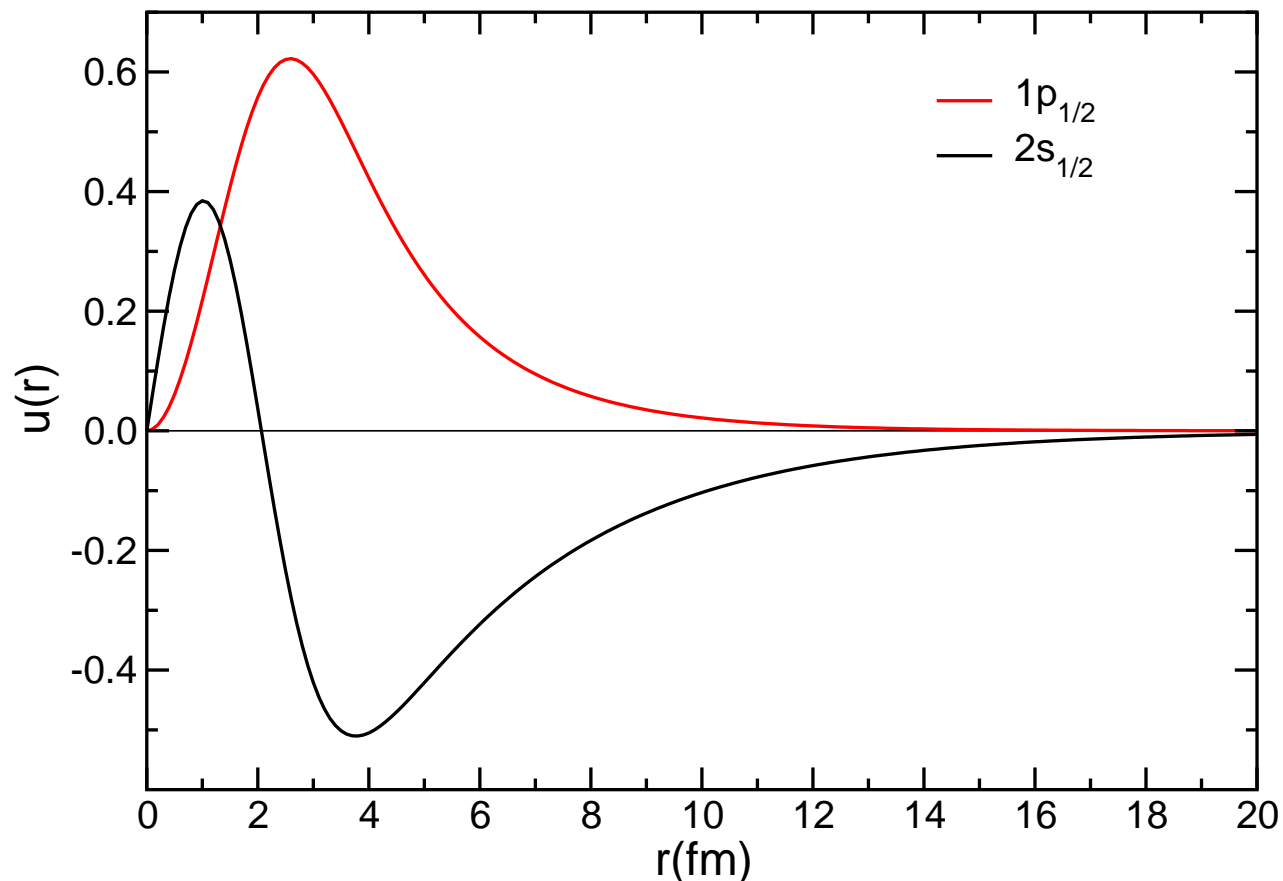
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# Illustrations of radial wave functions

Staying with our example of  $^{12}\text{C}(d,p)^{13}\text{C}$ , the ground and first excited states of  $^{13}\text{C}$  are known to be  $\frac{1}{2}^-$  and  $\frac{1}{2}^+$ , respectively and correspond to a neutron in the  $1p_{1/2}$  or  $2s_{1/2}$  shell model state outside the  $^{12}\text{C}$  “core”:



## Ambiguities and traps for the unwary ...

Having obtained our optical model potentials and chosen our binding potentials etc. for the internal wave functions, we may determine the spectroscopic factor for each state by normalising our DWBA calculation to the data (after first obtaining the correct  $L$  value for each state by comparison between the form of the measured and calculated angular distributions)

However, the reality is not quite so simple:

- 1) There are ambiguities in empirical optical model potentials – several different “families” of potentials may fit the same data equally well. This will affect (mostly) the values obtained for the spectroscopic factors
- 2) The “geometry” parameters (i.e. radius and diffuseness) of the binding potentials (for the transferred particle to the heavy core nucleus) are somewhat arbitrary – there is a large range of “reasonable” values, so that the derived spectroscopic factors can vary by up to 30 % ...
- 3) Check the definition of  $N$  in the code you use – some codes start from  $N=0$ , others from  $N=1$  (the calculated cross section scales with  $N$ )

# The adiabatic model – (d,p) and (p,d) only

Ambiguities in the optical model potentials and binding potential geometry apart, the DWBA runs into difficulties for (d,p) and (p,d) reactions for incident energies around 20 MeV [Johnson and Soper, Phys. Rev. C 1 (1970) 976]

These have been found to be caused by effects due to breakup of the weakly bound deuteron

The **adiabatic model** [Johnson and Soper, Phys. Rev. C 1 (1970) 976, Harvey and Johnson, Phys. Rev. C 3 (1971) 636] takes account of these effects in an approximate way by redefining the incident deuteron distorted wave – it still describes the motion of the centre of mass of the neutron and proton but they may not be in the form of a bound deuteron

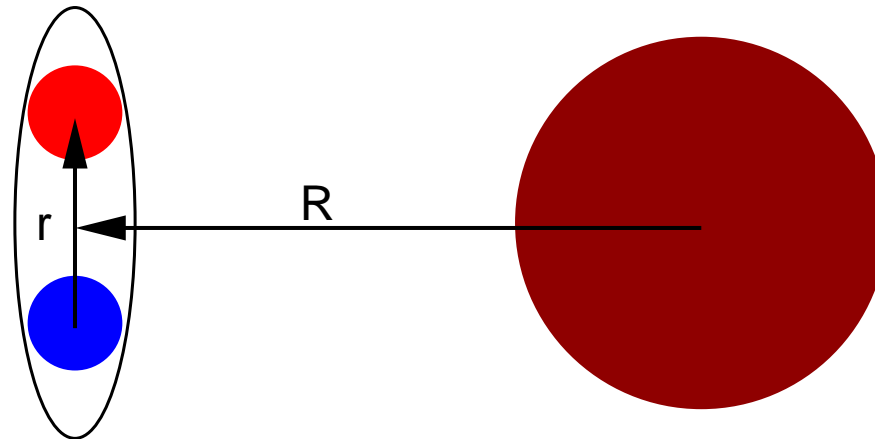
In practice, this is achieved by introducing the **adiabatic potential** into a standard DWBA code in place of the usual deuteron optical model potential

# The adiabatic potential

This is formally defined as:

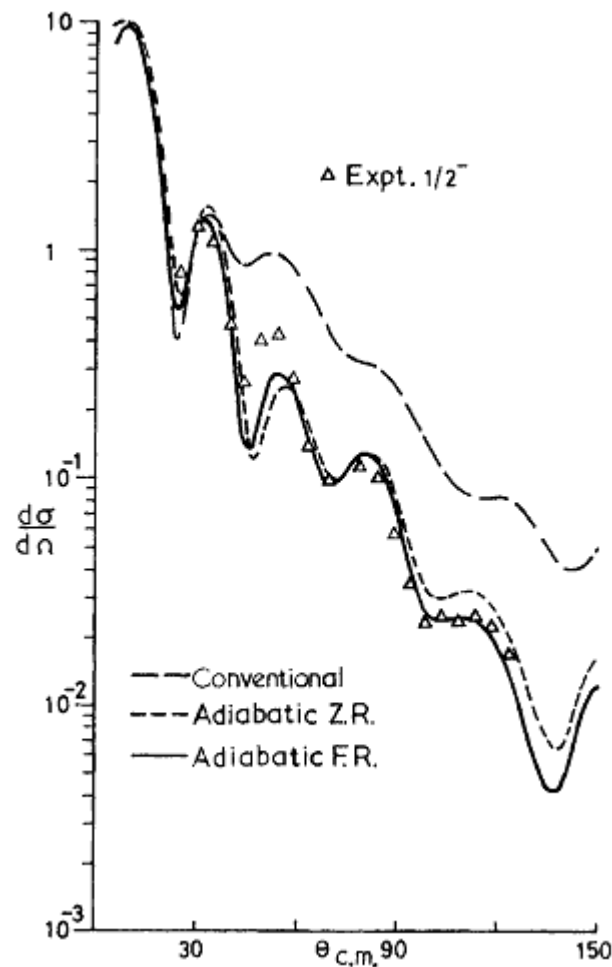
$$\bar{V}(\vec{R}) = D_0^{-1} \int d\vec{r} \left[ V_n \left( \vec{R} + \frac{1}{2}\vec{r} \right) + V_p \left( \vec{R} - \frac{1}{2}\vec{r} \right) \right] V_{np} \phi_d(r)$$
$$D_0 = \int d\vec{r} V_{np} \phi_d(r)$$

Where  $V_n$  and  $V_p$  are the proton and neutron optical potentials at half the incident deuteron kinetic energy and  $R$  and  $r$ , respectively, the radius vectors of the deuteron centre of mass relative to the target and the neutron relative to the proton:



## Adiabatic model versus DWBA:

The use of the adiabatic model can lead to significant improvement in the description of experimental data, e.g.  $^{54}\text{Fe}(d,p)^{55}\text{Fe}$  at 23 MeV:



Note that the adiabatic model in this form will **not** describe the deuteron elastic scattering (remember that the “deuteron” distorted wave was redefined), although this is possible within the framework of the adiabatic model theory [Johnson and Soper. Phys. Rev. C 1 (1970) 976]

Taken from Harvey and Johnson, Phys. Rev. C 3 (1971) 636

# The Coupled Channels Born Approximation

A CCBA calculation proceeds in the same way as for DWBA and requires the same ingredients, with the following additions:

- 1) The inelastic coupling (modelled using the coupled channels formalism) requires a Coulomb coupling strength,  $B(E\lambda)$ , and a nuclear coupling strength,  $\beta_\lambda$  (deformation parameter) or  $\delta_\lambda$  (deformation length)
- 2) The spectroscopic factors are replaced by spectroscopic **amplitudes**. These are the square roots of the spectroscopic factors, and can have a negative sign – interference effects between two routes to the same final state are now possible

Note that as the strong coupling to the inelastic state(s) is now taken into account explicitly, we must readjust the parameters of the entrance channel optical potential to recover the fit to the elastic scattering data



# Coupled Reaction Channels

A coupled reaction channels calculation proceeds as for CCBA with the same ingredients

However, the transfer couplings will now also have an effect on the elastic scattering (remember that they are no longer modelled using DWBA), hence further adjustment of the entrance channel optical potential is necessary

A further complication (shared with CCBA) compared to DWBA is that for a given final state there may now be several spectroscopic amplitudes (and their relative signs) in place of a single spectroscopic factor to be determined from the same data set

Finally, with CRC one must take account of the non-orthogonality of the entrance and exit channels – this should be corrected for and the correction is often important

## A practical example: $^{12}\text{C}(d,p)^{13}\text{C}$ at 30 MeV

We shall take the deuteron stripping reaction  $^{12}\text{C}(d,p)^{13}\text{C}$  at an incident deuteron energy of 30 MeV as a practical example, analysing the same data with progressively more sophisticated reaction models and noting the effect on the extracted spectroscopic factors

We shall also illustrate some of the sources of ambiguity in any analysis of direct reaction data, such as choice of distorting optical model potentials and binding potential geometry

We begin with a DWBA analysis. Our first requirement is a reaction model code. There are many available for DWBA calculations, two popular choices being DWUCK4 and DWUCK5. However, we shall use the code FRESCO [Thompson, Comput. Phys. Rep. 7 (1988) 167], a universal code which may also be used for CCBA and CRC calculations

## Standard DWBA with fitted optical model potentials

We analyse the transfer data of: H. Ohnuma *et al.*, Nucl. Phys. **A448** (1985) 205

Elastic scattering data from: G. Perrin *et al.*, Nucl. Phys. **A282** (1977) 221

Inelastic scattering data,  $^{12}\text{C}$   $2^+$ , from: J.M. Lind *et al.*, Nucl. Phys. **A276** (1977) 25

p +  $^{13}\text{C}$  elastic scattering data: P.D. Greaves *et al.*, Nucl. Phys. **A179** (1972) 1

d +  $^{12}\text{C}$  optical potential from Perrin *et al.*

p +  $^{13}\text{C}$  optical potential from fit to data of Greaves *et al.*

deuteron internal wave function calculated using the “soft core” potential of:

R.V. Reid, Jr., Ann. Phys. (NY) **50** (1968) 441

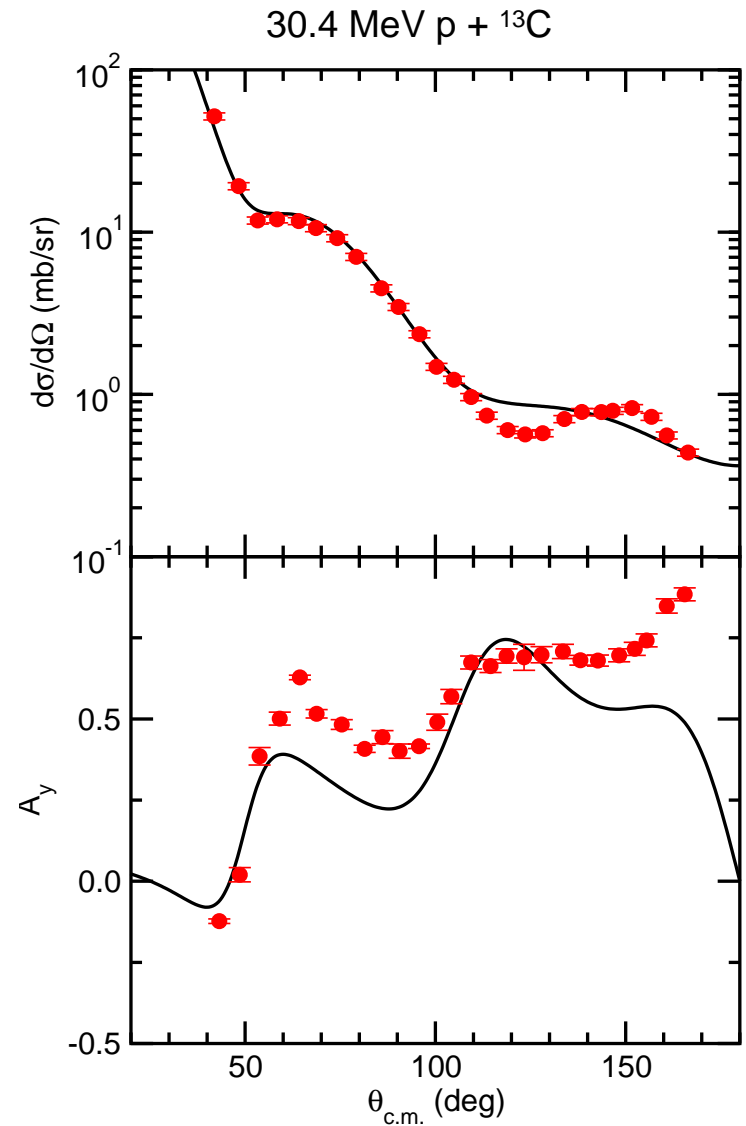
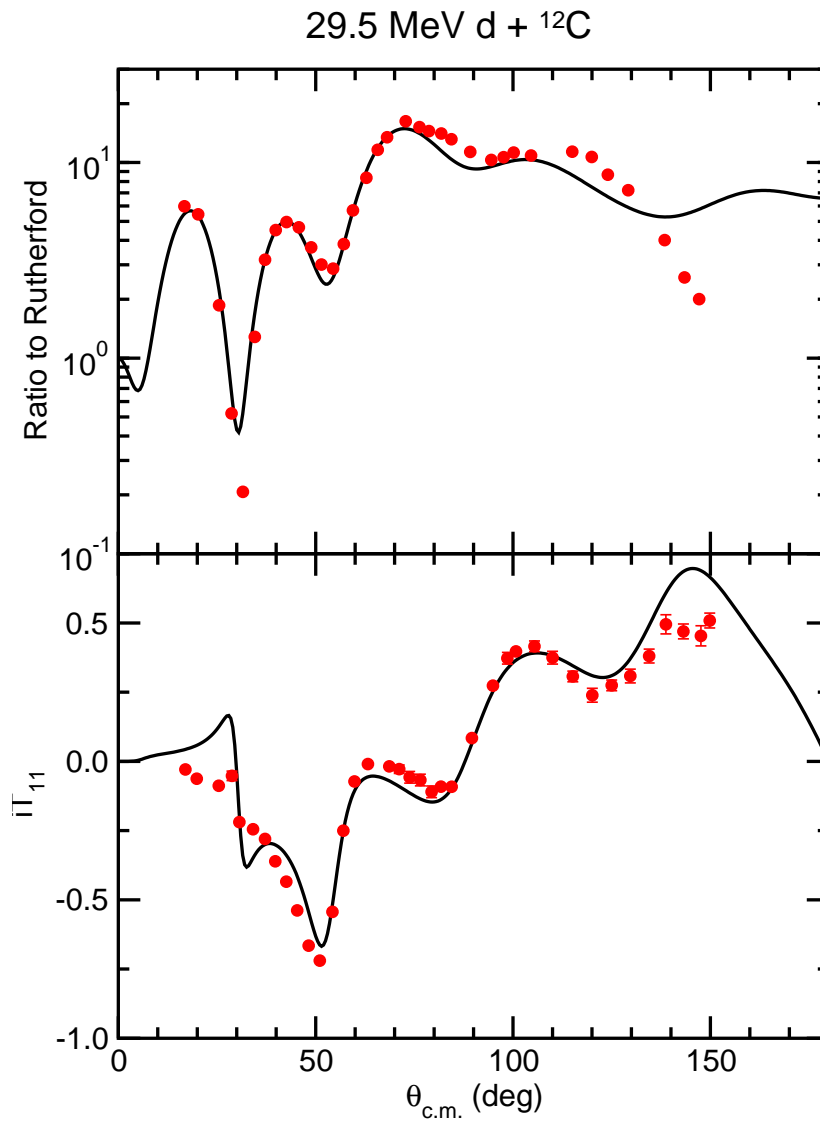
$^{13}\text{C}$  internal wave functions calculated by binding the neutron to the  $^{12}\text{C}$  core in a Woods-Saxon potential well of radius  $1.25 \times A^{1/3}$  fm and diffuseness 0.65 fm (depth adjusted to give the correct binding energy) plus a spin-orbit component of the same “geometry” with a fixed depth of 6 MeV

# Fits to the elastic scattering data

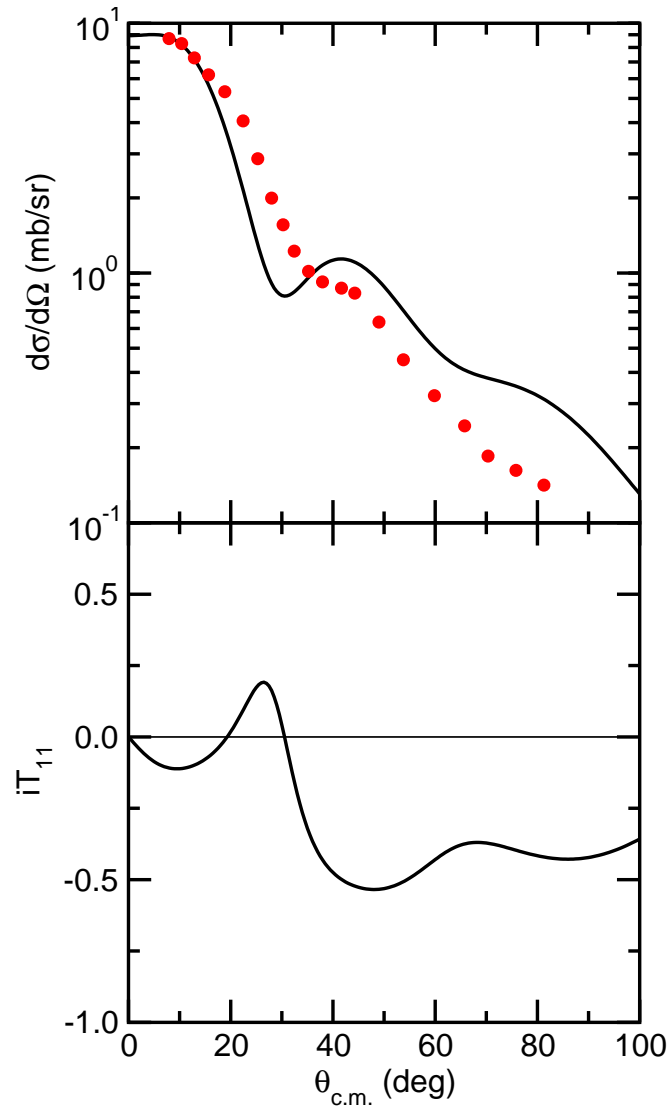
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# Fits to the transfer data: 0.0 MeV 1/2<sup>-</sup> state

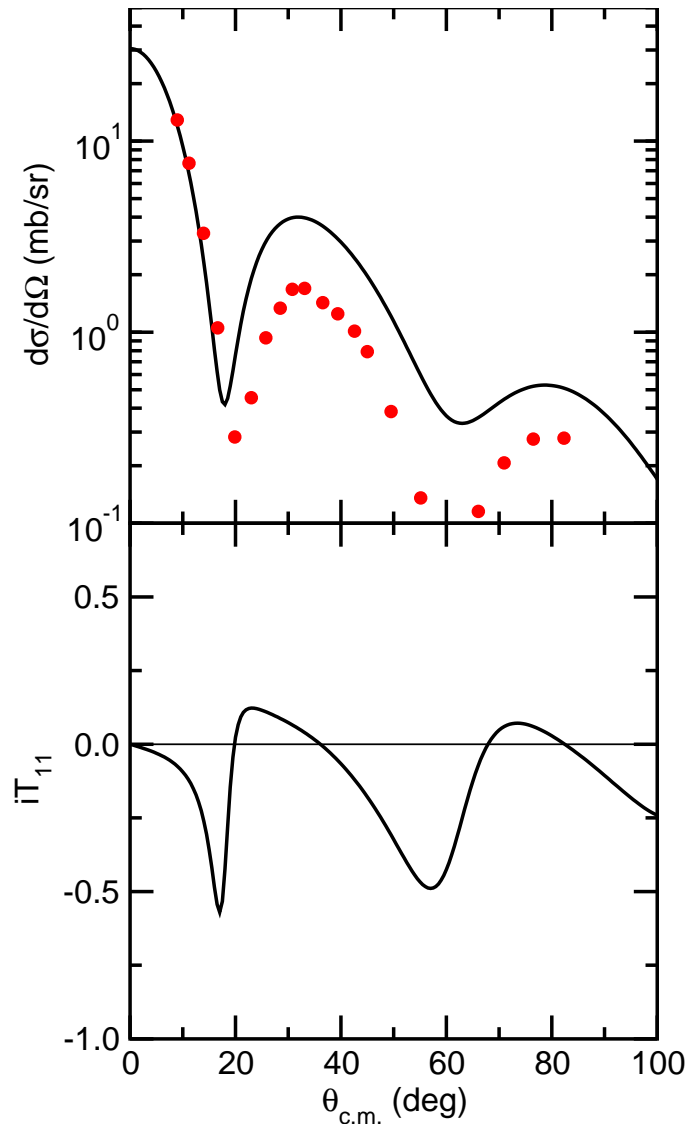


The fit to the data is far from perfect ...

We obtain the spectroscopic factor by adjusting the DWBA curve to best fit the data at forward angles (this is in general good practice) which yields a value of  $C^2S = 0.76$

This is an  $L = 1$  transfer – note the characteristic shape of the angular distribution

## Fits to the transfer data: 3.09 MeV $1/2^+$ state

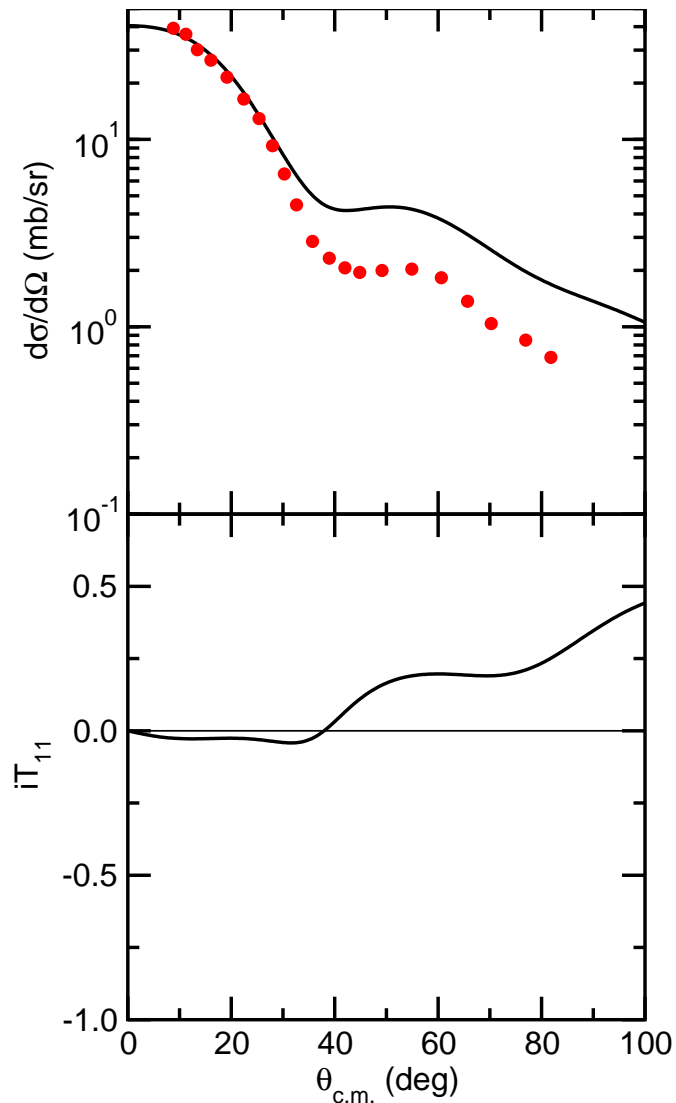


The fit to the data is somewhat better than for the  $1/2^-$  ground state, although there is a significant angle phase error in the position of the first minimum of the angular distribution

We obtain  $C^2S = 1.0$  – the value is probably too large due to the phase error which makes determining the normalisation of calculation to data problematic

This is an  $L = 0$  transfer – note the very characteristic shape (the phase error in the position of the first minimum is also highly characteristic of DWBA calculations for  $L = 0$  deuteron stripping!)

# Fits to the transfer data: 3.85 MeV $5/2^+$ state



The fit to the data is now good for angles smaller than about  $30^\circ$

We obtain  $C^2S = 0.77$

This is an  $L = 2$  transfer – for this reaction the shape of the angular distribution is somewhat similar to that for  $L=1$ , although the analysing powers are very different

## Standard DWBA with global optical model potentials

We now analyse the same data set but this time using global deuteron and proton optical model potentials in the entrance and exit channels. We are often constrained to do this with radioactive beam data due to lack of elastic scattering measurements – as we shall see, even for stable nuclei where global potentials should work reasonably well, this can lead to important differences in the extracted spectroscopic factors ...

For the  $d + {}^{12}\text{C}$  optical potential, we employ the global parameters of: Haixia An and Chonghai Cai, Phys. Rev. C **73** (2006) 054605

For the  $p + {}^{13}\text{C}$  optical potential, we employ the global parameters of: B.A. Watson, P.P. Singh, and R.E. Segel, Phys. Rev. **182** (1969) 977

All other elements of the calculations are as for the calculations with fitted optical model potentials presented previously

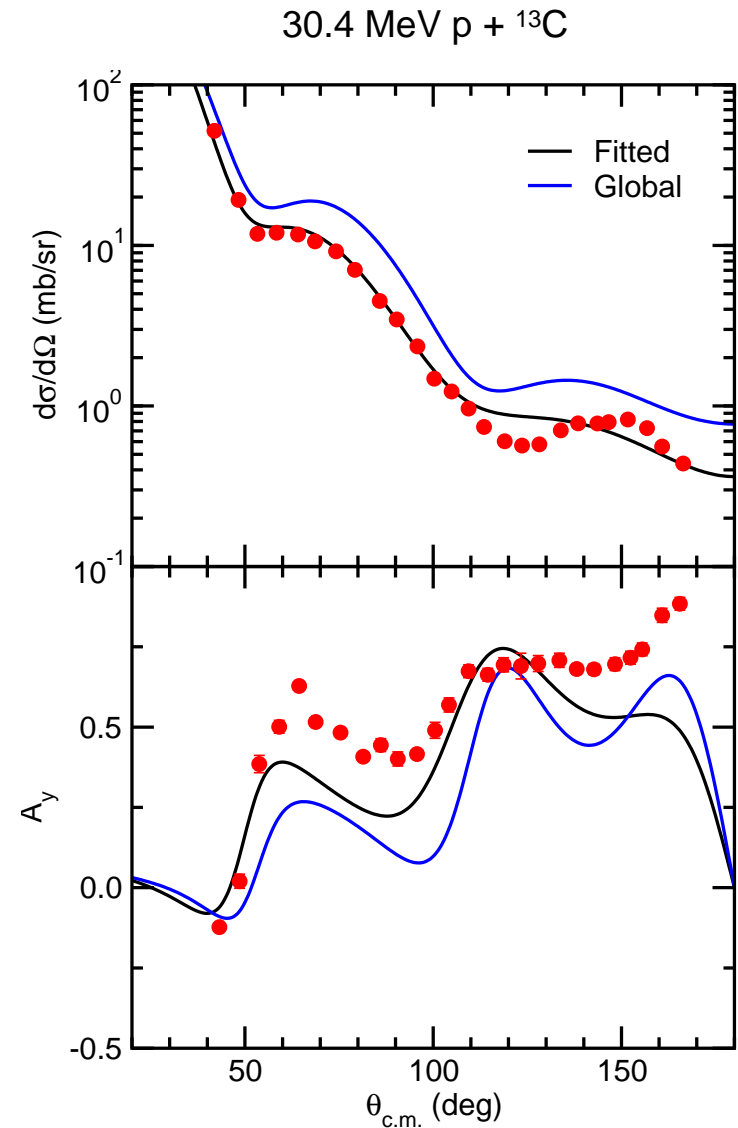
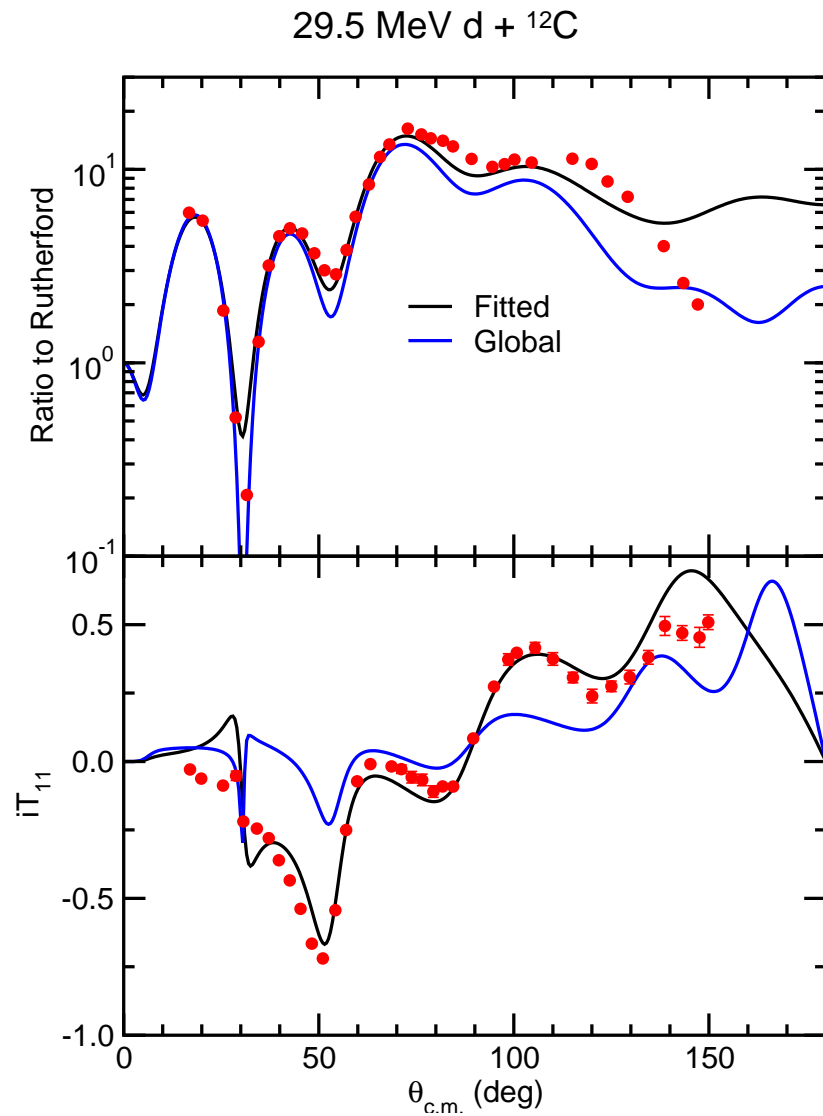


# Fits to the elastic scattering data

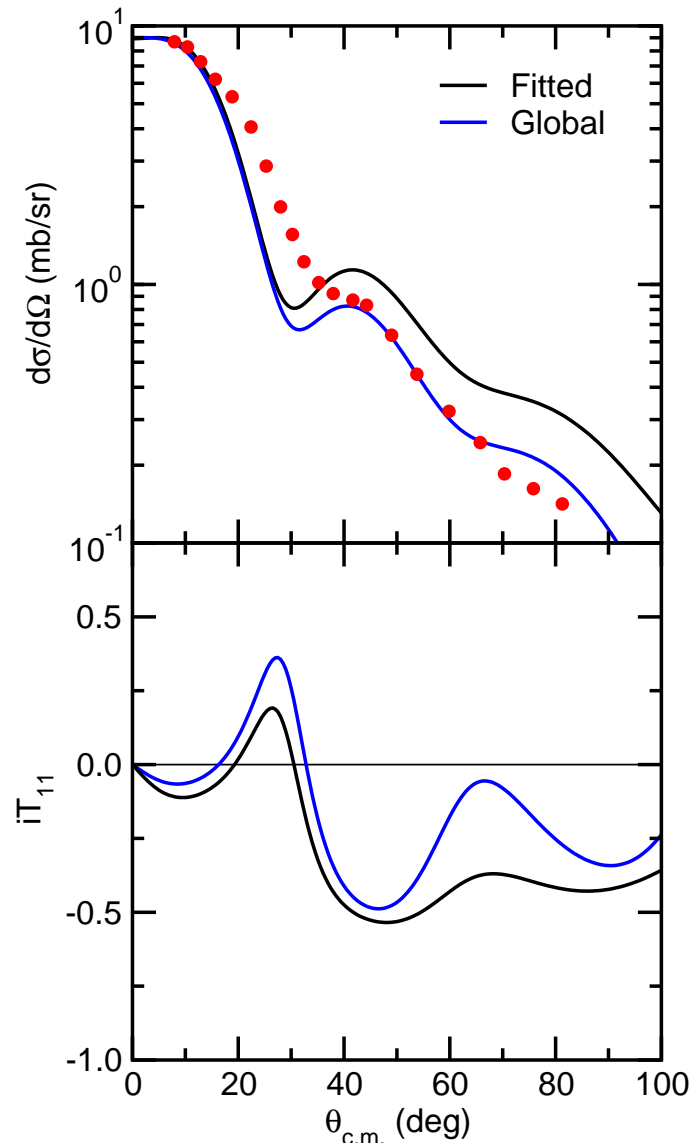
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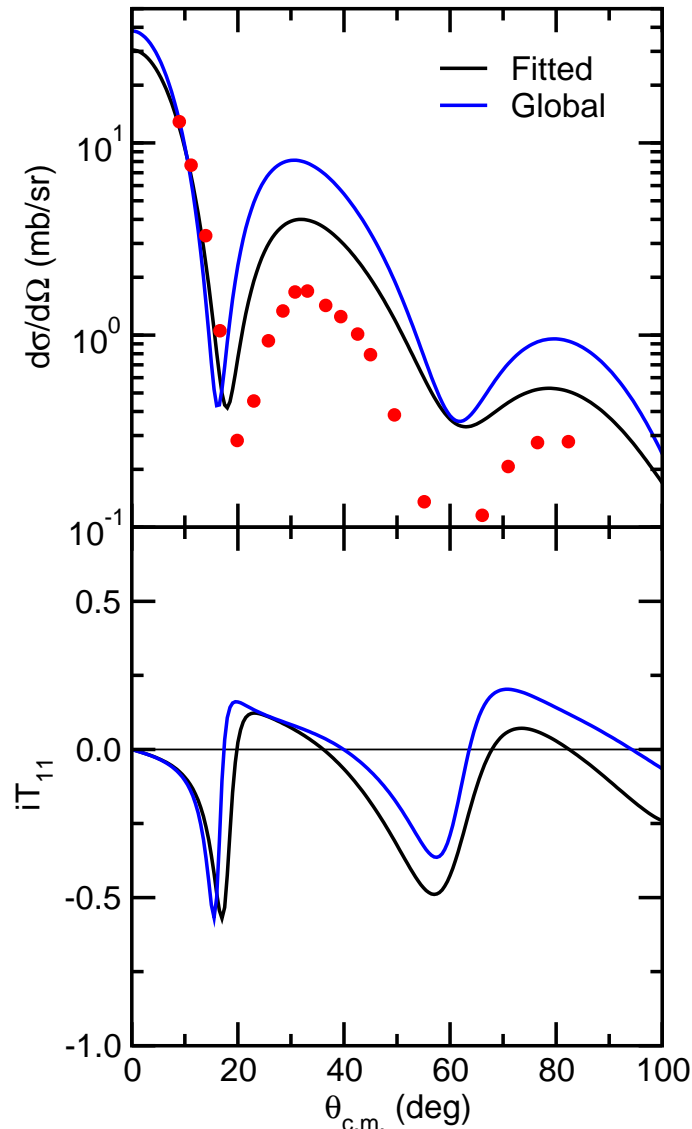
# Fits to the transfer data: 0.0 MeV $1/2^-$ state



The fit to the data is now better (but recall that the global  $p + {}^{13}\text{C}$  potential does not describe the elastic scattering data well)

We now obtain a spectroscopic factor of  $C^2S = 0.62$ , i.e. about 20 % smaller than with the fitted optical potentials

# Fits to the transfer data: 3.09 MeV $1/2^+$ state

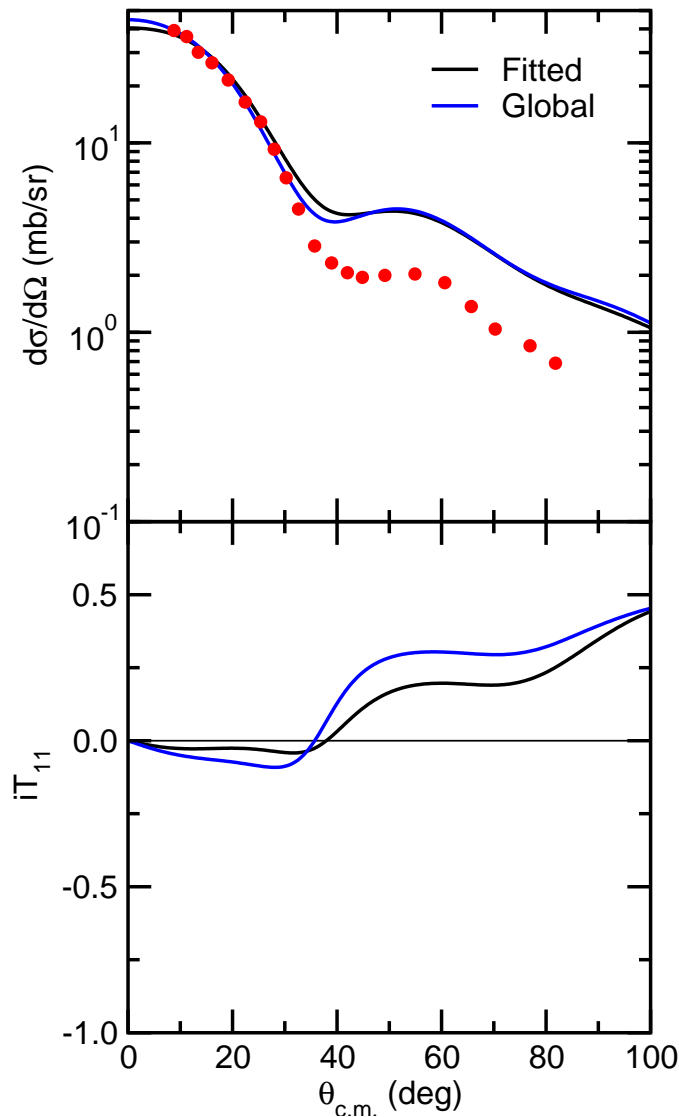


The fit to the data is now much worse – the angle phase error is now larger than with the fitted potentials

We now obtain a spectroscopic factor of  $C^2S = 1.69$ , i.e. about 70 % **larger** than with the fitted optical potentials!

This is mostly due to the increased angle phase error in the position of the first minimum

# Fits to the transfer data: 3.85 MeV $5/2^+$ state



The fit to the data is not significantly different; the global potentials give a slightly better description of the data at forward angles

We now obtain a spectroscopic factor of  $C^2S = 0.69$ , about 10 % smaller than with the fitted optical potentials

# Summary of DWBA calculations for $^{12}\text{C}(d,p)^{13}\text{C}$

The agreement with data is rather poor – using different fitted optical model potentials does not change this, nor does using global optical model parameters. However, the use of global optical potentials, even for stable nuclei, can lead to important differences in the extracted spectroscopic factors

This suggests that the DWBA, with its underlying assumptions that the transfers are individually weak (thus possible to treat within perturbation theory) and proceed in a single step, is not an adequate model of the reaction process in this case

As  $^{12}\text{C}$  has a strongly coupled first excited state (the 4.4 MeV  $2^+$ ) perhaps a CCBA calculation including transfer of the neutron to the  $^{12}\text{C}$  core in its excited state as well as its ground state will improve things?

Find out in the next lecture, where we shall also consider even more sophisticated models (and find that they are not a panacea for all ills!)

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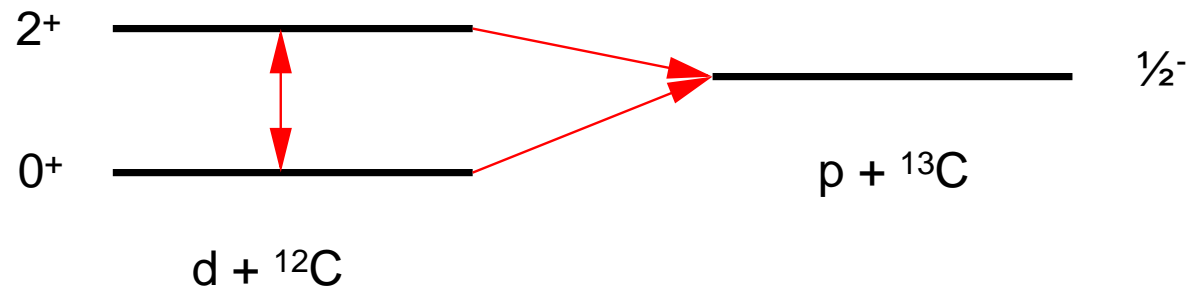
*and*

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Warsaw*

# CCBA with fitted optical model potentials

We saw in the last lecture that standard DWBA is unable to provide a satisfactory description of the data for  $^{12}\text{C}(d,p)^{13}\text{C}$  at  $E_d = 30$  MeV

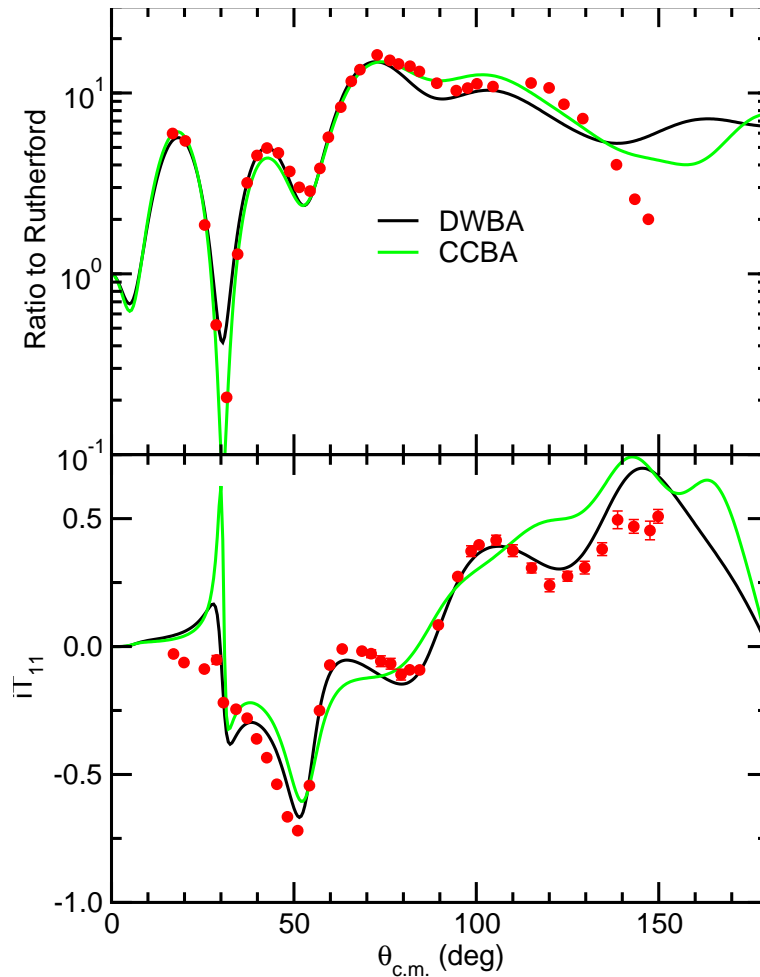
We shall now investigate the effect of adding transfer paths via the 4.4 MeV  $2^+$  first excited state of  $^{12}\text{C}$ :



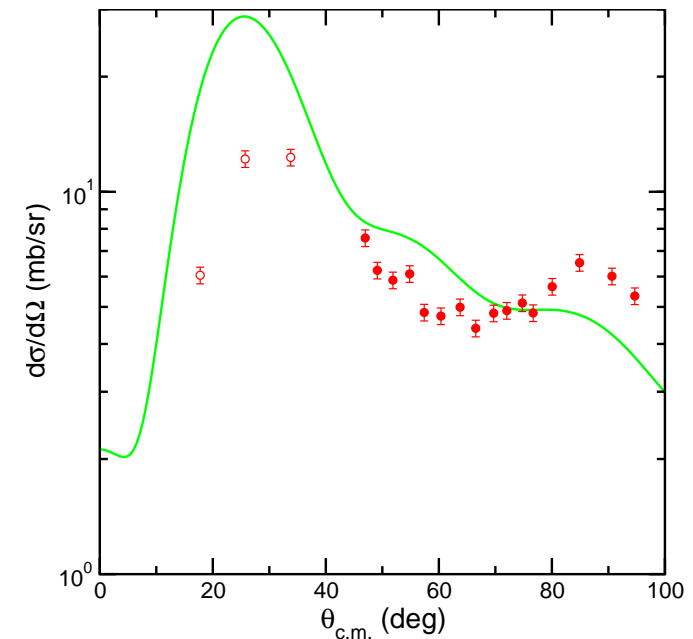
We take the Coulomb coupling strength,  $B(E2; 0^+ \rightarrow 2^+)$ , from: S. Raman *et al.*, At. Data Nucl. Data Tables **36** (1987) 1, with the nuclear coupling strength,  $\delta_2$ , extracted from the  $B(E2)$  using the collective model (this simplifying assumption will obviously need to be re-examined for exotic nuclei). All else as for the DWBA calculations

# Fits to the $d + {}^{12}\text{C}$ elastic and inelastic scattering data

As we now couple explicitly to the  ${}^{12}\text{C}$   $2^+$  state we must re-tune the entrance channel optical potential to recover the fit to the data:

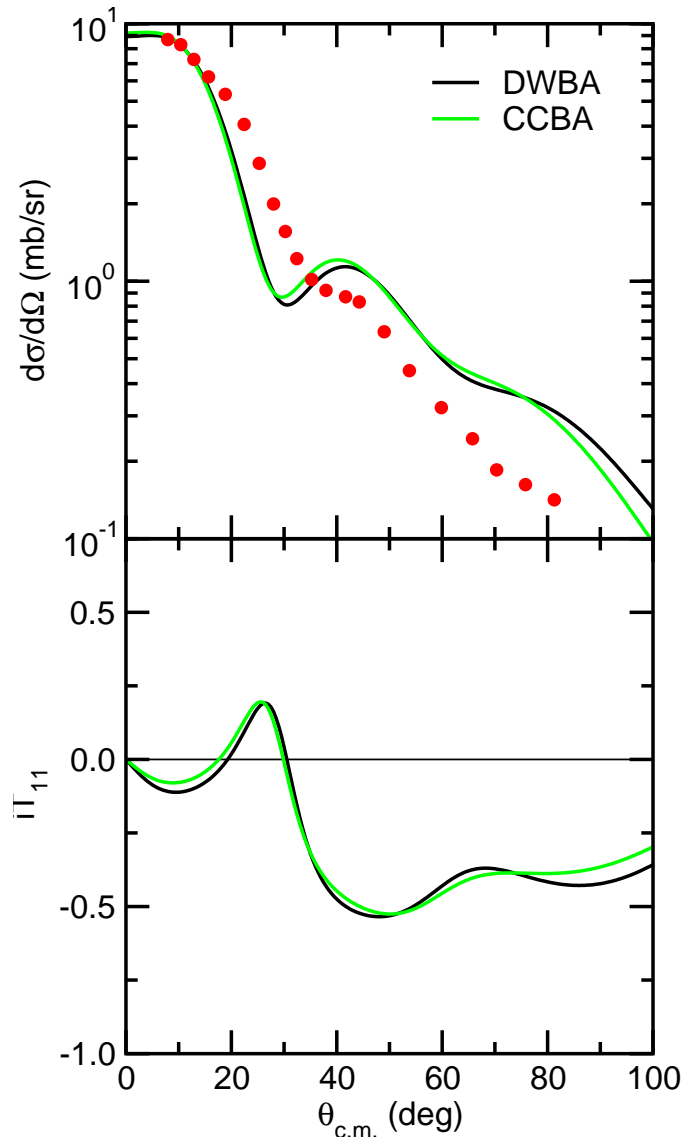


Apart from a slight deterioration in the description of the analysing power (the spin-orbit potential was not adjusted) the agreement with data is as good as for the optical model fit. Agreement with inelastic data is acceptable:





# Fits to the transfer data: 0.0 MeV 1/2<sup>-</sup> state

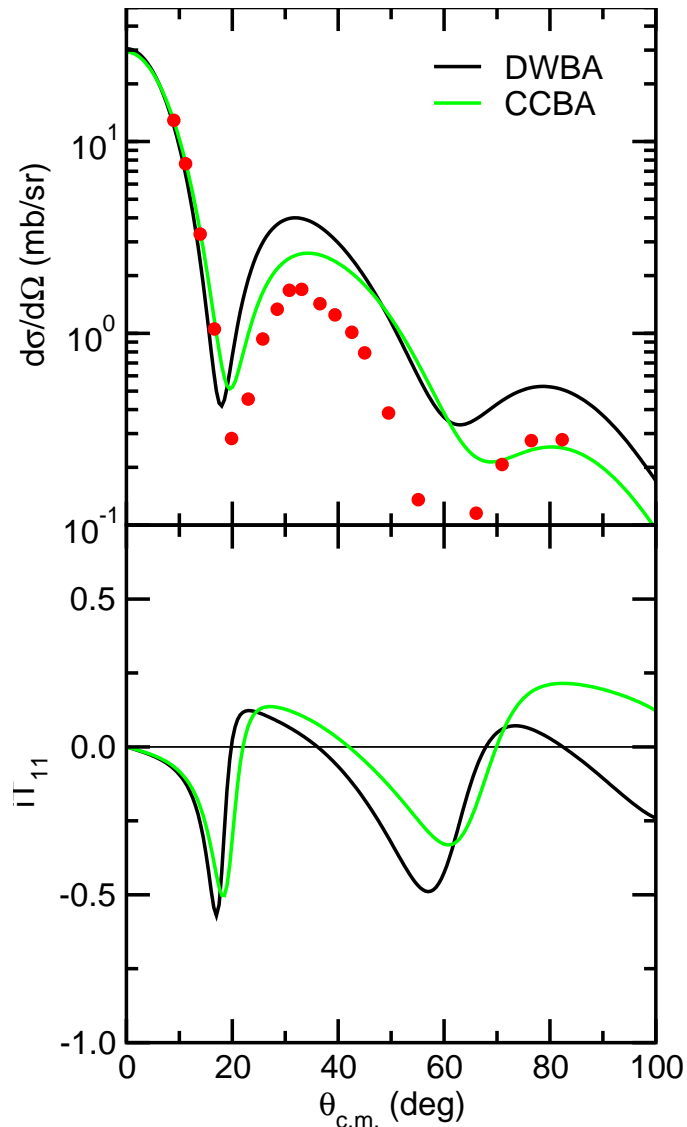


The description of the data is not significantly different from that with DWBA

We now extract **spectroscopic amplitudes** of 0.95 for  $^{12}\text{C}(0+) + 1p_{1/2}$  and -0.4 for  $^{12}\text{C}(2+) + 1p_{3/2}$

For comparison with DWBA, the  $^{12}\text{C}(0+) + 1p_{1/2}$  spectroscopic amplitude corresponds to  $C^2S = 0.90$  (DWBA value 0.76)

# Fits to the transfer data: 3.09 MeV $1/2^+$ state



The fit to the data is considerably improved compared to the DWBA – the two-step transfer via the  $^{12}\text{C}$   $2+$  state is able to move the first minimum to match the data

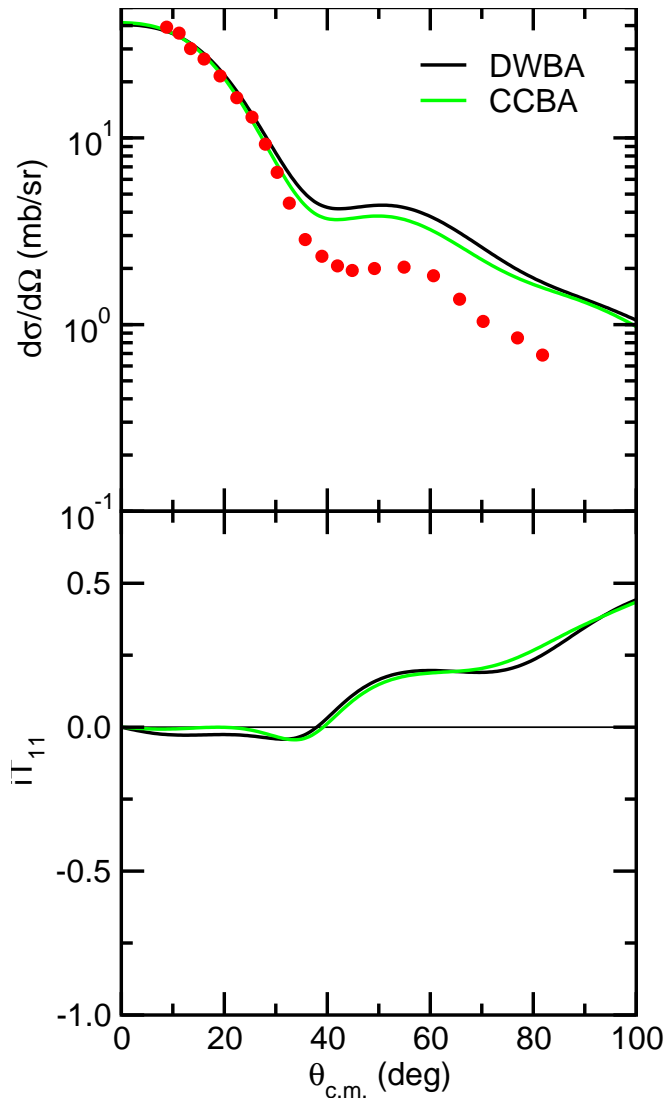
We extract the following spectroscopic amplitudes:

$$^{12}\text{C}(0+) + 2s_{1/2} = 0.91$$

$$^{12}\text{C}(2+) + 1d_{5/2} = -0.40$$

For comparison with DWBA, the  $^{12}\text{C}(0+) + 2s_{1/2}$  spectroscopic amplitude corresponds to  $C^2S = 0.83$  (DWBA value = 1.0)

# Fits to the transfer data: 3.85 MeV $5/2^+$ state



The agreement with data is not significantly better than for DWBA

We extract the following spectroscopic amplitudes:

$$^{12}\text{C}(0+) + 1d_{5/2} = 0.90$$

$$^{12}\text{C}(2+) + 1d_{5/2} = 0.70$$

$$^{12}\text{C}(2+) + 2s_{1/2} = -0.30$$

For comparison with DWBA, the  $^{12}\text{C}(0+) + 1d_{5/2}$  spectroscopic amplitude corresponds to  $\text{C}^2\text{S} = 0.81$  (DWBA value = 0.77)

# Summary of CCBA calculations

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We see that, with the exception of the transfer to the 3.09 MeV  $\frac{1}{2}^+$  state, CCBA does not improve the agreement between calculations and data

However, despite minor differences in the shape of the angular distributions between DWBA and CCBA there can be important differences in the extracted spectroscopic factors ...

Nevertheless, CCBA can account for the angle phase error in the calculated  $^{13}\text{C}$   $\frac{1}{2}^+$  angular distribution, considerably improving the fit to the data

In general, however, the spectroscopic amplitudes for two-step transfers via the  $^{12}\text{C}$   $2^+$  state are not very well determined by the data (the  $^{13}\text{C}$   $\frac{1}{2}^+$  state being the exception, as the position of the first minimum in the angular distribution is a clear signature)

**Thus, CCBA does not solve all our problems, and we must consider other influences, such as the effect of deuteron breakup**

# Breakup effects (1): the adiabatic model

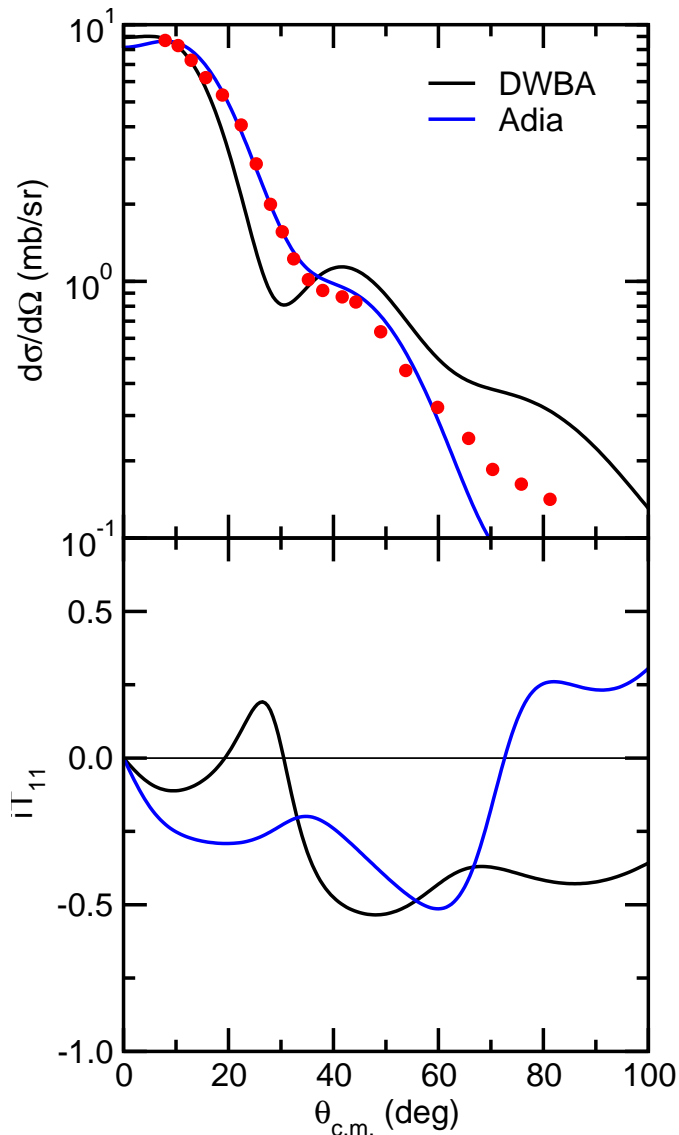
We now consider the influence of deuteron breakup on the  $^{12}\text{C}(d,p)^{13}\text{C}$  reaction by employing the adiabatic model, discussed in the previous lecture

The adiabatic model has not currently been formulated to allow excitation of the target to be taken into account properly, thus we shall consider one-step transfer only (as in the DWBA)

We take our  $V_p$  and  $V_n$  from optical model fits to  $p + ^{12}\text{C}$  and  $n + ^{12}\text{C}$  elastic scattering data at an incident nucleon energy of 15 MeV [Nodvik et *al.*, Phys. Rev. **125** (1962) 975 and Spaargaren and Jonker, Nucl. Phys. **A161** (1971) 354]

All else as for the DWBA and CCBA calculations

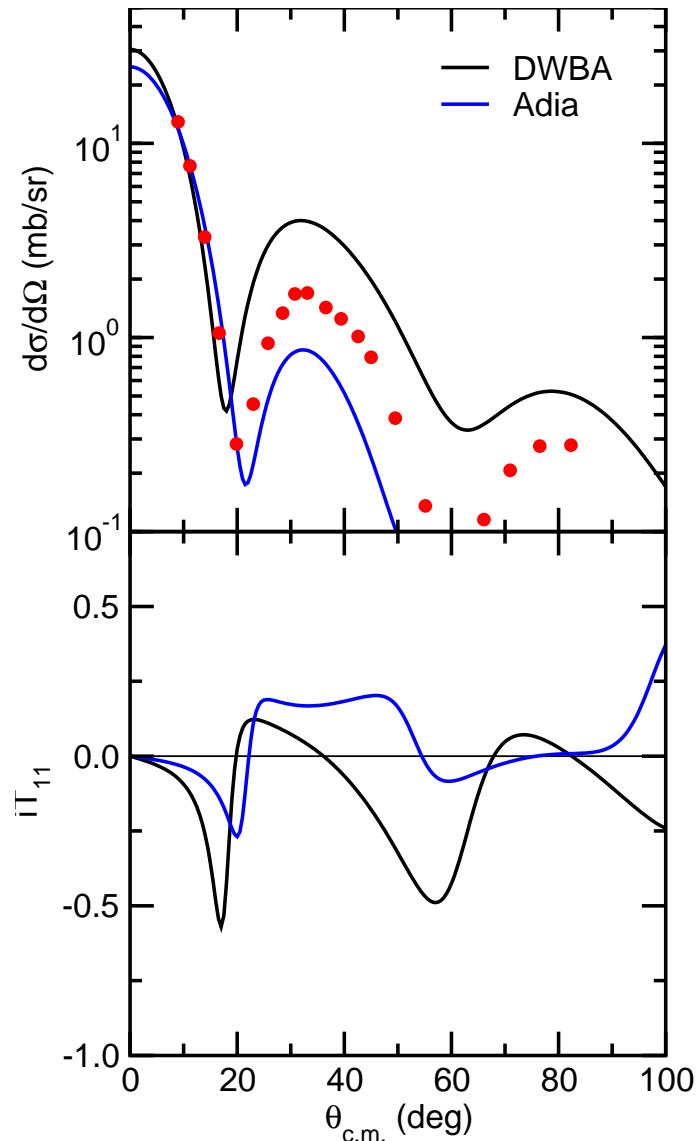
# Fits to the transfer data: 0.0 MeV 1/2<sup>-</sup> state



Very much improved description of the data compared to DWBA, and note the important effect on the analysing power  $iT_{11}$

The effect on the extracted spectroscopic factor is large:  $C^2S = 0.38$  compared to 0.76 or 0.62 in the DWBA (for fitted or global optical potentials, respectively) **representing a reduction of 50 % or 39%!**

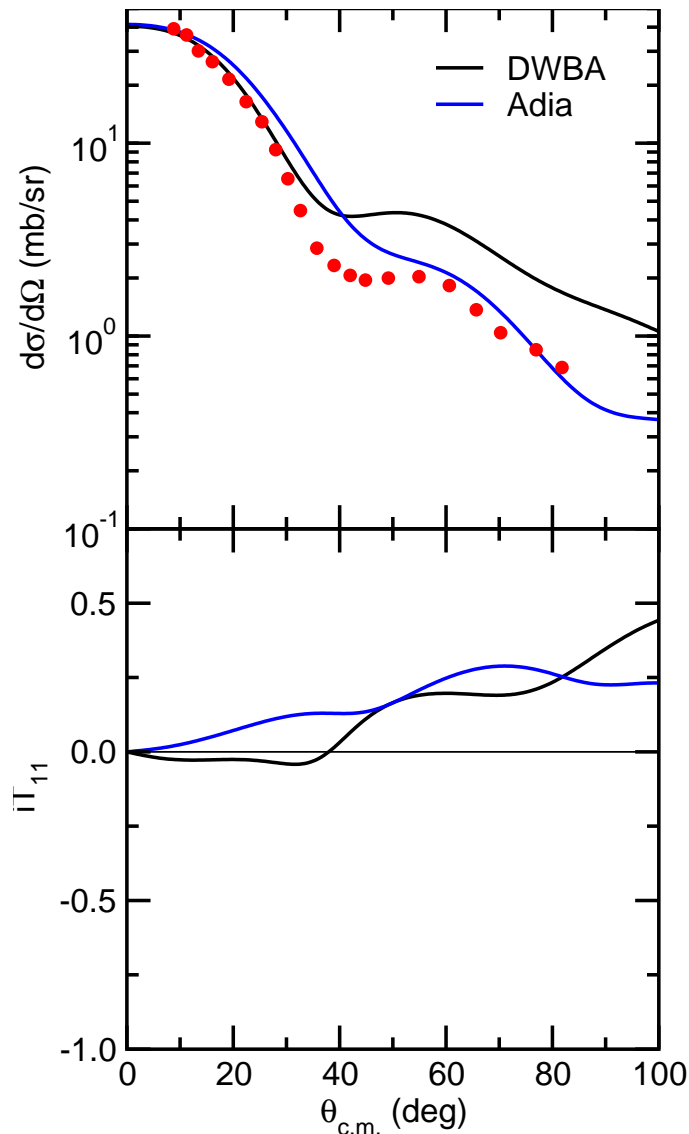
# Fits to the transfer data: 3.09 MeV $1/2^+$ state



The description of the data is somewhat better than for the DWBA, although the first minimum is perhaps shifted to slightly too large an angle. Again, note the important effect on the analysing power

The effect on the spectroscopic factor is again important:  $C^2S = 0.41$  compared to 1.0 or 1.69 for DWBA with fitted or global potentials, respectively, **a reduction of 59 % or 76 %!**

# Fits to the transfer data: 3.85 MeV $5/2^+$ state



The overall fit to the data is better than for DWBA, although at forward angles ( $\theta < 30^\circ$ ) the slope of the adiabatic model calculation is not steep enough

The effect on the analysing power is much less marked than for the  $L=1$  and  $L=0$  transfers

The effect on the spectroscopic factor is still significant:  $C^2S = 0.52$  compared to 0.77 or 0.69 for DWBA with fitted or global potentials, **a reduction of 32% or 25%**



# Summary of adiabatic model calculations

Overall, the adiabatic model gives much improved agreement with the data compared to either DWBA or CCBA

There are important effects on the shape of the analysing powers,  $iT_{11}$ , as well as improvements in the shape of the cross section angular distributions

There are dramatic differences in the extracted spectroscopic factors compared to those obtained with DWBA (using either fitted or global optical model potentials) with substantial reductions being obtained

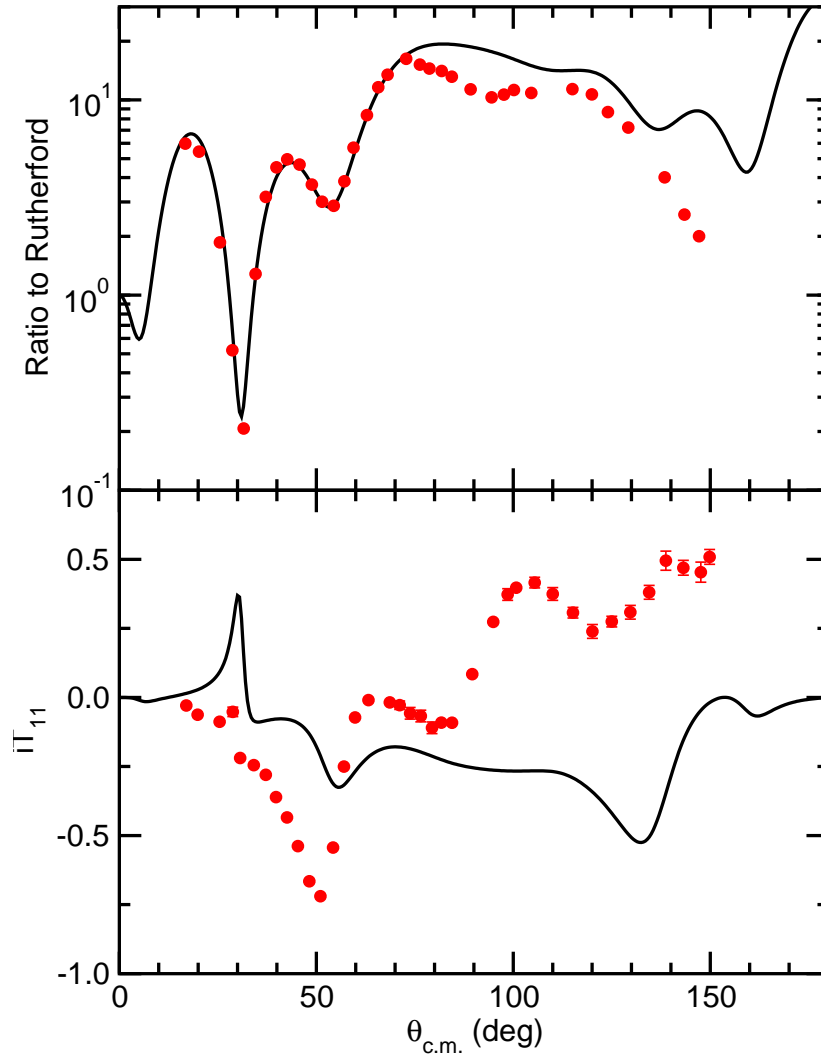
## Breakup effects (2): CDCC/CRC calculations

The adiabatic model is an approximate treatment of the effects due to deuteron breakup. A more sophisticated approach, the coupled discretised continuum channels (CDCC) method [Rawitscher, Phys. Rev. C **9** (1974) 2210], exists and may be combined with CRC (to model the transfer steps) to give the most complete calculation we are able to perform at the present time

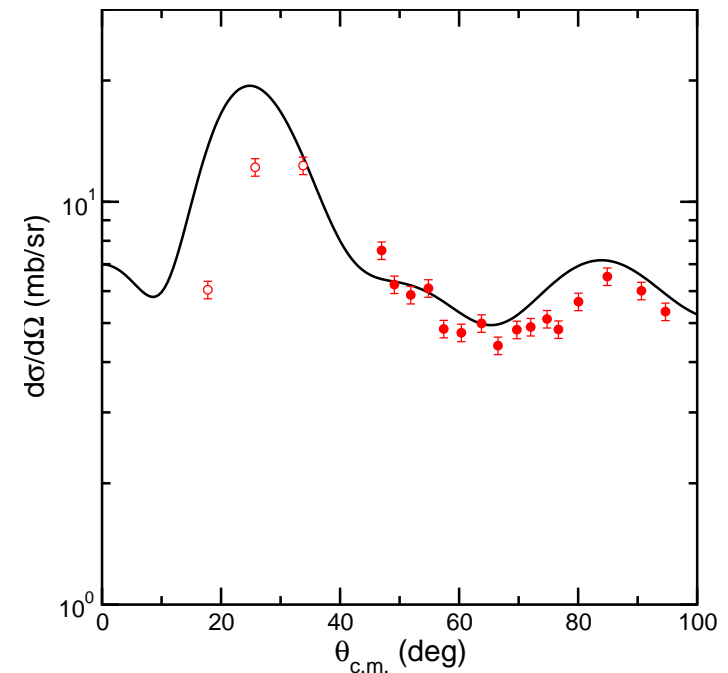
We shall not give details of the method here, it being beyond the scope of these lectures

Couplings to deuteron breakup, inelastic excitation of the  $^{12}\text{C}$   $2^+$  state and transfers via both the  $0^+$  and  $2^+$  states of  $^{12}\text{C}$  are included in the calculation that follows

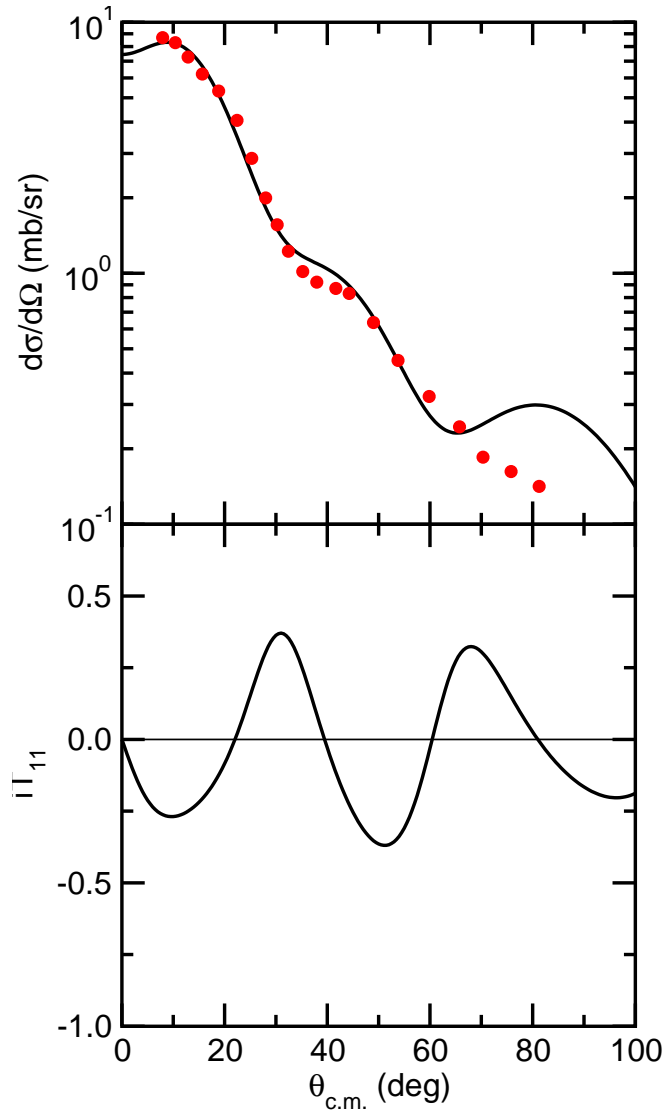
# Fits to the $d + {}^{12}\text{C}$ elastic and inelastic scattering data



Fit to the elastic scattering data is comparable to the optical model – the poor description of the analysing power is due to the absence of a static spin-orbit potential, known to dominate  $iT_{11}$  for deuteron elastic scattering; the inelastic scattering is well described:



# Fit to the transfer data: 0.0 MeV 1/2<sup>-</sup> state



The description of the data is much better than either DWBA or CCBA and similar to that of the adiabatic model

There are again important effects on the analysing power

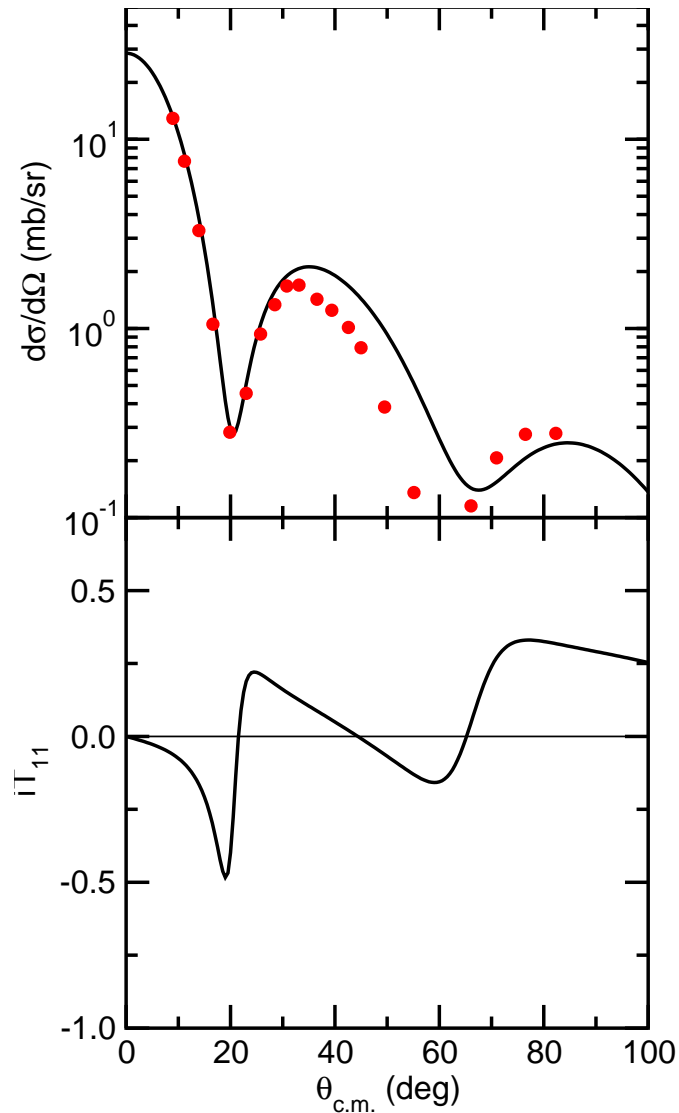
We extract the following spectroscopic amplitudes:

$$^{12}\text{C}(0+) + 1p_{1/2} = 0.81$$

$$^{12}\text{C}(2+) + 1p_{3/2} = 0.60$$

For comparison with DWBA, the  $^{12}\text{C}(0+) + 1p_{1/2}$  spectroscopic amplitude corresponds to  $\text{C}^2\text{S} = 0.66$  (DWBA value = 0.76)

# Fit to the transfer data: 3.09 MeV $1/2^+$ state



Description of data is again better than either DWBA or CCBA (although improvement over latter is slight)

Effect on analysing power compared to DWBA or CCBA is minor

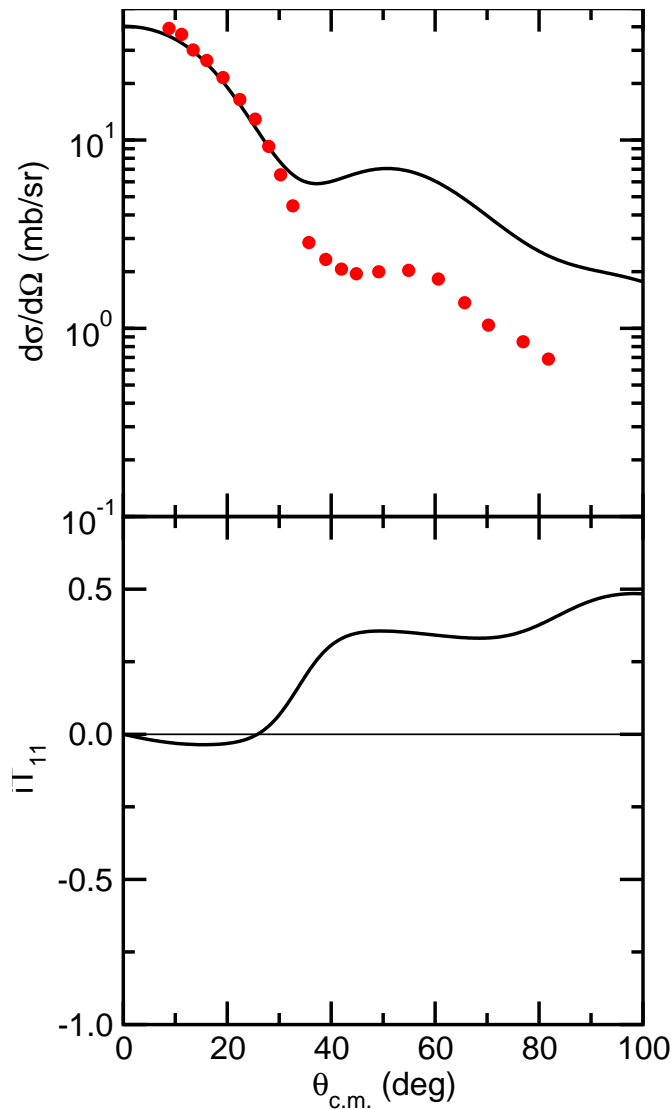
We extract the following spectroscopic amplitudes:

$$^{12}\text{C}(0+) + 2s_{1/2} = 0.77$$

$$^{12}\text{C}(2+) + 1d_{5/2} = -0.35$$

For comparison with DWBA, the  $^{12}\text{C}(0+) + 2s_{1/2}$  spectroscopic amplitude corresponds to  $C^2S = 0.59$  (DWBA value = 1.0)

# Fit to the transfer data: 3.85 MeV $5/2^+$ state



The agreement with data is slightly worse than for DWBA or CCBA

Effect on the analysing power somewhat larger than for the  $1/2^+$  state

We extract the following spectroscopic amplitudes:

$$^{12}\text{C}(0+) + 1d_{5/2} = 0.85$$

$$^{12}\text{C}(2+) + 1d_{5/2} = 0.80$$

$$^{12}\text{C}(2+) + 2s_{1/2} = 0.70$$

For comparison with DWBA, the  $^{12}\text{C}(0+) + 1d_{5/2}$  spectroscopic amplitude corresponds to  $C^2S = 0.72$  (DWBA value = 0.77)

# Summary of CDCC/CRC calculations

The CDCC/CRC combination provides by far the best overall description of the data, much better than either DWBA or CCBA, although it does not solve the problem with the  $^{13}\text{C}$  5/2+ data

We have seen how the choice of reaction model can have a significant influence on the shape of the calculated angular distributions and, more importantly in the context of these lectures, on the extracted spectroscopic factors

To recap, comparing DWBA and CDCC/CRC we obtain the following spectroscopic factors:

$$^{13}\text{C}(1/2-) \rightarrow ^{12}\text{C}(0+) + 1p_{1/2}, \quad C^2S(\text{DWBA}) = 0.76, \quad C^2S(\text{CDCC/CRC}) = 0.66$$

$$^{13}\text{C}(1/2+) \rightarrow ^{12}\text{C}(0+) + 2s_{1/2}, \quad C^2S(\text{DWBA}) = 1.00, \quad C^2S(\text{CDCC/CRC}) = 0.59$$

$$^{13}\text{C}(5/2+) \rightarrow ^{12}\text{C}(0+) + 1d_{5/2}, \quad C^2S(\text{DWBA}) = 0.77, \quad C^2S(\text{CDCC/CRC}) = 0.72$$

## Other influences on extracted spectroscopic factors

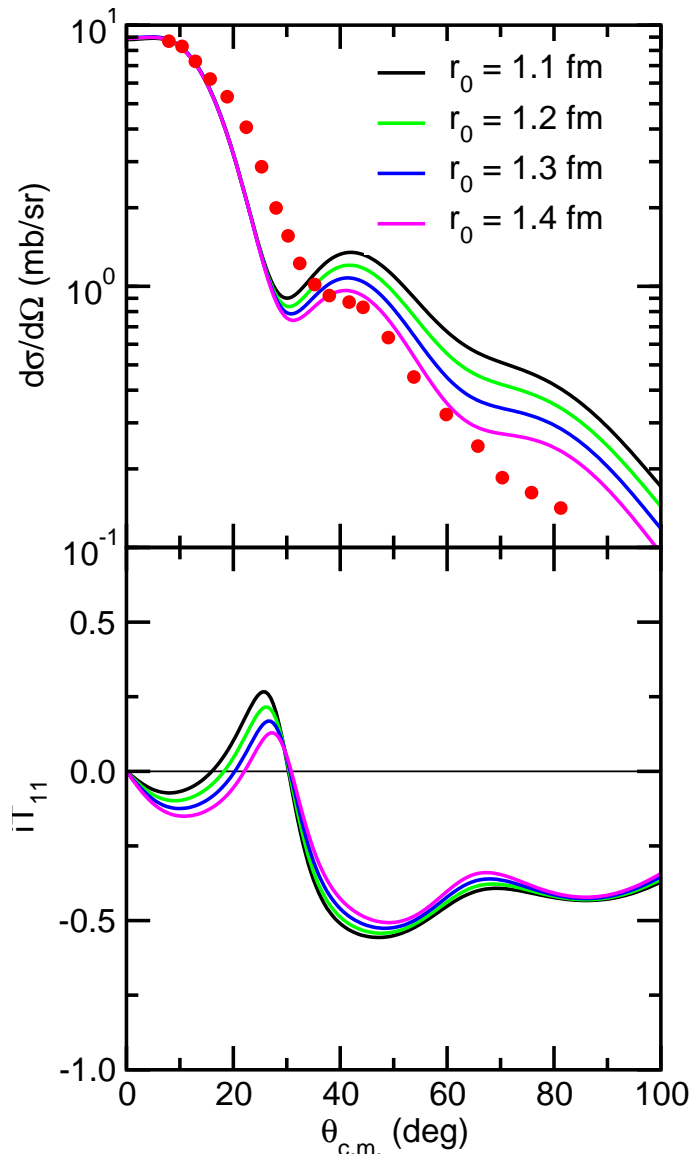
We saw on the previous slide that choice of reaction model can significantly influence the spectroscopic factors extracted from analysis of angular distribution data. However, the difference between the simplest useful model (DWBA) and the most sophisticated (CDCC/CRC) is only important (~40 %) for the L=0 transfer (the effect is ~10 % for L=1 and L=2 transfers) in this case

Perhaps the most important influence on the extracted spectroscopic factor is the choice of radius for the Woods-Saxon potential well that binds the transferred particle to the target nucleus

In all the calculations presented so far we have used the parameters  $R_0 = 1.25 \times A^{1/3}$  fm,  $a_0 = 0.65$  fm, often regarded as “standard”, although with little real justification. We now present a series of DWBA calculations for the same transfer with different reasonable choices of  $r_0$ , ranging from 1.1 fm to 1.4 fm, to show the effect of this choice on the extracted  $C^2S$



# Effect of choice of $r_0$ on extracted $C^2S$



The first thing to note is the effect on the shape of the cross section angular distribution for  $\theta > 30^\circ$  (part of the reason why we extract  $C^2S$  by normalising to forward angles)

However, the most important effect is on the extracted spectroscopic factor, which varies from  $C^2S = 0.92$  for  $r_0 = 1.1$  fm to  $C^2S = 0.61$  for  $r_0 = 1.4$  fm, **a reduction of ~34 %!**

The effect is much smaller for  $L=0$  transfer,  $C^2S$  ranging from 1.10 for  $r_0 = 1.1$  fm to 0.90 for  $r_0 = 1.4$  fm, **a reduction of ~18 %**

For  $L=2$  transfer, the effect is the same as for  $L=1$ ,  $C^2S$  ranging from 0.96 to 0.62, **a reduction of ~35 %!** Slight improvement in fit to data at  $\theta > 30^\circ$  for larger  $r_0$  values

# Summary so far

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We have seen that choice of reaction model and binding potential well radius both have important effects on the extracted spectroscopic factors, with the latter being, in general, rather more important

All things considered, an uncertainty of  $\sim \pm 30\%$  in the value of an absolute spectroscopic factor is not unrealistic – it could be even larger, as this is without considering uncertainties in the data, often quite large ( $\pm 20\%$ ) for radioactive beam data. Relative spectroscopic factors between states of the same nucleus are usually better determined, i.e. less sensitive to the details of the calculation

The interest in choosing a more sophisticated reaction model is that (usually) it will provide a better description of the shape of the angular distribution, thus facilitating the extraction of a spectroscopic factor, particularly if the angular coverage is sparse and does not extend very far towards  $\theta = 0^\circ$ , quite apart from effects on the magnitude of the cross section that do not change much the shape of the angular distribution

## Choice of reaction model: when is the DWBA appropriate?

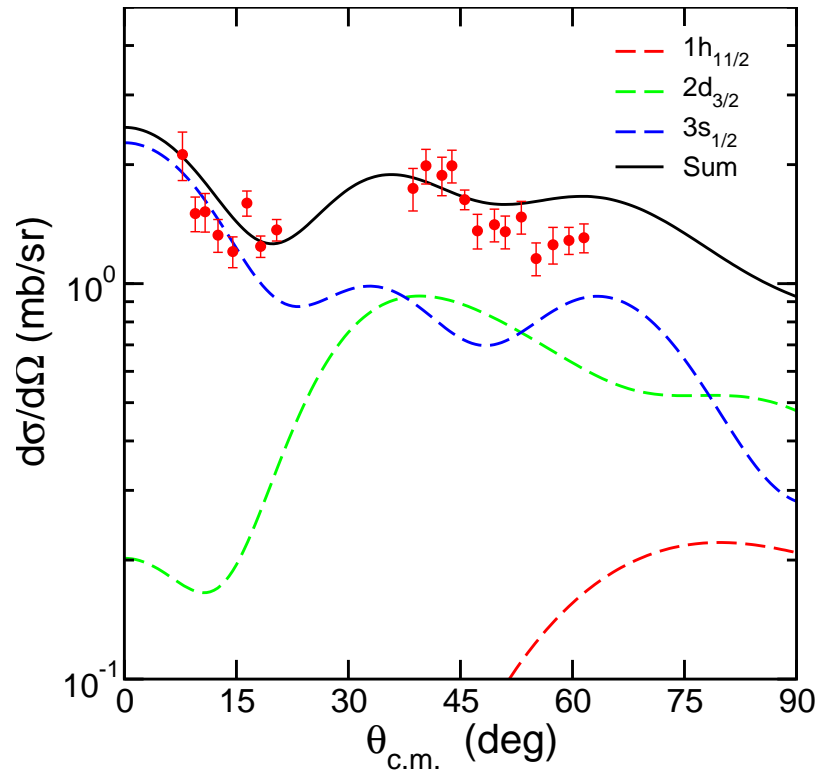
In general, staying with (d,p) reactions, the DWBA is an appropriate reaction model for heavy targets at low incident deuteron energy – exactly what constitutes “heavy” and “low” is a rather subjective choice, but a concrete example where DWBA and CDCC/CRC give identical results is  $^{124}\text{Sn}(d,p)^{125}\text{Sn}$  at  $E_d = 9 \text{ MeV}$

Data from [Jones et al., Phys. Rev. C 70 (2004) 067602], actually taken in inverse kinematics with a  $^{124}\text{Sn}$  beam

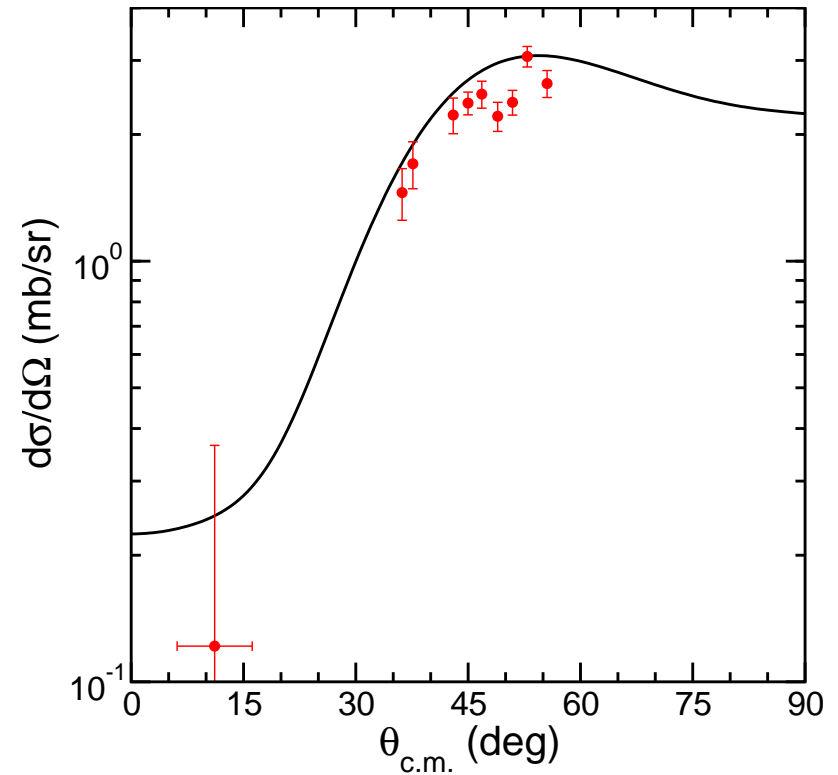
We repeat the original DWBA calculation and then perform a CDCC/CRC analysis, taking care to reproduce the  $d + ^{124}\text{Sn}$  elastic scattering predicted by the entrance channel optical model potentials used in the DWBA

All other input as in the DWBA calculation

# DWBA calculations for $^{124}\text{Sn}(d,p)^{125}\text{Sn}$ at $E_d = 9$ MeV



Mixture of 0.0 MeV 11/2-, 0.028 MeV 3/2+ and 0.215 MeV 1/2+ states



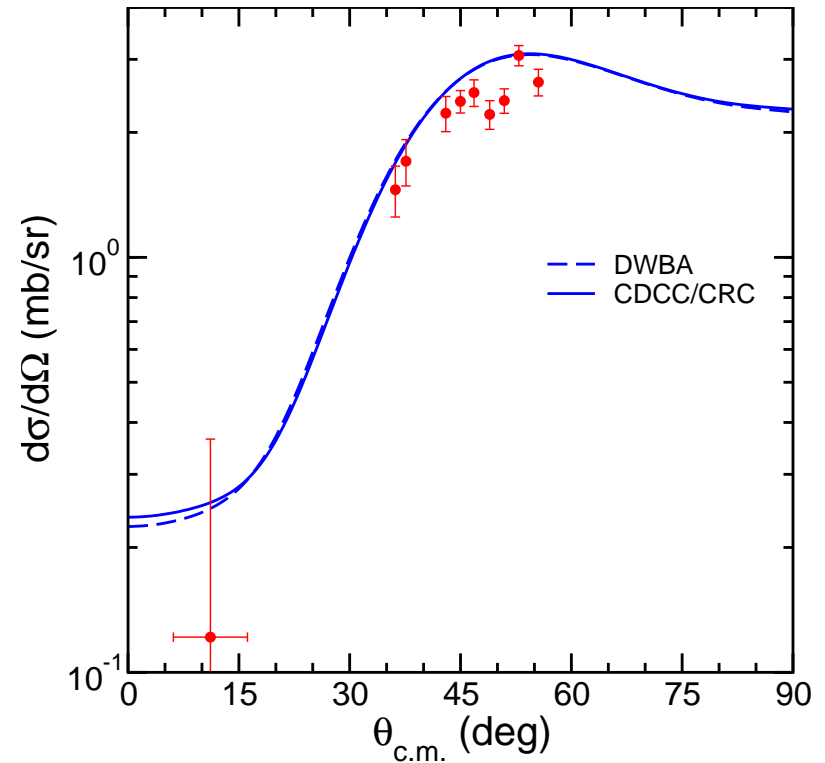
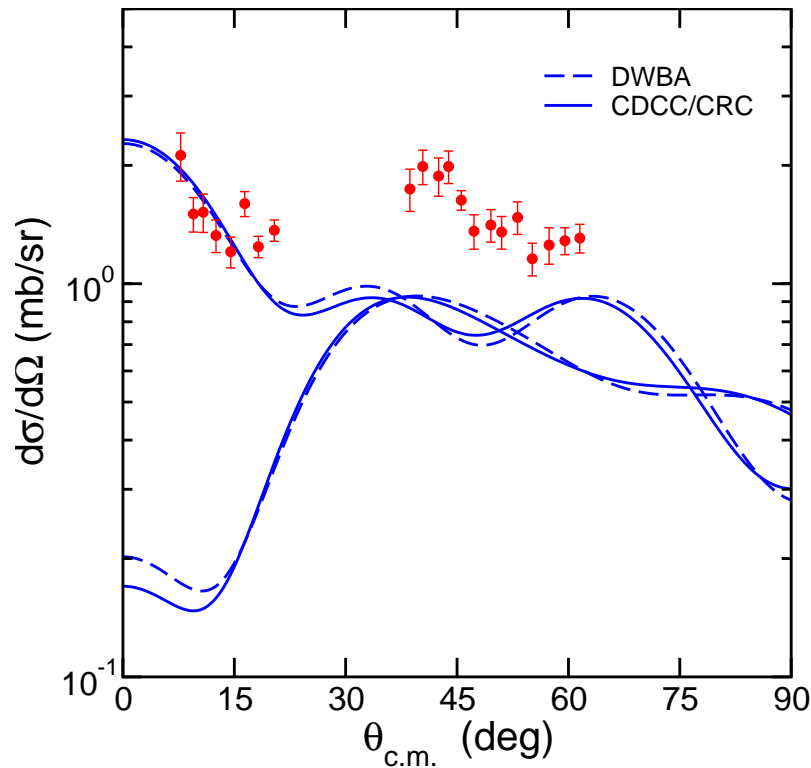
2.8 MeV 7/2- state

# Comparison of DWBA versus CDCC/CRC

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DWBA and CDCC/CRC give essentially identical results in this case, provided that the CDCC/CRC calculation reproduces the  $d + {}^{124}\text{Sn}$  elastic scattering predicted by the optical model potential used in the DWBA

# A counter example: the ${}^8\text{He}(p,t){}^6\text{He}$ reaction

We saw in the previous slide an example where the DWBA gives identical results to the more sophisticated CDCC/CRC model

We shall now present a counter example, where DWBA is unable to provide an adequate description of the available data and a more sophisticated reaction model is necessary

Data for the  ${}^8\text{He}(p,t){}^6\text{He}$  reaction are available at two widely spaced incident energies, 15.7 A.MeV [Keeley *et al.*, Phys. Lett. B **646** (2007) 222] and 61.3 A.MeV [Korshennikov *et al.*, Phys. Rev. Lett. **90** (2003) 082501]

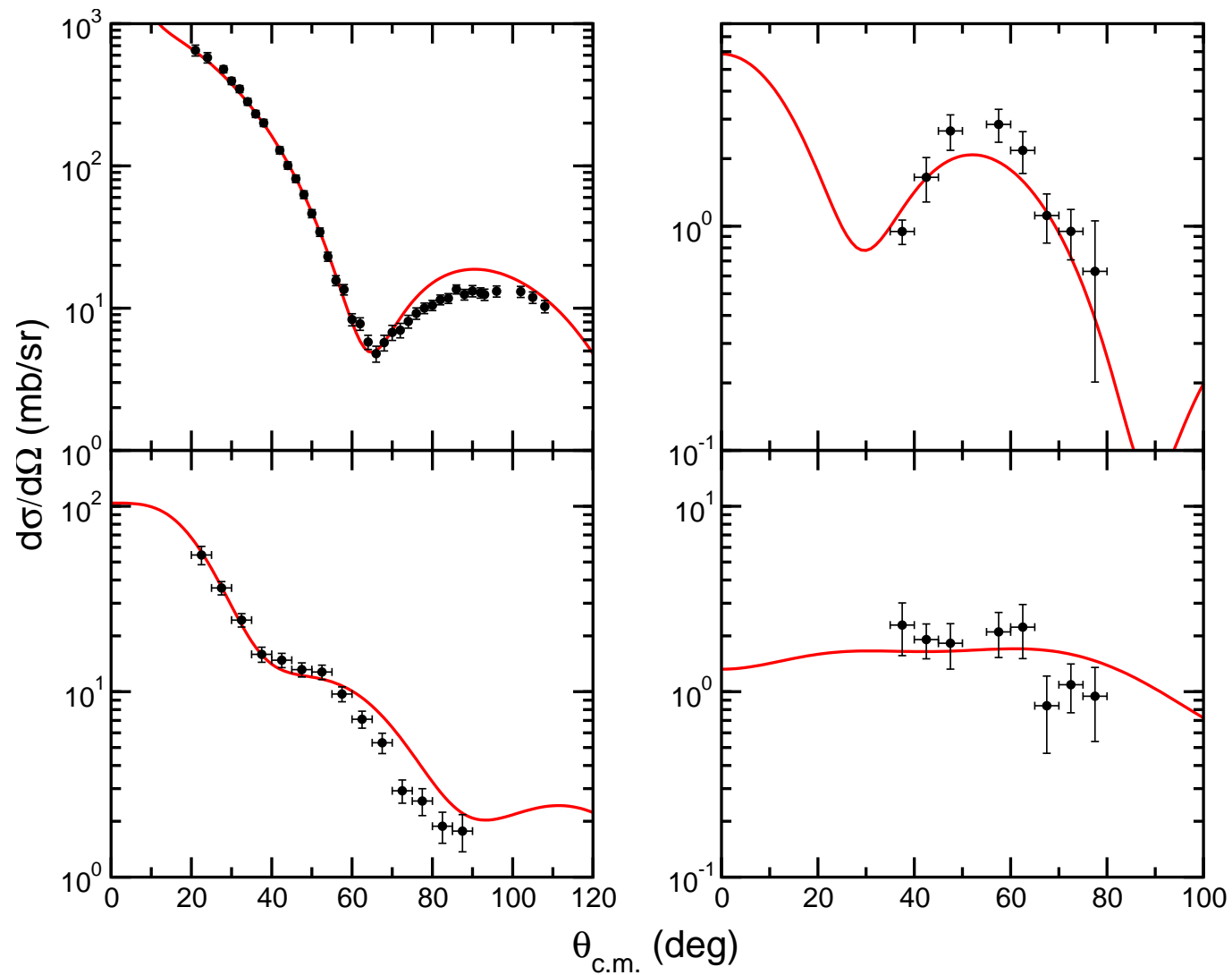
The CDCC/CRC combination (including the two-step mechanism via the  ${}^8\text{He}(p,d){}^7\text{He}(d,t){}^6\text{He}$  reaction) is able to provide a coherent picture of all these data, which the DWBA is unable to do

# CDCC/CRC fit to data at 15.7 A.MeV

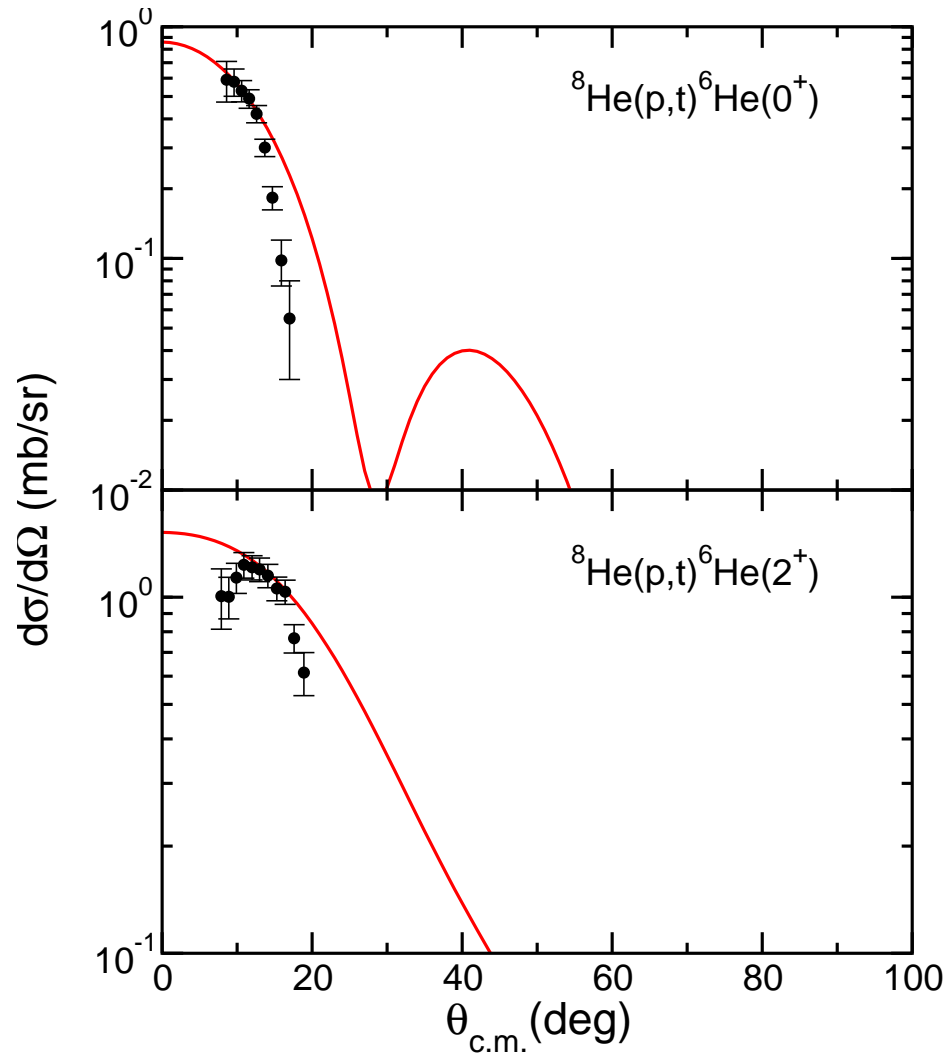
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# CDCC/CRC fit to data 61.3 A.MeV



Note that both calculations use exactly the same set of spectroscopic amplitudes

Description of the whole data Set is good

DWBA is unable to obtain a consistent description of the ensemble of the data with the same spectroscopic amplitudes at both energies – importance of accurate modelling of the reaction mechanism; no longer simple direct, one-step transfer



# Summary

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We have seen how choice of reaction model can significantly influence the nuclear structure information (the spectroscopic factors or amplitudes) that we wish to extract from nuclear reaction data

We have seen how DWBA can fail to give a satisfactory description of transfer data and that while the use of more sophisticated models can rectify some of the problems they are not a panacea for all ills – recall the  $5/2+$  state in  $^{13}\text{C}$

However, DWBA can work very well when the conditions underlying its basic premises are fulfilled e.g.  $^{124}\text{Sn}(d,p)^{125}\text{Sn}$  at low  $E_d$

Nevertheless, when these no longer hold, DWBA can give misleading results (as it does for the  $^8\text{He}(p,t)^6\text{He}$  case)

# Summary

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There remain several important sources of ambiguity in any determination of a spectroscopic factor quite apart from the use of different reaction models and uncertainties in the data

We have seen that different reasonable choices of the binding potential well radius can lead to large differences in the extracted spectroscopic factors – this is perhaps the largest single source of uncertainty. One could adopt wave functions from structure calculations (or adjust the potential well parameters to give equivalent wave functions) to avoid this problem; however, if the structure calculation does not well reproduce the spectrum of the nucleus in question has one really gained much?

The use of global rather than fitted optical potential parameters can also lead to important differences in the extracted spectroscopic factors – measure the elastic scattering! Not always possible, but it is good practice to do this for at least the entrance channel ...