## Non-empirical energy functionals from low-momentum interactions II. Low-momentum interactions from renormalization group methods

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Low-momentum interaction

Conclusions

#### Lecture series

#### Outline

- **Q** Introduction to energy density functional methods
- **2** Low-momentum interactions from renormalization group methods
  - Ideas underlying the renormalization group approach
  - Low-momentum interactions in the NN and NNN sectors
  - Ab-initio calculations of light nuclei
- **③** The building of non-empirical energy functionals

## Introduction

- Non-empirical energy functional
- Nuclear Hamiltonian
- Ideas underlying the renormalization group approach

#### Low-momentum interactions

- Low-momentum interactions in the NN sector
- Advantages for light-nuclei calculations
- Low-momentum interactions in the NNN sector

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## Performance of empirical EDFs

#### Performance of existing EDFs

- Tremendous successes for known nuclei
- Asymptotic freedom" as one enters "the next major shell"

### Crucial undergoing works

- Enrich the analytical structure of empirical functionals
  - Tensor terms, e.g. [T. Lesinski et al., PRC76, 014312]
  - B. G. Carlsson et al., PRC78, 044326]
  - $\rho_n \rho_p$  dependence to  $C_{qq}^{\tilde{\rho}\tilde{\rho}}(\vec{r})$ , e.g. [J. Margueron *et al.*, PRC77, 054309]
- Improve fitting protocols = data, algorithm and post-analysis

#### One can also propose a complementary approach...

- Data not always constrain unambiguously non-trivial characteristics of EDF
- Interesting not to rely entirely on trial-and-error and fitting data

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## Constructing non-empirical EDFs for nuclei

#### Long term objective

Build non-empirical EDF in place of existing models



Finite nuclei and extended nuclear matter



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## Constructing non-empirical EDFs for nuclei

#### Long term objective

Build non-empirical EDF in place of existing models



## Long term project and collaboration

#### Design non-empirical Energy Density Functionals

- Bridge with *ab-initio* many-body techniques
- Calculate properties of heavy/complex nuclei from NN+NNN
- Controlled calculations with theoretical error bars

	SPhN	T. Duguet, J. Sadoudi, V. Soma
	IPNL	K. Bennaceur, J. Meyer
	TRIUMF	A. Schwenk, K. Hebeler, S. Baroni
	NSCL	S. K. <mark>Bogner, B. G</mark> ebremariam
	OSU	R. J. Furnstahl, L. Platter
	ORNL	T. Lesinski
	JULICH	A. Nogga

#### Take-away message

#### Renormalization group methods

- O "The" nuclear Hamiltonian H is a low-energy effective theory of QCD
- **Q** H is characterized by an intrinsic "resolution scale" in energy/momentum  $\Lambda$
- **③** One can optimize  $\Lambda$  of  $H(\Lambda)$  to suit a certain purpose

#### Low momentum interactions

- **Q** Obtained by lowering the "resolution scale"  $\Lambda$  of conventional potentials
- Softer than conventional potentials
- Simplify tremendously A-body calculations
- Good starting point to build non-empirical energy functionals

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Coordinates  

$$\vec{k} \equiv \frac{\vec{k}_1 - \vec{k}_2}{2}$$
  
 $\vec{K} \equiv \vec{k}_1 + \vec{k}_2$ 

### Two-body (iso)spin

$$\begin{split} |SM_S\rangle & \text{with } S=0,1 \text{ and } -S \leq M_S \leq +S \\ |TM_T\rangle & \text{with } T=0,1 \text{ and } -T \leq M_T \leq +T \end{split}$$

#### Two-body momentum $\otimes$ spin $\otimes$ isospin basis (no CIB/CSB)

$$\langle \vec{k}_1 \vec{k}_2; SM_S; TM_T | V^{NN} | \vec{k}_1' \vec{k}_2'; SM_S; TM_T \rangle \equiv \langle \vec{k} | V_{ST} | \vec{k}' \rangle (2\pi)^3 \, \delta^3(\vec{K} - \vec{K}')$$

#### Main points

- $\blacksquare$   $V^{NN}$  decomposes in four spin-isospin channels (S, T)
- Focus on relative motion  $\langle \vec{k} | V_{ST} | \vec{k}' \rangle$

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## Matrix elements representation of $V^{NN}$

## Partial-wave expansion of $\langle \vec{k} | V_S T | \vec{k}' \rangle$

$$\begin{aligned} \langle \vec{r} | \vec{k} \rangle &= e^{i\mathbf{k} \cdot \mathbf{r}} &= 4\pi \sum_{LM_L} i^L Y_{M_L}^{L*}(\hat{k}) Y_{M_L}^L(\hat{r}) j_L(kr) = 4\pi \sum_{LM_L} i^L Y_{M_L}^{L*}(\hat{k}) \langle \vec{r} | k L M_L \\ \mathcal{Y}_{(LS)J}^M(\hat{k}) &\equiv \sum_{M_LM_S} \langle (LS) M_L M_S | JM \rangle Y_m^\ell(\hat{k}) | SM_S \rangle \end{aligned}$$







$$\langle \vec{k} | V_{ST} | \vec{k}' \rangle = \frac{\pi}{2} (4\pi)^2 \sum_{LL'JM} i^{L'-L} \mathcal{Y}^{M}_{(L'S)J} (\hat{k}') \mathcal{Y}^{M}_{(LS)J} (\hat{k}) V^{JST}_{LL'} (k,k')$$

Low-momentum interactions

## Degrees of freedom, energy scales and nuclear Hamiltonian

# Any $H(\Lambda) = T + V^{NN}(\Lambda) + V^{NNN}(\Lambda) + \dots$

- **Q** Describing point-like nucleons is *effective*
- O Includes a resolution scale  $\Lambda$  that cut-offs  $k\gtrsim\Lambda$
- **③** Has necessarily up to A-body components



## Low-momentum interactions

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- Has necessarily up to A-body components



#### High-momentum/short-range physics explicitly encoded characterized by $\Lambda$

• Most  $H(\Lambda)$  have  $\Lambda \gg \Lambda_{\text{data, physics}} \approx 2.1 \text{ fm}^{-1} \Leftrightarrow E_{lab} \approx 350 \text{ MeV}$ 



## Low-momentum interactions

### Degrees of freedom, energy scales and nuclear Hamiltonian

## Any $H(\Lambda) = T + V^{NN}(\Lambda) + V^{NNN}(\Lambda) + \dots$

- Describing point-like nucleons is *effective*
- **2** Includes a resolution scale  $\Lambda$  that cut-offs  $k \gtrsim \Lambda$
- **3** Has necessarily up to A-body components



#### High-momentum/short-range physics explicitly encoded characterized by $\Lambda$

Short-range modeling is essentially arbitrary!



[S. Bogner et al., PR386, 1]



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#### Conclusion: run down $\Lambda$ to $\sim \Lambda_{\rm data}$

Take  $\Lambda < \Lambda_{\chi}$  to further decouple unconstrained/irrelevant high-k modes

• Keep  $\Lambda > m_{\pi}$  to maintain  $\chi$ -EFT hierarchy  $V^{NN} \gg V^{NNN} \gg \cdots$ 

Huge by-product:  $H(\Lambda_{low})$  is softer and makes A-body calculations simpler

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Renormalization-group flow equation in the NN sector

#### Phase-shifts at given $\Lambda$ (uncoupled partial-wave)

Lipmann-Schwinger equation for the T-matrix with  $k, k' \leq \Lambda$ 

$$T_{L}^{JST}(k,k';E;\Lambda) = \frac{m}{\hbar^{2}} V_{LL}^{JST}(k,k';\Lambda) + \frac{2}{\pi} \mathcal{P} \int_{0}^{\Lambda} k''^{2} dk'' \frac{V_{LL}^{JST}(k,k'';\Lambda) T_{L}^{JST}(k'',k;E;\Lambda)}{E - \hbar^{2} k''^{2}/m}$$

Phase-shifts obtained from the "fully-on-shell" T-matrix

$$\tan \delta_L^{JST}(k;\Lambda) \equiv -k T_L^{JST}(k,k;\hbar^2 k^2/m;\Lambda)$$



Low-momentum interactions 0 = 0 = 0 = 0

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Renormalization-group flow equation in the NN sector

Lower  $\Lambda$  of  $V^{NN}$  (uncoupled partial-wave)

■ Maintain  $\delta_L^{JST}(k;\Lambda)$  for  $k \leq \Lambda$  and  $E_{\text{Deuteron}} = 2.2$  MeV invariant

$$\frac{\mathrm{d} T_L^{JST}(k,k';\hbar^2 k^2/m;\Lambda)}{\mathrm{d}\Lambda} = 0$$

This leads to a RG flow equation for  $V_{LL}^{JST}(k,k';\Lambda)$  with  $k,k' \leq \Lambda$ 

$$\frac{\mathrm{d}}{\mathrm{d}\Lambda} V_{LL}^{JST}(k',k;\Lambda) = \frac{2}{\pi} \frac{V_{LL}^{JST}(k',k;\Lambda) T_{LL}^{JST}(\Lambda,k;\hbar^2 \Lambda^2/2m;\Lambda)}{1 - (k/\Lambda)^2}$$

• Keeping  $\delta_L^{JST}(k;\Lambda)$  independent of  $\Lambda$  implies  $\partial_{\Lambda} V^{NN}(k,k';\Lambda) \neq 0$ 

Boundary conditions to solve RG flow equation

• Start from an existing model  $V^{NN}(k, k'; \Lambda_{high})$  (0 for k or  $k' \ge \Lambda_{high}$ )

$$V_{LL}^{JST}(k,k';\Lambda_0) \equiv V_{LL}^{JST}(k,k';\Lambda_{\text{high}})$$

**2** Run down  $\Lambda$  down to  $\sim \Lambda_{data}$ 

### Renormalization-group flow equation in the NN sector

#### Example of (non-unitary) RG flow in ${}^{1}S_{0}$ channel

Evolution performed through Lee-Suzuki transformation in practice

Start from AV18 NN potential



#### Results

V<sup>NN</sup>(k', k; Λ) changes because H(Λ) is not observable; (Ψ(Λ)|H(Λ)|Ψ(Λ)) is
 Non-unitary evolution but not a trivial cut of matrix elements beyond Λ!
 Matrix elements beyond Λ are renormalized onto those below Λ

**\blacksquare** No off-diagonal matrix element coupling low-k and high-k left

### Renormalization-group flow equation in the NN sector

#### Example of (unitary) RG flow in ${}^{1}S_{0}$ channel

It exists a unitary RG method: the Similarity Renormalization Group (SRG)Start from AV18 NN potential



#### Comparison with non-unitary evolution

• Unitary evolution only adds diagonal band to keep  $\delta_L^{JST}(k)$  unchanged at all k

 ${\it @}$  Low-energy sector + decoupling of low- and high-k modes the same

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## Renormalization-group flow equation in the NN sector

#### Dependence on $\Lambda_0$

- Start from various  $V^{NN}(\Lambda_{high})$
- Universal  $V^{NN}(\Lambda \approx 2) \equiv V_{\text{low k}}$
- Model-dependence screened-out

#### Phase shifts $\delta_L^{JST}(k)$

Unchanged for  $k \leq \Lambda$  at all  $\Lambda$ 

#### Deuteron properties

- Binding energy invariant
- Wave-function unchanged

#### $T^{NN}(\Lambda) =$ vacuum interaction

- Energy/density independent
   It is not a C matrix in diaming
- Low/high k not decoupled by G



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#### Deuteron binding energy and wave function





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#### Dependence on $V^{\overline{N}N}(\Lambda_0)$ Comparison with G-matrix in ${}^{3}S_{1}$ channel Start from various $V^{NN}(\Lambda_{high})$ k' (fm<sup>-1</sup>) 3 4 5 2 Universal $V^{NN}(\Lambda \approx 2) \equiv V_{\text{low k}}$ 0.5 Model-dependence screened-out k (fm<sup>-1</sup>) 8 k' (fm<sup>-1</sup>) 2 3 4 5 -0.5 Phase shifts $\delta_L^{JST}(k)$ 0.5 k (fm<sup>-1</sup>) 8 $\blacksquare Unchanged for k < \Lambda at all \Lambda$ $\Lambda = 2.0 \; {\rm fm}^{-1}$ -0.5 k' (fm<sup>-1</sup>) Deuteron properties 4 $\Lambda = \Lambda_{\text{high}}$ Binding energy invariant 0.5 Wave-function unchanged k (fm<sup>-1</sup>) s -0.5 $V^{NN}(\Lambda) =$ vacuum interaction Energy/density independent G matrix [S. Bogner, private communication] It is not a G-matrix in disguise! $\blacksquare$ Low/high k not decoupled by G

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### Advantage to use $\Lambda_{low}$ for nuclear structure calculations

## Why bother if things like $\delta_L^{JST}(k)$ and $E_{\text{Deuteron}}$ remain unchanged?

Because non-observable features are  $\Lambda$ -dependent

- Correlations in wave-functions  $|\Psi(\Lambda)\rangle$
- **②** Convergence in calculations, e.g. of  $E = \langle \Psi(\Lambda) | H(\Lambda) | \Psi(\Lambda) \rangle$
- Take advantage to perform technically simpler calculations!

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#### In NN sector

V<sup>NN</sup>(Λ<sub>low</sub>) smoother in k-space
 V<sup>NN</sup>(Λ<sub>low</sub>) smoother in HO basis

#### In NNN sector

- Λ<sub>low</sub> provides better convergence
   Same for larger Λ-body system
- What about  $\Lambda$ -dependence of  $E_{^{3}\mathrm{H}}$ ?



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## Induced many-body forces

#### RG evolution induces up to A-body forces in A-body system

- $\textcircled{\sc 0}$  Well known from Lee-Suzuki effective-interaction theory
- Most easily seen from SRG [S. Bogner et al., AP323, 1478]

$$\frac{\mathrm{d}H(\Lambda)}{\mathrm{d}\Lambda} = -\frac{4}{\Lambda^5} \left[ \left[ T, H(\Lambda) \right], H(\Lambda) \right] \quad \propto \quad \left[ \left[ \sum c^{\dagger} c, \sum c^{\dagger} c^{\dagger} c c c \right], \sum c^{\dagger} c^{\dagger} c^{\dagger} c c c \right] \\ \propto \quad \dots + \sum c^{\dagger} c^{\dagger} c^{\dagger} c c c + \dots$$

## • Is it a problem?

- Not if remain of "natural size", i.e. follow  $\chi$ -EFT power counting
- $\blacksquare$  A-dependence of A-body observables used to assess truncation errors
- **③** A-body forces are a priori present at  $\Lambda_0$  anyway
  - Might as well develop (S)RG machinery to make them soft
  - Recently achieved through SRG method [E. D. Jurgenson et al., PRL103, 082501]
- What are the results?

## Induced many-body forces in three- and four-body systems

## For $V^{NN}$ only

- Tjon line suggests missing  $V^{NNN}$
- $\bigcirc \ \Delta E_{^{3}\mathrm{H}} \sim 0.6 \text{ MeV for } \Lambda \in [2,\infty]$
- **③** Non-negligible but  $\ll \langle V^{NN}(\Lambda) \rangle_{\Lambda}$

## Including induced $V^{NNN}$

- Break away from Tjon line
- $\ \, {\bf O}_{\Lambda} E_{^3{\rm H}} = 0 \ {\rm for \ all} \ \Lambda$

$$\langle V^{NNN}(\Delta\Lambda) \rangle_{\Lambda} \sim \langle V^{NNN}(\Lambda_0) \rangle_{\Lambda_0}$$

• But net  $V^{NNN}(\Lambda_{low})$  much softer!

#### Induced $V^{NNN}$

- Negligible in <sup>4</sup>He
- To be monitored in heavier nuclei!

## Tjon line: $E_{^{3}\mathrm{H}}$ versus $E_{^{4}\mathrm{He}}$



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### Induced $V^{NNNN}$

- $\partial_{\Lambda} E_{4_{\mathrm{He}}} \approx 0 \text{ for } \Lambda \in [2, \infty]$
- Negligible in <sup>4</sup>He
- To be monitored in heavier nuclei!

## NCSM calculation of <sup>3</sup>H



## Induced many-body forces in three- and four-body systems

## For $V^{NN}$ only

- Tjon line suggests missing  $V^{NNN}$
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### Conclusions

#### New paradigm for nuclear interactions

- **Q** The "hard core" is not an absolute feature of  $H(\Lambda)$
- ${\it \bigcirc}\ H(\Lambda_{\rm high})$  models hurt, i.e. A-body is highly non-perturbative
- Solution As the doctor says "if it hurts, just don't do it!"

#### Use $H(\Lambda_{\text{low}})$ for low-energy studies

- Keep  $m_{\pi}, k_F < \Lambda_{\text{low}} \leq \Lambda_{\text{data}}$
- **②** Simplifies tremendously ab-initio calculations of light-nuclei
- $\bigcirc$   $V^{NNN}$  unavoidable
  - What about  $V^{NNNN}$  in heavy nuclei?
  - What about the power counting at finite density?

#### What about EDF calculations now?

- See how low-momentum interactions do for infinite nuclear matter EOS
- Build EDF from many-body perturbation theory
- Approximate such still-too-complicated EDF
  - Constrain Skyrme- or Gogny-like energy functionals

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## Selected bibliography



#### S. Kehrein,

The flow equation approach to many-particle systems, 2006, Springer-Verlag, Berlin

- S. D. Glazek, K. G. Wilson, Phys. Rev. D48 (1993) 5863
- S. K. Bogner, T. T. S. Kuo, A. Schwenk, Phys. Rept. 386 (2003) 1
- S. K. Bogner, R. G. Furnstahl, R. J. Perry, Phys. Rev. C75 (2007) 061001
- S. K. Bogner, R. G. Furnstahl, R. J. Perry, A. Schwenk, Phys. Lett. B649 (2007) 488
- S. K. Bogner, R. G. Furnstahl, R. J. Perry, Ann. Phys. 323 (2008) 1478
- E. D. Jurgenson, P. Navratil, R. G. Furnstahl, Phys. Rev. Lett. 103 (2009) 082501