

GIANT RESONANCES AND ISOSPIN ASYMMETRY

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In order to grasp the main issues on giant resonances (GR), the following questions are addressed : how can GR be ordered ? What is the effect of isospin symmetry breaking on GR ? How to describe and to interpret GR ? What is the link between their microscopic and macroscopic descriptions ? Which nuclear structure properties GR can help to study ? What is the status of GR in neutron-rich nuclei ?

1 Introduction

The purpose of this course is to provide an up-to-date overview of Giant Resonances (GR) properties, focusing on concepts which are not always detailed in textbooks. The relationship between GR and isospin symmetry will also be investigated. It should be noted that GR belong to the most probable excitations in a given nuclei. Therefore they are among the easiest modes to measure (which doesn't mean that it is easy to do so), and provide information on both nuclear structure and nuclear matter.

The bibliography given at the end is minimal on purpose, but shall provide complementary answers to questions related to GR.

2 Properties of giant resonances

2.1 Definitions

In order to clearly state the problem, definitions related to excited states are hereby provided. An excited state with total angular momentum J_f and isospin T_f is built with an excitation called R (the giant resonance in the present case) on top of an initial state (J_i, T_i) . The definitions are illustrated on Fig. 1. The excitation R is characterised by the following quantum numbers : J_R (total angular momentum), L_R (orbital angular momentum), S_R (spin), π_R (parity) and T_R (isospin). Giant resonances being nuclear excitation, the following relations hold, due to conservation of angular momentum, parity and isospin :

$$\vec{J}_f = \vec{J}_i + \vec{J}_R \quad (1)$$

$$\vec{T}_f = \vec{T}_i + \vec{T}_R \quad (2)$$

$$\pi_f = \pi_i \cdot \pi_R = \pi_i \cdot (-1)^{L_R} \quad (3)$$

The total angular momentum of the giant resonance (GR) and its parity are further characterised by :

$$\vec{J}_R = \vec{L}_R + \vec{S}_R \quad (4)$$

$$\pi_R = (-1)^{L_R} \quad (5)$$

The minimal set of quantum numbers which completely characterise the GR is therefore (L_R, S_R, T_R) . It allows to deduce the values of J_R and π_R using Eq. 4 and 5.

If the initial state is the ground state of an even-even nucleus, which is often the case, we have $J_i^\pi=0^+$. Therefore Eq. 1, 2, 3 become :

$$\vec{J}_f = \vec{L}_R + \vec{S}_R \quad (6)$$

$$\pi_f = (-1)^{L_R} \quad (7)$$

The GR is the transition from the initial state to the final state, and it should not be confused with the final state itself. However Eq. 6 and 7 shows that this confusion can be tolerated. In this case the excited state can be called a GR state : starting from a 0^+ ground state, the final state has the same characteristics than the GR and therefore can be denoted as a GR state. But if the initial state is not a 0^+ ground state, then the final states have different quantum numbers than the GR.

It should be further noted that there is a difference between a GR excitation and an electromagnetic excitation. An excited state accessible through a GR is also sometimes accessible through an electromagnetic excitation, but this is not always the case. For instance the so called IS-GMR state cannot be accessed through electromagnetic excitation (E0 electromagnetic transition does not exist). Therefore GR states can be accessed using nuclear interaction, through direct reactions, and in some cases also by electromagnetic excitation.

Fig. 2 displays a typical excitation spectrum exhibiting the GR pattern at high energy, corresponding to the $L_R=2$, $S_R=0$ and $T_R=0$ quantum numbers. Above the elastic peak at 0 MeV and the first 2^+ excited states, a large bump, around 10-20 MeV corresponds to the giant quadrupolar resonance. The GR is therefore a high energy mode, with a large cross section. The y axis is the transition probability from the ground state to the 2^+ states, calculated for a given multipolar operator (here quadrupolar). It allows to calculate the cross section, which depends on the probe, which can be electromagnetic (real or virtual photons) or nuclear (p, α , ...).

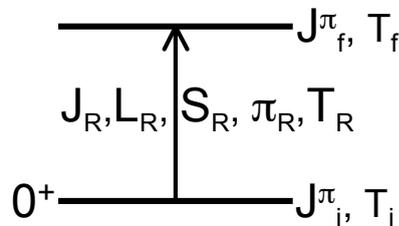


FIG. 1 – Definition of the quantum numbers of the initial state i , the giant resonance R , and the final state f .

What are all the possible values of L_R , S_R and T_R characterising a GR? On this purpose, it should be noted that the experimental data on GR is well described if GR are considered like a superposition of many particle-hole (ph) pairs built upon the mean-field ground state (Fig. 3). This microscopic description of GR allows to determine the possible values of S_R and T_R . A particle (a proton or a neutron) and a hole have both :

$$\vec{s}_p = \frac{\vec{1}}{2} \tag{8}$$

$$\vec{s}_h = \frac{\vec{1}}{2} \tag{9}$$

Therefore, since a GR is of particle-hole nature :

$$\vec{S}_R = \vec{s}_p + \vec{s}_h = \vec{0} \text{ or } \vec{1} \tag{10}$$

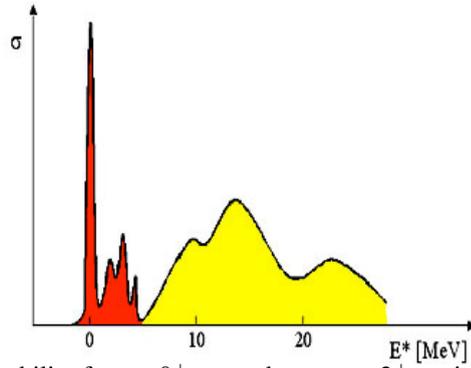


FIG. 2 – Transition probability from a 0^+ ground state to a 2^+ excited state. The GR has $L_R=2$, $S_R=0$ and $T_R=0$ quantum numbers

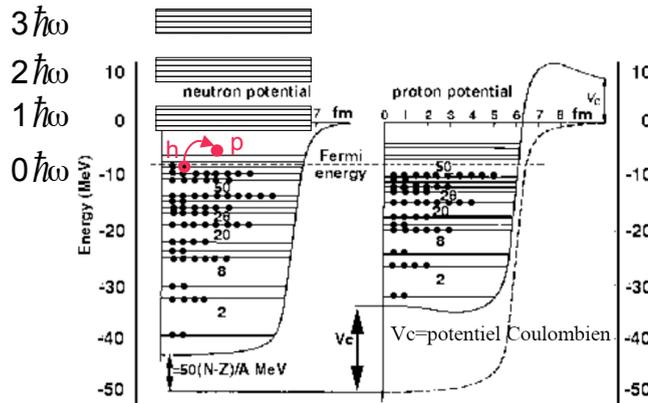


FIG. 3 – Particle-hole description of a Giant Resonance. Several ph configurations contribute. The major harmonic oscillator shells are indicated.

The same holds for the isospin of the GR, T_R . Nucleons belong to isospin doublets,

$$\vec{T}_R = \vec{t}_p + \vec{t}_h = \frac{\vec{1}}{2} + \frac{\vec{1}}{2} = \vec{0} \text{ or } \vec{1} \quad (11)$$

Hence **the values of both the spin and isospin of the GR are well determined : 0 or 1**. The values of the orbital angular momentum L_R cannot be constrained in the same way, and can take in principle all values. However, the cross section drops if L_R is very large because

$$\vec{L}_R = \vec{l}_p + \vec{l}_h \quad (12)$$

so that L_R is limited by the angular momentum values of the single particles states (s,p,d,f,g,h...) of the shell structure (Fig. 3).

In the case of the parity of the GR, π_R , another constraint holds : it is well known that the parity of the single particle states usually alternate between adjacent major shells : s shell ($N=2, \pi = 1$), p shell ($N=8, \pi = -1$), sd shell ($N=20, \pi = 1$), pf shell ... Since the parity of the giant resonance obeys to :

$$\pi_R = \pi_p \cdot \pi_h, \quad (13)$$

the parity of the GR depends on the major shell on which the nucleon is promoted (particle state). On Fig 3, it can be seen that if the nucleon is promoted within the same major shell than the initial one, one gets $\pi_R=1$. These excitations are called $0\hbar\omega$, referring to the major shell of the harmonic oscillator involved on top of the valence shell. The next shell corresponds to $1\hbar\omega$, and therefore the parity of the GR is $\pi_R=-1$. Similarly the parity of the giant resonances, fixed by the conservation laws (Eq. 3) drives the possible major shell to be excited : $\pi_R=-1$ imposes $1\hbar\omega$ or $3\hbar\omega$ excitations whereas $\pi_R=1$ imposes $0\hbar\omega$ or $2\hbar\omega$ excitations. It should be noted that, consequently, at first order, GR energy centroids follow the empirical rule $\hbar\omega \sim A^{-1/3}$ of the harmonic oscillator.

2.2 Classification of the Giant Resonances

Having characterised the possible S and T values of the GR, it is convenient to order them. Fig. 4 shows a standard classification where GR are ordered by the possible S and T values (the R index is omitted in the following for convenience). The L and parity values of the GR are also given on this figure. The parity values can be deduced from L using Eq. (5).

2.2.1 The L dependence

The S=0, T=0 mode corresponds to an excitation which does not distinguish protons from neutrons nor spin up from spin down. The transition operator for a given L_R value is therefore :

$$Q_{LM} = \sum_{i=1}^A r_i^L Y_{LM}(\hat{r}_i) \quad (14)$$

Macroscopically this corresponds to a vibration in phase of all the nucleons. For L=0 it corresponds to the dilatation-compression (breathing) mode, whereas for L=2, Eq (14) shows that the

nucleus will be axially deformed, vibrating from a prolate to an oblate, through the spherical shape. The L=1 mode does not correspond to an excitation since it describes the translation of the whole nucleus Eq (14).

2.2.2 The S dependence

The S=0 mode is called electric, and the S=1 magnetic, because it involves the spin. **This naming should not be confused with electric or magnetic transitions for photons** : most of the electric GR can be excited by electromagnetic excitations (so-called EL or ML transitions) but not all of them as stated above : the GMR cannot be accessed through EL transitions. Let us further detail the relationship between the photon quantum number and the GR one. In order to excite a nucleus in a state corresponding to the GR, we must have :

$$J_R = J_\gamma \tag{15}$$

where J_γ is the total angular momentum of the photon : it corresponds to the L of EL or ML electromagnetic transition.

The same holds for the parity. We must have

$$\pi_\gamma = \pi_R = (-1)^{L_R} = (-1)^{J_\gamma} \text{ or } (-1)^{J_\gamma+1} \tag{16}$$

the condition corresponds to the case of an electric or magnetic photon, respectively. Therefore, L_R (characterising the GR) is not always equal to J_γ . For instance, in the case of an ISSGDR, we have $L_R=1, S_R=1$ (Fig. 4). Therefore $J_R = 0, 1$ or 2 . The parity $\pi_R = (-1)^{L_R}$ is -1 . This imposes that E1

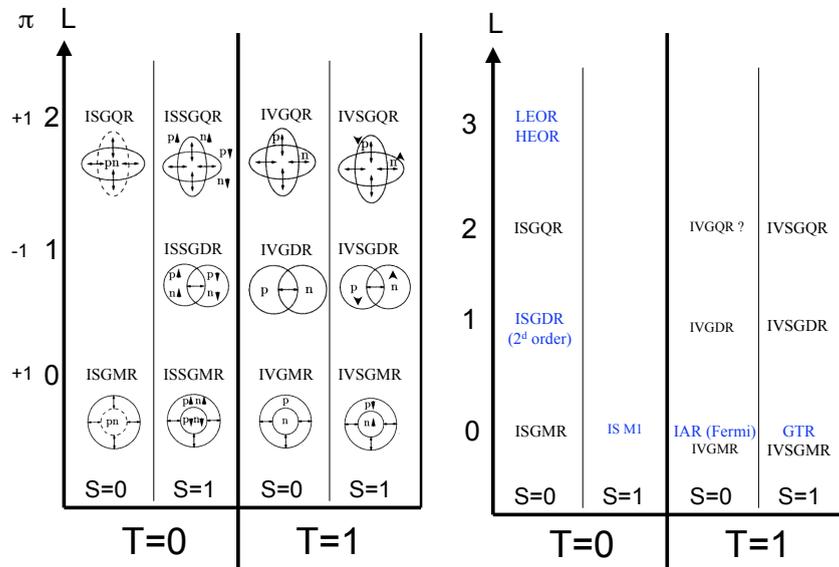


FIG. 4 – Spin, isospin and angular momentum ordering of Giant Resonances (the R index is omitted). Left : macroscopic picture. Right : experimental status

and M2 electromagnetic transitions can excite this mode (M0 photon does not exist). Therefore so called magnetic ISSGDR ($S_R=1$) can be excited by an electric (E1) gamma transition.

2.2.3 The T dependence

T=0 GR are called isoscalar (IS) giant resonances, whereas T=1 are called isovector (IV) GR. In the case of the T=1 mode, the transition operator depends on t_3 which is the isospin projection on the third axis in isospin space : $t_3=1/2$ for neutrons and $-1/2$ for protons. This non trivial demonstration require to tensorial algebra and can be found in the textbook of Bohr and Mottelson. Therefore the transition operator is expressed as :

$$Q_{LM} = \sum_{i=1}^A t_3(i) r_i^L Y_{LM}(\hat{r}_i) \quad (17)$$

Hence, transition strength of the T=1 mode, requires that the neutrons and protons of the excited nucleus vibrates out of phase. Otherwise, if the neutrons and protons are in phase they will almost cancel (because of the relative minus sign in Eq. 17), providing a very low IV strength and the IS strength which will be more important. Note that the IV T=1, L=1 mode corresponds to an excitation of the nucleus, because it is the translation of neutron with respect to proton.

The same formalism holds for the spin S_R : magnetic S=1 GR corresponds to vibration of spin up nucleons against spin down one. Finally both IV and magnetic excitations can be combined, as seen on Fig. 4.

2.2.4 Experimental status

What is the experimental status on GR ? The right part of Fig. 4 displays an overview of the experimental status in stable nuclei where the size of the name represents the quality of the data. Some states are new compared to the left part of Fig. 4. Let us review them briefly : the ISGDR mode is a second order effect which corresponds to the next term in the developpement of the Bessel (l+1) function describing the transition operator 14. It corresponds to a compression wave travelling back and forth through the nucleus along a definite direction and is called the squeezing mode. Another mode is the L=3 GR, divided into a Low Energy Octupole Resonance (LEOR) and a High energy one (HEOR). This splitting is due to the $\pi_R=-1$ condition which imposes either $1\hbar\omega$ or $3\hbar\omega$ excitations (Fig. 3), corresponding to LEOR and HEOR, respectively. It should be noted that the cross section for exciting S=1, T=0 GR is low compared to S=1,T=1 or S=0, T=0 modes ; therefore these GR lie in the experimental background, and only the S=1, T=0, L=0 mode have been measured so far in the S=1 channel. This excitation is called the M1 mode because of Eq. 15 and 16 : considering photons, only M1 transitions can access to this GR.

2.3 Breaking the isospin symmetry in GR

There are 3 causes of isospin symmetry breaking in GR : i) the weak interaction allows for charge exchange modes (neutron \leftrightarrow proton), ii) the possible neutron excess in nuclei and iii) the Pauli principle.

2.3.1 The weak interaction

Another type of GR, which cannot be described macroscopically, is due to the non-conservation of T_3 by the weak interaction, allowing to transform a proton in a neutron and vice-versa. In that case the GR can be considered as a ph excitation built on top of the mean field, but in which the hole is a neutron and the particle a proton (or vice-versa). Therefore this is a $T=1$ mode since T_3 is 1 or -1. Fig. 5 shows the corresponding picture. These GR are dominant in the "charge-exchange" $L=0$ mode. The $S=0$ mode (Fermi transition) corresponds to a particle-hole where the particle and the hole have the same quantum numbers. Therefore the final nucleus state should belong to the same isobaric analog multiplet than the initial one (exchange of one proton in neutron or vice-versa). Hence these GR are called Isobaric Analog Resonances (IAR). The $S=1$ $T=1$ mode for charge exchange is called Gamow-Teller mode (Fig 5). In this $S=1$ mode, the total spin of the hole can differ from the particle one (for instance from $9/2$ to $7/2$), therefore these magnetic charge exchange GR are sometimes called spin-flip mode. It should be noted that the difference between the IAR and IVGMR in the charge exchange $S=0$ channel (or GTR and IVSGMR in the charge exchange $S=1$ channel) is that IAR (GTR) are $0\hbar\omega$ GR whereas IV(S)GMR are $2\hbar\omega$ modes. The right part of Fig. 5 shows a measurement of the IAR and GTR in a charge-exchange reaction. The GTR is broader than the IAR, because more ph configuration can contribute, since $S=1$ in the GT case.

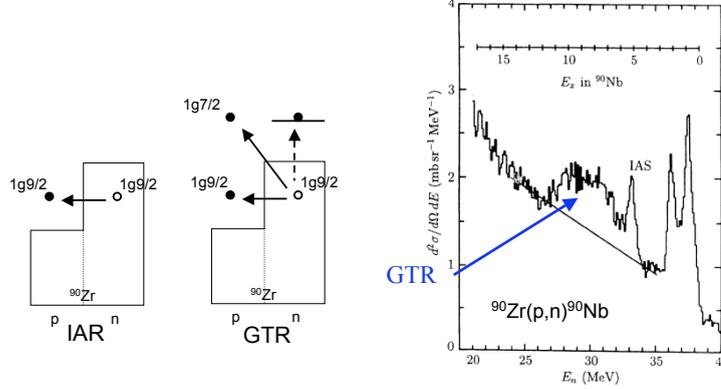


FIG. 5 – Particle-hole description of charge-exchange Giant Resonances. Left : Isobaric Analogue (Fermi) Resonance. Middle : Gamow-Teller Resonance. Right : Experimental spectrum of the IAR (labelled as IAS) and the GTR. The expectation energy of the final nucleus ^{90}Nb is given in the upper axis.

2.3.2 The neutron excess

Another cause of isospin symmetry breaking in a $T=1$ transition is due to neutron excess. The ground state of a $N=Z$ nucleus has $T_{3i}=0$ and usually $T_i=0$: the T value is minimum because $T_+ = \sum t_{+i}$ acting on the ground state gives 0, since all the neutron states accessible from a proton are occupied. Hence T_3 is maximum and therefore $T_3=T$. Note that this rule is less valid in the case of $N=Z$ nucleus, where there is no neutron excess to block the promotion of a proton.

In a $N=Z$ nucleus, a $T_R=1$ transition therefore leads to $T_f=1$ (because $T_i=0$), and the excited states in the $Z-1$, Z , $Z+1$ nuclei are isobaric analog states (they belong to the same isomultiplet).

In the case of an $N > Z$ nuclei, $T_i = (N-Z)/2$, and is therefore different from zero. Hence the $T_R=1$ IV GR leads to $T_f = T_{i-1}, T_i, T_{i+1}$ transitions. These states cannot be isobaric analog because they belong to different T_f values in the $Z-1, Z, Z+1$ nuclei. Therefore, in the case of neutron excess, the IV $T=1$ excitations are not feeding isobaric analog states, which is a case of isospin symmetry breaking.

Finally, transition from (N,Z) to $(Z-1, N+1)$ nuclei are hindered because of the Pauli principle : in the output channel states can be occupied by neutrons.

In summary the T_3 degeneracy of GR is raised (isospin symmetry breaking) because of i) the weak interaction processes, ii) the n excess of the initial nuclei and iii) the Pauli principle. It is therefore necessary to generalise our GR picture by raising this T_3 degeneracy in the $T=1$ sector (Fig 6) On this figure, for instance, $GT+$ GR have different transition rates than $GT-$. **Therefore a given GR is characterised by a (T, T_3, L, S) value.** The size of the GR names still provide the present experimental status.

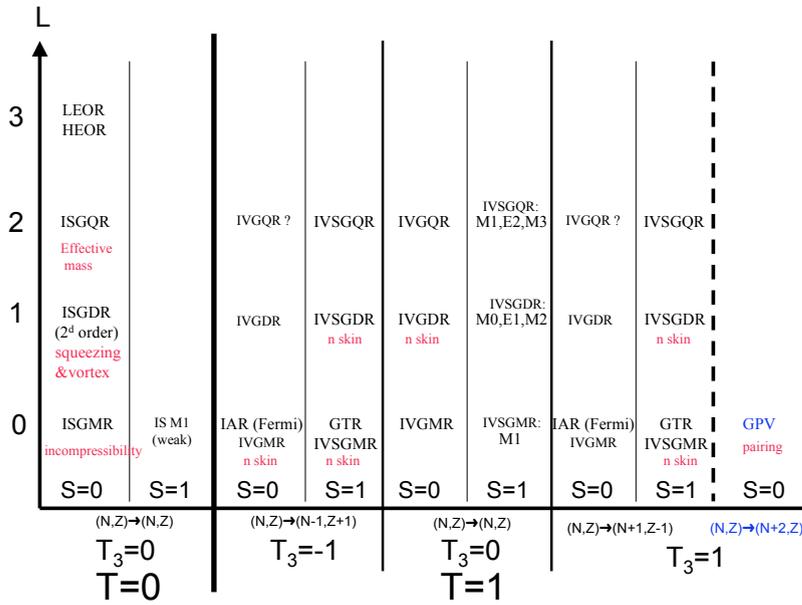


FIG. 6 – Spin, isospin and angular momentum ordering of Giant Resonances. The explicit T_3 dependence is displayed, as well as the nuclear properties which can be studied with the corresponding GR

2.4 How to measure a given GR ?

As stated in the introduction, GR are convenient excitations to detect because of their large cross section, and their abilities to provide information on nuclear structure, nuclear matter quantities, and the energy density functional. Which probe should be used to measure a given GR ? Photons, protons, alphas, nuclei ? This important question should be addressed considering the GR landscape of Fig. 6. First the (L, S) values of the GR cannot strictly be selected by the probe : all values can contribute, but their cross section vary, depending on L and S , and it is therefore possible to restrict the possible (L, S) value in a given experiment. The remaining (L, S) values are

further disentangled using angular distributions as depicted on Fig. 7. This (L,S) identification is called multipole decomposition analysis. For instance the ISGMR is peaked at 0 deg. in the center of mass frame, whereas the ISGQR is more constant below several degrees. Therefore selecting the detection angle allows to focus on one (or few) (L,S) values.

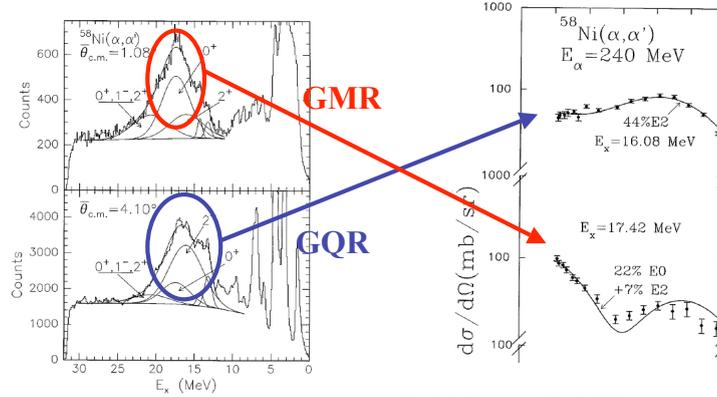


FIG. 7 – Multipole decomposition analysis showing the IS GMR and IS GQR angular distributions. Adapted from D.H. Youngblood et al. Phys. Rev. Lett. 76 (1996) 1429

2.4.1 Isospin selectivity of a probe

The GR (T,T₃) value can be selected by the probe. Considering a probe q (alpha or proton, etc.) with isospin (T_q,T_{3q}), the question is : What is the relationship between T of the GR and T_q ? Namely, what is the isospin character of a GR which can be excited by the probe q ? For instance, is there a specific probe to excite the T=0,T₃=0 resonances (IS GMR, IS GQR, IS ...) ? To answer to this question, often raised in experiment preparation, let us write the conservation of isospin (we will consider here the inelastic scattering case) :

$$A + q \rightarrow A^* + q \quad (18)$$

$$\vec{T}_i + \vec{T}_q = \vec{T}_f + \vec{T}_q \quad (19)$$

$$T_{3i} + T_{3q} = T_{3f} + T_{3q} \quad (20)$$

The last equations implies T_{3f}=T_{3i}, which is normal because of inelastic scattering (same initial and final nucleus : T₃=(N-Z)/2).

Besides, the GR definition (Eq. 2) implies :

$$\vec{T}_i + \vec{T}_R = \vec{T}_f \quad (21)$$

$$T_{3i} + T_{3R} = T_{3f} \quad (22)$$

Therefore T_{3R}=0 : again, same initial and final nucleus in the case of inelastic scattering. The point is : what is the relationship between T_R and T_q ? To answer we use the graphical representation of the triangular relation : Eq. (19) implies

$$|T_i - T_q| \leq T_S \leq T_i + T_q \quad (23)$$

where T_S is the modulus of the vectorial sum of \vec{T}_i and \vec{T}_q . In quantum mechanics, this relationship is translated by the triangle (Fig. 8, left), where the 3 sides of the triangle play a symmetric role (the triangular relation can be used in any order). The same triangle holds for the output channel (Fig. 8, middle). Moreover, Eq. 21 allow to draw a triangle between T_i , T_f and T_R . One sees that T_R , T_q and T_q form a triangle (Fig. 8, right). Therefore we get the master relation :

$$\boxed{0 \leq T_R \leq 2T_q} \quad (24)$$

$$T_{3R} = 0 \quad (25)$$

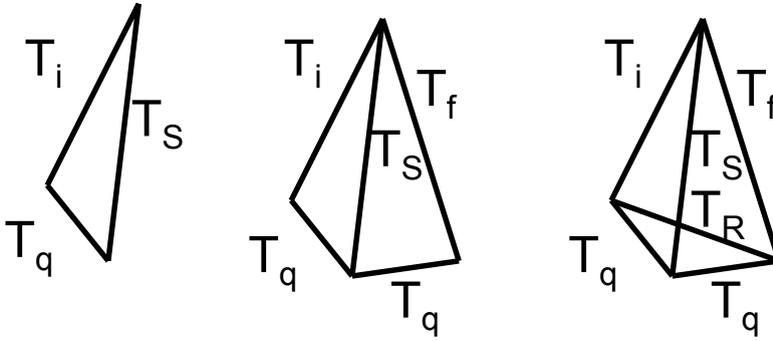


FIG. 8 – Building a kite in 3 steps to find the relationship between the isospin of the GR T_R and the isospin of the probe T_q

Hence a $T_q=0$ probe (like alpha or deuteron) only excite $T_R=0$ IS GR such as the ISGMR or the ISGQR. In the case of a proton ($T_q=1/2$), both $T_R=0$ and $T_R=1$ GR can be excited : it is not selective. Moreover Eq. (24) also implies that there cannot be selective $T_R=1$ nuclear probes, because $T_R=0$ is always possible. That is why IV GR are usually probed using gamma excitation, which do not obey to the above relations. This original "kite" relationship 24 should be reminded when asking which probe should be used to detect a given excitation, during the preparation of an experiment.

2.4.2 Detection setup for stable and exotic nuclei

Experimentally, the measurement method of GR is different when considering stable or unstable nuclei. Fig. 9 depicts the comparison of the two detection setups for the ISGMR. In the case of a stable nuclei, the probe is sent on the target nucleus (direct kinematics). The 0 deg. recoiling alphas are measured in the focal plane of the spectrometer, whereas the beam is deviated in a Faraday cup in the focal plane. The position of the recoiling alphas in the focal plane detectors provide their energy and scattering angle, allowing to deduce the GMR energy spectrum and its angular distribution. In the case of the ISGMR, a 0 deg. mode is relevant because the cross section is peaked at this angle (Fig. 7). In the case of unstable nuclei, the ISGMR is measured in inverse

kinematics. The recoiling alphas have a very low energy (few hundreds keV) and therefore the use of an active target is useful. In such a device, the detection gas of a Time Projection Chamber-like detector also plays the role of the target. In the present case the gas was Helium, to perform (α, α') scattering. This setup allows for a 4π solid angle coverage, and also for a low alpha energy measurement : there is no straggling as in a (solid target+external detector) setup.

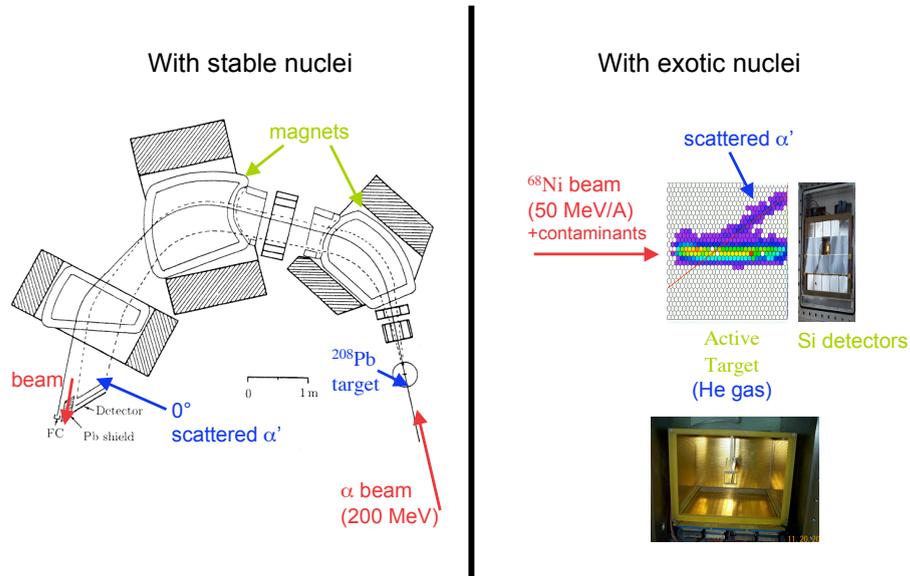


FIG. 9 – Schematics of the method of measurement of GR in the case of stable (left) and unstable (right) nuclei

2.5 Properties related to GR : why to study them ?

GR measurements allow to probe properties of nuclei and nuclear matter, which is useful in the case of the investigation of the structure of neutron-rich nuclei. Fig. 6 displays the selectivity of these properties with respect to the (L,S,T,T₃) ordering of GR. For instance the ISGMR mode can be related to the incompressibility of nuclear matter, allowing to constrain the equation of state, used in supernovae collapse. However the link between the compression of a finite nuclear system (the nucleus) and the infinite one (nuclear matter) is not straightforward and the determination of the incompressibility modulus has been debated these last decades. It seems that a microscopic method should be used, but on which data ? The incompressibility deduced from the GMR measurement in ²⁰⁸Pb is different from the one extracted from Sn measurements. Moreover, relativistic models provide a different value that non relativistic ones. Therefore the method should be rethought, and measurements on isotopic chains of the ISGMR extended. This points out an historical analogy between magicity and incompressibility : with the advent of data on neutron-rich and unstable nuclei, incompressibility has to be considered from a more general point of view, than the one developed on stable nuclei only, like in the case of magicity.

The neutron skin can also be measured from several giant resonances, such as the IVGDR. Examples will be given in chapter 4. The IVGDR is sensitive to the oscillation of the neutron skin

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with respect to the core. The measurement of the corresponding angular distributions using direct reactions (about few tens of MeV/A), allows to probe this effect through a DWBA analysis.

Another example on Fig. 6 is the Giant Pairing vibration. It corresponds to a giant resonance in the pairing channel and will not be detailed here. It can be obtained by two particles transfer reactions but has not been detected so far. Other examples are the effective mass through the ISGQR, squeezing and vortex modes through the ISGDR.

3 Description of giant resonances

In this section state-of-the-art models for the description of GR are presented. Key ideas will be given, but not the full demonstration of these models. These derivations can be found in numerous references (including those given at the end of this course). On the contrary, we would like to focus on what may be missing in the literature in order to grasp the main ideas of describing giant resonances microscopically.

It should be noted that the microscopic approach is required in order to provide a global understanding of the GR phenomenon, both in stable and exotic nuclei. For instance, so-called pygmy modes are naturally predicted by microscopic models, without using any additional parameter than the functional already designed on stable nuclei.

3.1 Microscopic description of GR

3.1.1 Method

As explained in the previous section, GR are well described by the superposition of particle-holes (ph) pairs. An approach which exactly relies on this feature belongs to the mean field model. The point is to provide a GR description, which only depends on the energy density functional (EDF) $E[\rho]$. Starting from the EDF, which can be obtained from a nucleon-nucleon interaction in a non-relativistic framework (Skyrme, Gogny ...) or from a Lagrangian in the relativistic one (DDME2, PKA1, ...), one can write the action of the system :

$$S = \int_{t_1}^{t_2} dt \int d\vec{r} (i\hbar \Psi^*(\vec{r}, t) \partial_t \Psi(\vec{r}, t) - E[\rho]) \quad (26)$$

The minimisation of Eq. (26) leads to the Schrödinger (or Dirac in the relativistic case) equation. However, due to the many body problem, an approximation has to be performed in order to solve the motion equation. Restricting the Hilbert space to perform the minimisation of S on Slater determinants only, is a way to perform such an approximation. In practice, the action S is rewritten using this constraint :

$$S = i\hbar \sum_{i=1}^A \int_{t_1}^{t_2} dt \int d\vec{r} (\varphi_i^*(\vec{r}, t) \partial_t \varphi_i(\vec{r}, t) - E[\rho]) \quad (27)$$

where φ are the single particle states. The least action principle ($\delta S=0$) leads in this case to the motion equation :

$$i\hbar \partial_t \varphi_i = \frac{\delta E[\rho]}{\delta \rho} \varphi_i \hat{=} h \varphi_i \quad (28)$$

This equation is called, for historical reasons, the Time Dependent Hartree Fock (TDHF) equation. It should be noted that the mean field $h \hat{=} \frac{\delta E[\rho]}{\delta \rho}$ is completely determined from the EDF.

Looking to the small amplitude limit of equation 28, that is harmonically perturbing the ground state, the linear response theory is obtained, driven by the equation :

$$\Pi = \Pi_0 + \Pi_0 \frac{\delta h}{\delta \rho} \Pi \quad (29)$$

The energies and transition probabilities from the ground state to the excited state (Fig. 1) can be extracted from the perturbed response function Π . Eq. (29) shows that Π is calculated from the unperturbed one Π_0 and the residual interaction $\frac{\delta h}{\delta \rho} = \frac{\delta^2 E[\rho]}{\delta \rho^2}$. Π_0 is itself built on the single-particle states obtained from the mean field description of the ground state (static limit of Eq. (28)). Therefore the EDF $E[\rho]$ determines both the residual interaction $\frac{\delta h}{\delta \rho}$ and the mean field (Π_0) and hence the excited energy spectrum, obtained from the response Π . No additional parameter is needed. The linear response theory is also called RPA (Random Phase Approximation) for historical reasons.

3.1.2 Achievements

Although the RPA equations were known since several decades, the achievement of RPA models having the EDF as only parameter, has known recent progress. The first RPA model with the EDF as only parameter was developed in the 70's. Figure 10 shows the quadrupolar response of the spherical doubly magic ^{208}Pb nuclei. On the X axis is the excitation energy of the nucleus, and the Y axis is the transition probability from the ground state towards the considered excited state. The large ISGQR peak is clearly visible around $E^*=11$ MeV. The GQR is therefore microscopically predicted and in agreement with the ISGQR data on ^{208}Pb . Doubly magic nuclei are spherical, without any pairing effects, and spherical RPA models are adapted to describe them. These specific nuclei would correspond to dots on the (Z,N) nuclear chart (Fig. 11)

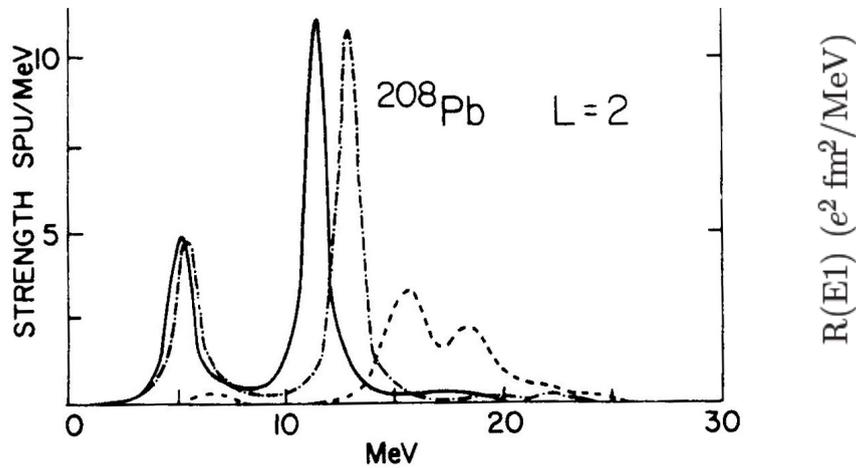


FIG. 10 – Linear response theory (RPA) for the 2^+ response function in ^{208}Pb . The solid and the dotted dashed lines correspond to the use of two different functionals. The dashed line correspond to the unperturbed response function. From G.F. Bertsch and S.F. Tsai, Phys. Rep. 18 (1975) 125

It is only a decade ago that such a description has been extended to spherical nuclei with pairing (Quasiparticle-RPA, QRPA). This allows to study isotopic chains, with a magic proton or neutron number to preserve sphericity. On the nuclear chart this would correspond to a line (Fig 11) and therefore allows to study the evolution of GR from stability to exotic nuclei. Why having waited so long since the advent of the RPA ? Two major practical reasons can be invoked : first the advent of the measurements on exotic nuclei in the 90's opened data sets involving isotopic chains.

Models had to be developed to interpret this data. Second, the computing power to run such QRPA codes based on a single EDF was available from the 90's.

Finally, a couple of years ago, the first models able to microscopically predict the GR, in a superfluid deformed nuclei, and based on a single EDF were built. Fig. 12 shows the IVGDR in ^{20}Ne calculated with a deformed relativistic QRPA model. Such models have also recently been developed with the Gogny and Skyrme functionnals. This achievement allows in principle to study the whole nuclear chart on even-even nuclei (Fig. 11), namely to understand the evolution of GR in their full generality. Many works in that direction are currently under progress and this is an exciting time for GR in exotic nuclei.

3.2 Intuitive picture of a GR

3.2.1 Transition densities

It is important to have a clear idea of the present phenomenon under study : what is happening to the nucleus when it reaches an excited state corresponding to a GR, and what is the link between

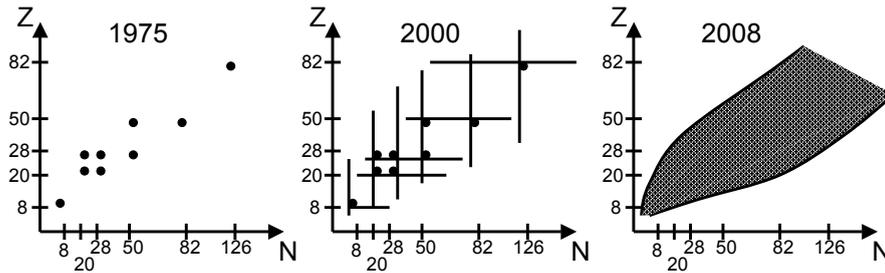


FIG. 11 – Nuclei in which the GR can be described microscopically, with the spherical RPA (left), the spherical QRPA (center) and the deformed QRPA (right)

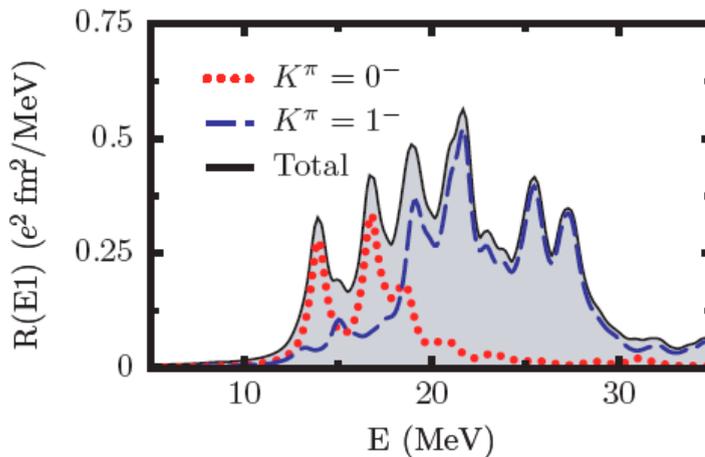


FIG. 12 – ^{20}Ne IVGDR response calculated with the relativistic QRPA with an axial deformation. $K=0$ corresponds to a translation along the symmetry axis, and $K=1$ in the perpendicular direction. From D. Peña Arteaga and P. Ring, Phys. Rev. C77 (2008) 034317

this picture and the microscopic theory ? This question is not often raised in textbooks. The RPA consists of smoothly perturbing the nucleus' ground state, that is harmonically. Due to the harmonic perturbation, the nucleus densities undergo oscillations. This is the vibrational mode and the densities therefore reads (considering the L=0 Giant Monopole case for the sake of simplicity) :

$$\rho(r, t) = \rho(r) + \delta\rho(r)\text{Cos}(\omega t) \quad (30)$$

One can therefore imagine the nucleus vibrating around its ground state equilibrium position, with an $\omega=E/\hbar$ pulsation, where E is the excitation energy corresponding to the GR. Let us now interpret the amplitude of the vibration, driven by $\delta\rho$ (Eq. 30). This quantity is usually called the transition density. Fig. 13 displays the picture at the time $t=0$, which here corresponds to the maximum of the elongation. A typical transition density $\delta\rho$ for the GMR is displayed on this figure (below $\delta\rho$). The use of Eq. 30 at $t=0$ leads to a density of the nucleus which is depleted in the inner part, and extended to a larger area as in Fig. 13. This corresponds to a dilatation of the nucleus (right panel of Fig. 13). As a function of time, the GMR corresponds to the so-called breathing mode. It should be noted that typical values of $\delta\rho$ are significantly smaller than the one of ρ because the vibration is harmonic (small amplitude limit). Therefore one should imagine a GR like a small amplitude vibration around the ground state, at high frequency (high E), implying most of the nucleons (collective mode). This last point on collectivity will be illustrated below. Note also that the present example is given in the case of the L=0 radial mode. In the case of other multiplicities, the interpretation can be generalised by introducing spherical harmonics in the equations above.

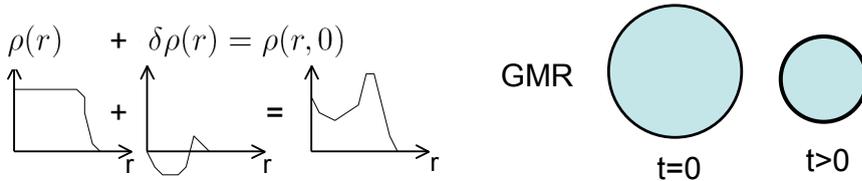


FIG. 13 – Left : Behaviour of the nuclei density at $t=0$ in the GMR case. Right : picture of the GMR as a function of time

3.2.2 Microscopic insight

Therefore to describe a GR, both its energy and transition densities are needed. These quantities are provided by the RPA response function since the transition probability from the ground state to the excited state are derived from the transition density by :

$$\left| \langle GR | \hat{Q}_L | g.s. \rangle \right|^2 = \left| \int \delta\rho(\vec{r}) \hat{Q}_L(\vec{r}) d\vec{r} \right|^2 \quad (31)$$

where \hat{Q}_L is the transition operator of Eq. 14 and 17.

More precisely, the RPA model provides the following form for the transition density :

$$\delta\rho(r) = \sum_{mi} (X_{mi} - Y_{mi}) \varphi_i^*(r) \varphi_m(r) \quad (32)$$

where φ_i and φ_m are the particle and the hole wave function, respectively, obtained from a mean field Hartree-Fock calculation based on the same EDF than the RPA one. Equation 32 relates the pictorial interpretation of the GR to the microscopic approach. Namely, it displays the collective contribution of particle-hole pairs to the excitations, built on the mean field. The X and Y coefficients are solutions of the RPA equation : they allow to calculate the transition densities and therefore to predict the transition probability from the ground state to the excited state (Eq. 31, Fig. 10,12). It should be noted that transition densities are usually the largest around the spatial surface of the nucleus (usually for $L>0$). But this doesn't mean that only valence nucleon participate : all nucleons which have a spatial component contributing around the nucleus surface are involved, as stated by Eq. 32. Moreover, there can also be smaller contributions to the transition density from the inner spatial part of the nucleus, as we will see in examples given in the next chapter. Therefore GR are collective modes because they involve a large number of ph pairs.

3.2.3 How to interpret a giant resonance ?

The transition densities allow to have an insight in the GR mode. Equation 31 shows that $\delta\rho$ can also be interpreted as the spatial contribution of the nucleus to the excitation, corresponding to the given L_R , S_R values of the GR. Also for the isospin, the neutron and proton transition densities should be considered. To build the IS or the IV GR, the operator (Eq. 31) will make $\delta\rho_n$ and $\delta\rho_p$ added or subtracted, respectively (see previous chapter). If the neutron and proton transition densities have the same sign (or are in phase, i.e. their maximum are spatially close), their sum will be important, but their difference will be small. Therefore such a state will have a large IS contribution to the GR, and a small IV one. On the contrary, if $\delta\rho_n$ and $\delta\rho_p$ are of opposite sign (or out of phase), their sum will be small, whereas their difference will be important, and these state will be of more IV than IS type. The neutron and proton transition densities are sufficient to interpret an excitation mode, without invoking the IS or IV nature of the excitation. The IS/IV paradigm comes from the experimental probe, as we have seen in the previous section. But to understand the excitation, the neutron/proton paradigm should be considered, instead of the IS/IV one.

All demonstrations of this section can be found in the bibliography. What is original here (and we hope useful) lies in the interpretation of GR. Table 1 summarises the three independent quantities that characterise a GR, and more generally a nuclear excitation : energy, strength and collectivity.

Property of the GR	Microscopic quantity	Macroscopic picture
Energy	$\omega=E/\hbar$	Frequency of the spatial oscillation
Strength	cross section σ	Amplitude of the oscillation
Collectivity	RPA Amplitudes X_{mi} and Y_{mi}	Number of nucleons involved in the oscillation

TAB. 1 – The three main properties of a nuclear excitation and their interpretation

3.3 Specific features of the RPA vs. other models

3.3.1 The EDF and the RPA

It may be useful to characterise the RPA compared to other nuclear structure models. A crucial point is that the RPA is based on the same EDF used at the Hartree-Fock level. Hohenberg and Kohn have proven a theorem which shows the existence of such a functionnal in many body systems. Therefore EDF are a legitimate starting point. How to derive this functionnal is still an open question. Besides in nuclear physics, the coefficient of the functionnal are determined by a fit of the results of the Hartree-Fock calculations on experimental masses and radii. Therefore these EDF somewhat effectively include effects beyond the mean field. Finally, the mean-field approach breaks symmetries (translation, rotation, particle number), which has some implications on using an EDF depending on the density calculated in the center of mass of the nucleus.

3.3.2 RPA vs. the shell model

Two important families of models in nuclear physics are mean field type (HF, RPA and beyond) and shell model calculations. In the first one, part of the interaction is neglected, whereas in the second, some of the orbitals are neglected. In the case of GR, which are high energy collective modes, HF-RPA models are especially well adapted : orbitals far from the Fermi level are taken into account, and the harmonic type of the excitation makes is adapted for first order approximation of the interaction. Also RPA provides transition densities, which gives a crucial insight to describe the excitation. The simplicity of the HF+RPA method also allows to easily relate the EDF (interaction) to observables and to study dedicated effects (spin orbit, tensor, ...).

On the contrary, in the shell model approach, the core vs. valence division of space hinders predictions of densities. The restriction of the valence space is not well adapted to high-energy and collective excitations such as GR, which imply many various orbitals. On the other hand shell model calculations are much more accurate to predict low-lying states, whereas the RPA predictions are less precise at low energies (because for instance of pairing effects). This description of low energy states can be much improved in the mean-field type family by the use of the Generator Coordinate Method (GCM, similar to 5DCH) : this method mixes HF solutions with various deformations in order to better describe low-lying states. A very good agreement with the data can be obtained. On the contrary, the RPA mixes the 1p 1h configurations of the HF solution and is more adapted for the high lying collective state such as GR. Moreover the RPA model is convenient to describe charge exchange GR ($T_{R=1}, T_{3R}=\pm 1$) on the same footing, by considering a particle as a proton and the hole as a neutron for instance, in Eq. 32.

4 Giant Resonances in exotic nuclei

In this chapter, several results on GR are discussed, taking advantage of the data on exotic nuclei. The aim is to understand these excitation modes in their full generality, from stable to exotic nuclei. A possible specificity of GR in exotic nuclei is the appearance of modes in the low energy tail of the GR, corresponding to a excitation lower in energy and lower in transition strength compared to the GR. Therefore, in these soft modes, the nucleus both vibrates at a lower frequency and with a smaller amplitude than the GR. The nature of these low-lying modes have been investigated and their collectivity is currently investigated.

So far, the experimental status of GR in unstable nuclei is a recent field of study, currently expanding : the IVGDR has been measured in neutron rich Oxygen and Tin isotopes at GSI, in ^{26}Ne at Riken, and the IS GMR and GQR have been measured in ^{56}Ni at GANIL.

Fig. 14 summarises the GR ordering in the case of exotic modes. Exotic modes can occur in exotic nuclei, but also in stable nuclei for some of them. Namely the GPV, the vortex mode and the scissors modes can be studied in stable nuclei.

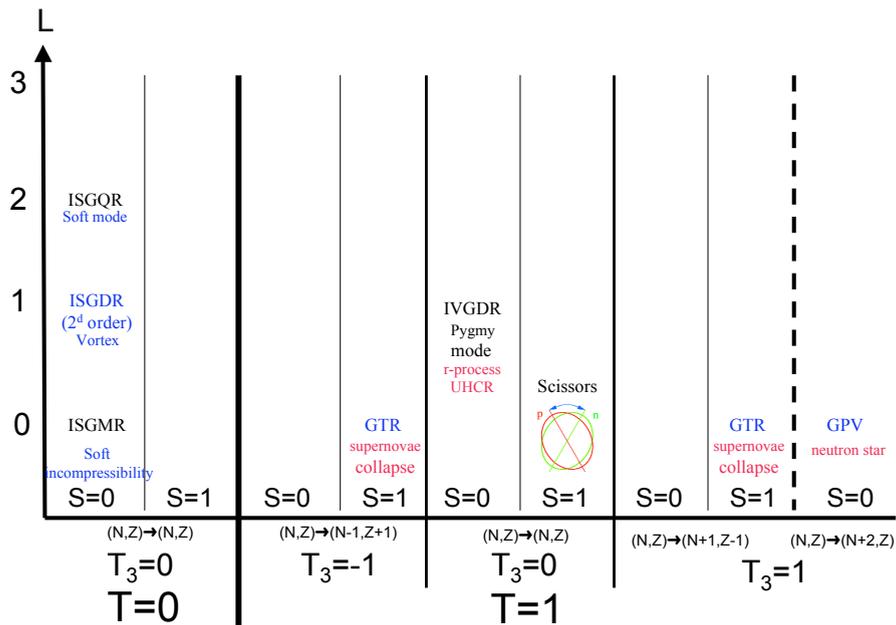


FIG. 14 – Spin, isospin and angular momentum ordering of exotic modes

Fig. 15 shows the IS GMR evolution from the doubly magic ^{40}Ca nucleus, to the much more neutron rich ^{60}Ca one, calculated in the RPA framework. The appearance of a low energy mode could be interpreted as a soft compression mode of a neutrons skin around a neutron+proton core, but the inspection of the transition density would be required in order to validate this assertion. Fig. 15 also shows the evolution of the ISGQR in very neutron-rich Ca nucleus, where there is no appearance of a soft mode, but rather a significant global shift of the centroid of the GQR towards lower energy. The soft GMR and GQR modes have not been measured yet, and these predictions open some relevant experimental perspectives.

4.1 The pygmy dipole mode

On the contrary, there is data on the soft IV dipole modes. Fig. 16 shows the evolution of the IVGDR response in oxygen isotopes, calculated within the relativistic QRPA framework : an increase of the low lying strength around 10 MeV is observed. The right panel shows the IVGDR response calculated in ^{132}Sn , also predicting low lying strength, which has recently been measured. Transition densities are helpful in order to interpret this mode. The top right insert of Fig. 16 shows the neutron and proton transition densities for the excitation at 15.3 MeV, corresponding to the GDR : $\delta\rho_n$ and $\delta\rho_p$ are opposite. Therefore adding them (T=0 IS response) will give almost no strength, whereas subtracting them (T=1 IV response) will provide an important response. That is why the GDR is contributing to the IV strength and not the IS, at first order.

However, the transition densities of the state located at 7.8 MeV, are in phase in the inner part of the nucleus, and out of phase at the surface. Therefore, this mode could be interpreted as neutron and proton in phase in the inner core, whereas the neutron skin is oscillating around it. Are these low-lying states collective ? On this purpose, the X and Y coefficients of Eq. 32 should be studied. A important number of significant X and Y coefficients contributing to the transition density makes the excitation collective, whereas if only few of them contribute, it is not collective. In the case of heavy nuclei like ^{132}Sn both non-relativistic (Skyrme, Gogny) and relativistic models predict a collective low E1-strength that shall be called pygmy mode. However, in the case of light nuclei, non-relativistic models predict non-collective low-lying strength (for instance in Oxygen

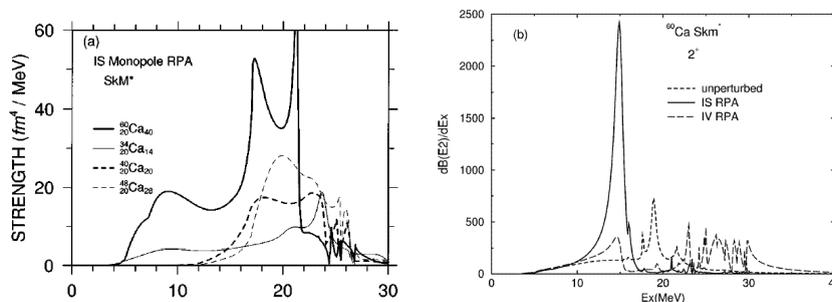


FIG. 15 – Left : Evolution of the ISGMR response in neutron-rich Ca isotopes Right : Same for the ISGQR response. From H. Sagawa and H. Esbensen, Nucl. Phys. A 693, 448 (2001)

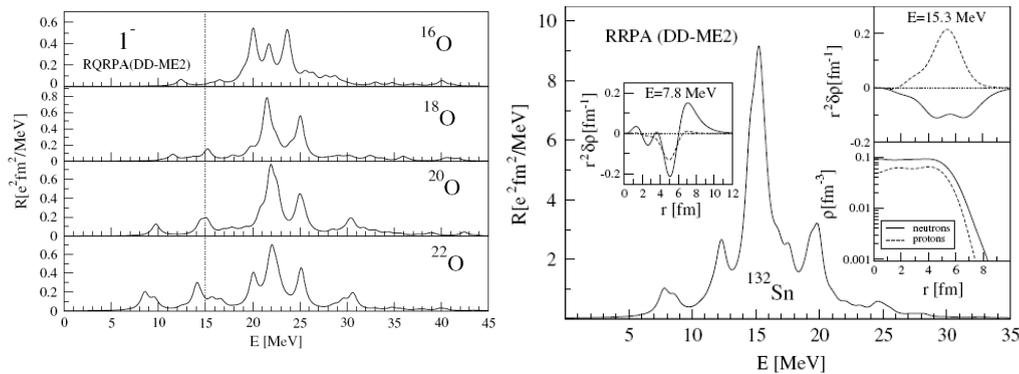


FIG. 16 – Left : Evolution of the IV dipole response in oxygen isotopes Right : IV dipole response in ^{132}Sn . From N. Paar et al., Phys. Rev. C 67, 034312 (2003)

isotopes), whereas relativistic one still predict a collective mode.

It could be inferred that pygmy modes are a specificity of unstable nuclei. However recent work on stable nuclei showed that these low-lying modes can also be found in some neutron-rich stable nuclei : low-lying modes are characteristic of neutron-rich nuclei, rather than stable/unstable ones. Stable/unstable refers to the weak interaction, and has no deep link with nuclear structure. Fig. 17 shows the experimental low-lying excitation spectrum of ^{140}Ce stable nucleus, obtained with an isoscalar probe (alpha, see the kite relationship) and by an IV one (real gamma excitation) : real gamma excitation excites the protons, and therefore induces a relative proton vs. neutrons motion in the nuclei. The high energy part of the spectrum ($E^*>6.5$ MeV) is excited by the IV probe only whereas the low-lying part is more of IS and IV nature. The interpretation comes from a similar behavior than in ^{132}Sn . Namely, the neutron and proton in the core are in phase (see the transition densities of the low-lying mode on Fig. 16), providing the IS response, whereas the neutron skin translates around this core, providing the IV part. At higher energies, the strength becomes part of the low energy tail of the GDR, that is of pure IV part.

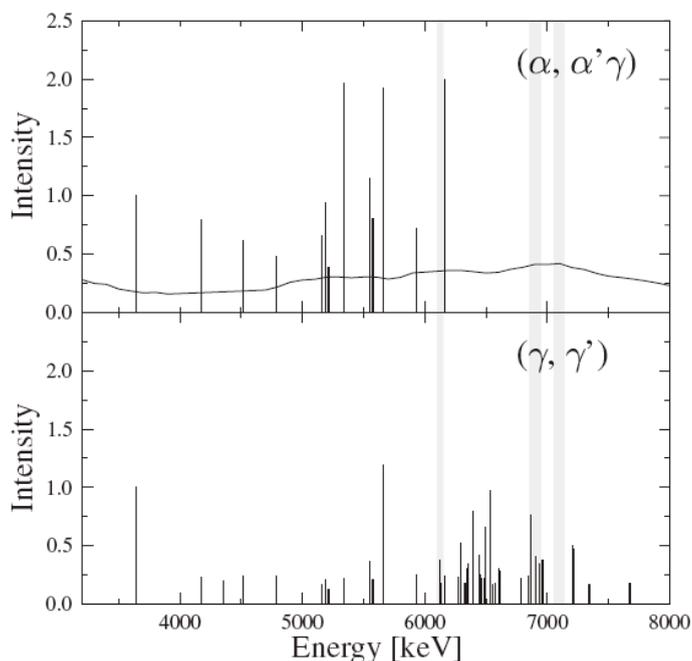


FIG. 17 – Response of the IS excitation in ^{140}Ce (top) and the IV one (bottom). From D. Savran et al, Phys. Rev. Lett. 97, 172502 (2006)

As mentioned in the previous section, a probe used to measure a GR is aimed to be selective for $T=0$ (IS) or $T=1$ (IV). On the other hand, the interpretation of the GR and the pygmy benefits from neutron and proton transition densities. One should therefore be able to switch from the experimental representation (IS/IV) to the theoretical one (n/p), which is one purpose of this course.

The question of the existence of a proton pygmy mode is raised in a proton-rich nuclei. However, due to the Coulomb barrier, it is more difficult for proton to form a skin at the surface of the nucleus. Nevertheless Fig. 18 shows the IV GDR response of ^{32}Ar , calculated with the relativistic

QRPA. A proton pygmy mode is predicted, and transition densities shows that a proton skin is oscillating around a neutron+proton core. Other nuclei, like ^{46}Fe are also predicted with a proton pygmy mode. Experiments have been recently performed in order to detect this predicted mode.

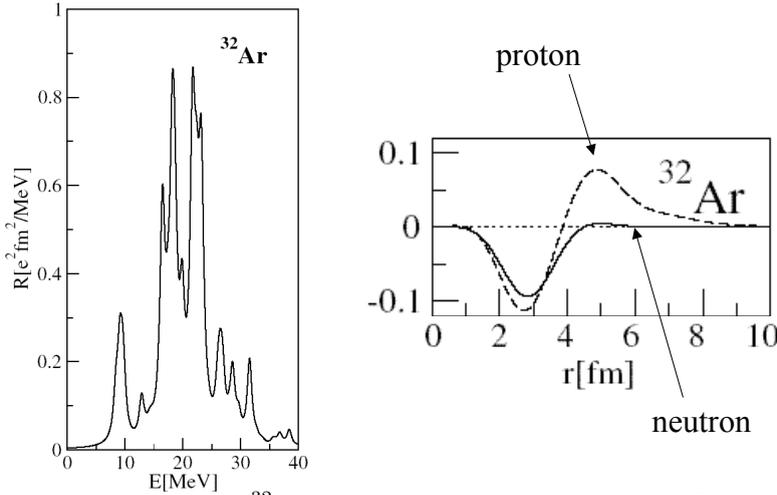


FIG. 18 – IV dipole response in ^{32}Ar calculated with relativistic QRPA. The transition densities of the mode around 8 MeV is displayed on the right. From N. Paar et al., Phys. Rev. Lett. 94, 182501 (2005)

Another issue is the impact of deformation on the pygmy mode, related to the fate of GR in extremely neutron-rich nuclei. On this purpose, it is necessary to use a QRPA approach for deformed nuclei. In these nuclei with a large neutron skin, the spin-orbit potential is expected to be weaker (as a derivative of the density) and this is one cause of magic number evolution. Another effect of this weakening is that semi-magic nuclei like the $Z=50$ Tin chain may become deformed : in relativistic QRPA predictions, Sn isotopes start to be deformed at $A=142$. Figure 19 shows the pygmy strength as a function of A . It should increase linearly with the neutron number, which is expected for spherical nuclei. However in the case of deformed nuclei, the increase is quenched : the neutron skin is thinner on one axis and thicker on the perpendicular one, with respect to the skin width in spherical nuclei (Fig. 19). The increase of the skin on one axis in deformed nuclei is not sufficient to compensate the decrease of the skin on the other axis as far as the pygmy mode is concerned.

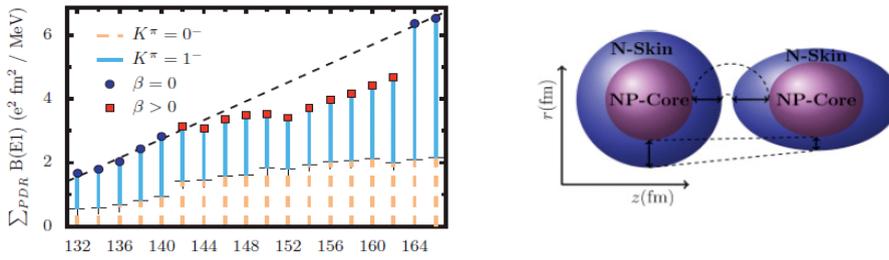


FIG. 19 – Low energy IV dipole strength in Sn isotopes, decomposed from the axis of symmetry ($K=0$) and the one perpendicular ($K=1$). From D. Peña Arteaga, E. Khan, and P. Ring, Phys. Rev. C 79, 034311 (2009)

4.2 Measurement of the neutron skin with GR

Another illustration on how GR are useful for nuclear structure studies is the measurement of the neutron skin with GR. This can be achieved using three different methods. First the IVGDR mode is sensitive to this skin. Comparing the measured angular distributions (for instance by electromagnetic excitation), with a calculated one allows to validate the transition densities used to generate the transition potential input in the DWBA calculations. Another method uses the IVSGDR (Fig. 6) : it can be shown that the sumrule (total strength of the response) can be expressed from the neutron radii minus the proton one. Since the GR exhausts the vast majority of the strength, measuring it provides the sumrule.

A third way to measure neutron skin with GR is related to symmetries. SU(4) Wigner supermultiplets are labelled by S and T values. The member of these multiplets are degenerated if the symmetry is respected. Translated into giant resonances, this means that the IAR and the GTR states built from the same initial state should be degenerated in energy. Since the SU(4) Wigner symmetry is explicitly broken by the L.S term (which depend on S), the degeneracy raising between the IAR and the GTR should be driven by the magnitude of the spin orbit term. On the other hand, the neutron skin weakens the L.S term. There should therefore be a correlation between the IAR/GTR degeneracy raising and the neutron skin. Fig. 20 depicts this effects on Sn isotopes : the more neutron-rich nucleus the weaker is the degeneracy raising. This is due to the weakening of the L.S. term in neutron-rich nuclei, due to the more diffuse nucleus surface. Therefore the measurement of the IAS and IAR centroids provide information on the neutron skin.

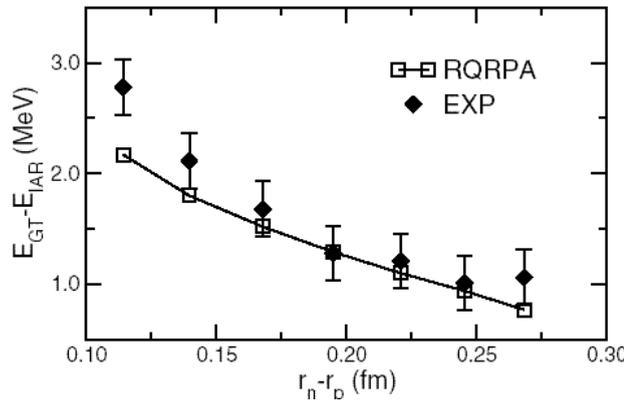


FIG. 20 – Evolution of the energy shift between the GTR and IAR centroids in the Sn isotopic chain. From D. Vretenar et al., Phys. Rev. Lett. 91, 262502 (2003)

It should be mentioned that several other exotic modes are predicted. They can be found in the bibliography. For instance the L=1, T=0 mode is made of a squeezing mode at high energy and a vortex mode at lower energy.

5 Conclusion

Giant Resonances are collective high-energy modes with a large cross section, providing information on nuclear structure (neutron skin, incompressibility, etc.). It should be reminded that they are one of the easiest excitation to provoke in the nucleus. The neutron excess breaks the isospin symmetry of these transitions, which are therefore ordered by their (L,S,T,T3) values.

In exotic nuclei, new modes of excitation have appeared such as the pygmy mode. However this low-lying modes also occur in some neutron-rich stable nuclei. The effect of the deformation on this modes is currently investigated. Many other modes are predicted and remain to be detected, such as the proton pygmy. Experimentally exotic nuclei are useful in order to build isotopic chain to study the evolution of GR with neutron excess.

Many other aspects of GR have not been mentioned in this course : Giant Pairing Vibrations, deformation and GR, fine structure, width, decay, hot GR, multiphonon, tools (macroscopic models, sumrules). GR play also a crucial role in at least four astrophysical sites : in the r-process (for the synthesis of elements), in the collapse of supernovae (in electron capture or neutrino emission), in the propagation of ultra high energy cosmic rays (by stripping 10^{19} eV nuclei so they arrive as proton on Earth), and in the crust of neutron star (affecting their cooling properties).

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