

Introduction - Strong interaction in the nuclear medium: new trends

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ABSTRACT

Recent achievements in nuclear forces theory open new perspectives for the next decade of low energy nuclear physics, bringing together people from very different communities. Although many developments remain to be done, the possibility to directly use QCD to describe nuclear system is a major challenge that is within reach. In this introduction to the 2009 International Joliot-Curie School (EJC2009), new trends in the strong nuclear interaction are summarized starting from quarks and ending with finite or infinite nuclear systems. At different energy scales, selected new concepts and ideas have been discussed in a rather simple way. Recent advances in the theory of nuclear forces, thanks to chiral perturbation and effective field theories, have led to a new generation of strong nuclear interaction particularly suited to low energy nuclear physics. The interesting aspects of new interactions compared to conventional forces are underlined. Recent achievements in ab initio theories that directly start from the bare nucleon-nucleon interaction and their key role to understand the three-body force are illustrated. Finally, future perspectives for standard nuclear physics theories, namely Shell Model and Energy Density Functional, are discussed.

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I. INTRODUCTION

Two fundamental questions of present days nuclear physics are (i) How to understand the very rich structure of atomic nuclei in terms of interaction between nucleons? (ii) How to relate the strong nuclear interaction to the underlying fundamental Chromodynamics (QCD) that governs the physics of quarks and gluons. These two questions illustrate the many facets of nucleon-nucleon interactions (see Fig. 1). "Low energy" nuclear scientists mainly address (i) and often consider the strong interaction as a "fundamental" interaction and nucleons as elementary (often point-like) particles. From the "High energy" nuclear physics point of view, nucleons, being formed by quarks and gluons, can obviously not be considered as elementary particles and the strong interaction itself should be understood from the more fundamental Standard Model. The nuclear force is at the crossroad of these two visions. Recently, important progresses have been made in the

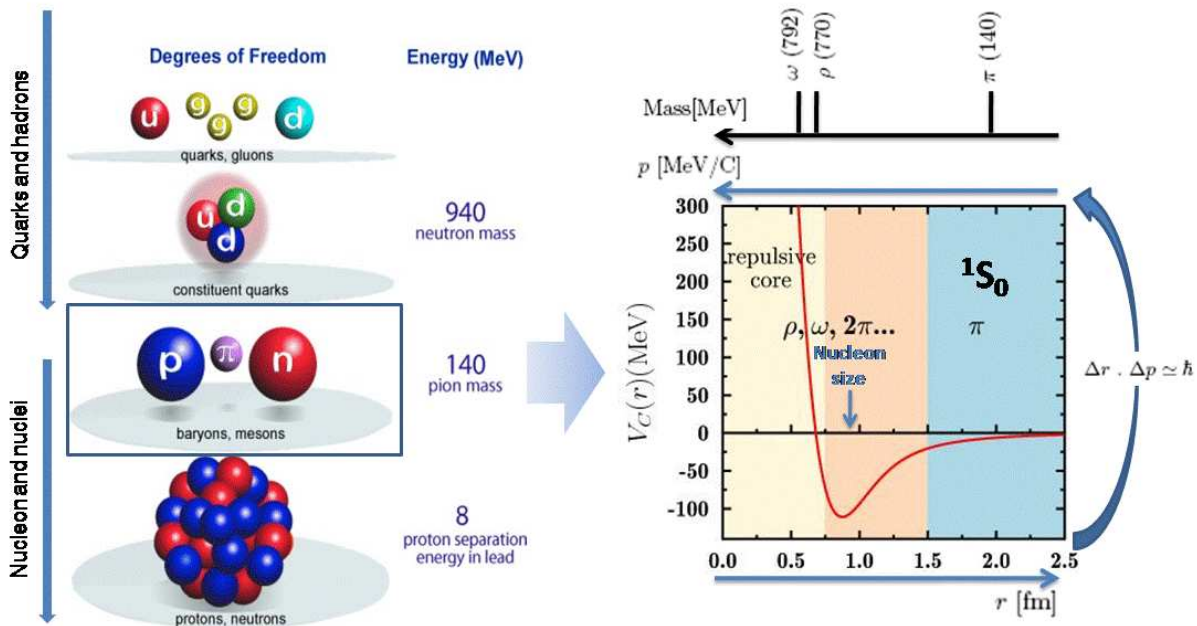


FIG. 1: (Color online) Left: The strong nuclear interaction is at the crossing point between "High energy" nuclear physics dedicated to quarks and how they organize into nucleons and "Low energy" nuclear physics dedicated to nucleons and how they organize into nuclei. Right: Typical example of central nuclear interaction as a function of relative distance between the two nucleons. The relative distance can directly be related to the energy range involved in the interaction. Using the Heisenberg uncertainty, the typical momentum exchange at a given r is given by $p \propto \hbar/(2r)$ while the scale of energy involved is proportional to the square of p . A meson is involved in the interaction if its mass is in the energy scale associated to a given r . For instance, the π is the only meson that contributes to the long-range part of the interaction. At shorter relative distance, more and more mesons contribute to the interaction.

understanding of the nuclear strong interaction directly from QCD. Conjointly, new experimental results have pointed out our lack of knowledge of the interaction in the nuclear medium. Selected issues directly related to the strong interaction and its effects are listed below:

- Lattice QCD is a promising tool to calculate ab initio hadrons and mesons properties on a mesh. Recently, first applications of Lattice QCD have been made possible [1].
- The use of Effective Field Theory in combination with Chiral perturbation theory provides a systematic and elegant framework to construct the nuclear forces for QCD Lagrangian [2].

- New "soft" interactions, like the so-called V_{lowk} , deduce from conventional or more recently similarity renormalization group technique have been proposed [3]. These interactions open new perspectives by getting rid of the hard core while keeping the low energy nuclear physics unchanged.
- With recent soft- and hardware progresses, systematic ab initio calculations for light nuclei starting directly from the bare nucleon-nucleon interaction are now possible.
- Comparison of these ab initio theory with high precision measurements have confirmed and precised the importance of three-body forces.
- Specific components of the force (spin-orbit, tensor interaction, 3-body) have been largely debated in the low energy nuclear physics community [4] underlying the necessity to use less phenomenological approaches.
- Standard theories of low energy nuclear physics, like the nuclear Shell Model [5] or the Energy Density Functional [6], that generally start from interactions directly adjusted to reproduce data, are now being revisited using a "bottom-up" approach, i.e. starting from the QCD ingredients and ending with the nuclei properties.

The aim of the 2009 Joliot-Curie international School is to present new advances in the comprehension and use of the nuclear interaction. It covers not only the physics of quarks, nucleons, nuclei and hypernuclei as well as stars. These subjects are certainly far too broad to expect that a single physicist that is often specialized in much narrower fields. However, it is essential that we learn enough from each others to be able to communicate and cross fertilize.

The list of new aspects quoted above will be extensively discussed in the different lectures of this school. The objectives of the present introduction are (i) To provide a roadmap for the school, that, I hope, will help the reader to make connection between different lectures (ii) To provide simple summary on the ongoing research. The price to pay is to use shortcuts and simplified pictures that might appear too schematic for specialists. People interested in detailed discussions can directly read dedicated lectures of this school. (iii) to emphasize the new aspects that will impact the future of nuclear physics. Being from the Low energy branch of nuclear physics, the overview given below is highly personal and obviously subjective. The present lecture is organized as follows: First, basic ingredients of conventional nuclear forces are recalled. Then, new perspectives offered by the Effective Field Theory and Chiral Perturbation Theory are discussed. Last, implications for the description and understanding of nuclei are underlined.

II. CONVENTIONAL NUCLEAR FORCES

The aim of the present section is not to give a broad review on nuclear interaction that have been proposed in the past. An excellent review can be found for instance in [7, 8] as well as a complete description of conventional nuclear forces. Here, I recall some basic notions that will be helpful for the present discussion.

Conventional forces generally start from semi-phenomenological parametrisations that could be written schematically as:

$$V_{NN} = v_{NN}^{\text{EM}} + v_{NN}^{\pi} + v_{NN}^{\text{Rep}}. \quad (1)$$

The first part denotes the Electromagnetic (EM) contribution. v_{NN}^{π} stands for the One-Pion Exchange potential responsible for the long-range attraction. v_{NN}^{Rep} corresponds to the strong repulsive short range interaction. It is generally taken as a phenomenological parametrization. Altogether, most recent nuclear interactions have around 30 parameters that are adjusted on experimental nucleon-nucleon scattering cross sections.

A. Classification of nuclear forces

In figure 1, the central part of the interaction in the 1S_0 channel is displayed. For non experts, the meaning of this denomination is recalled here. The properties of the nuclear interaction can be studied by introducing the eigenstates of the associated two-body problem, denoted here by $\Psi_{NN} = \Psi(\mathbf{r}_1\sigma_1\tau_1; \mathbf{r}_2\sigma_2\tau_2)$ where σ (\uparrow or \downarrow) and τ (n or p) denote respectively the spin and isospin quantum numbers. By coupling the two spins on one side and the two isospin on the other side, the total spin S and isospin T and associated projections M_S and T_z are generally introduced. As an illustration, the different components of the projections are given in figure 2. Since the nuclear

Spin		Isospin		
$ S M_S\rangle$		$ T T_z\rangle$		
$ 0 0\rangle$	$ \uparrow\downarrow\rangle - \downarrow\uparrow\rangle$	$ 0 0\rangle$	$ \text{np}\rangle - \text{pn}\rangle$	singlet
$ 1 1\rangle$	$ \uparrow\uparrow\rangle$	$ 1 1\rangle$	$ \text{nn}\rangle$	triplet
$ 1 0\rangle$	$ \uparrow\downarrow\rangle + \downarrow\uparrow\rangle$	$ 1 0\rangle$	$ \text{np}\rangle + \text{pn}\rangle$	
$ 1 -1\rangle$	$ \downarrow\downarrow\rangle$	$ 1 -1\rangle$	$ \text{pp}\rangle$	

FIG. 2: (Color online) Illustration of the different projection of total spin (left) and total isospin (right)

interaction is spherical symmetric, the orbital angular momentum L that couples to the spin S as $J = L + S$ can be introduced, where J is the total angular momentum. Then, a given channel of the interaction is denoted by:

$$^{2S+1}[L]_J \quad \text{with} \quad [L = 0, 1, 2, \dots] = S, P, D, \dots$$

Some of the most common channels and their content in terms of isospin and total angular momentum are given in figure 3.






S	T	n/p state	$J = 0$	$J = 1$	$J = 2$
0	1	nn, pp, (np + pn)	1S_0		1D_2
1	0	(np - pn)		$^3S_1, ^3D_1$	
0	0	(np - pn)		1P_1	
1	1	nn, pp, (np + pn)	3P_0	3P_1	$^3P_2, ^3F_2$

FIG. 3: (Color online) Illustration of the most common channels generally used for the nuclear interaction. The forbidden channels due to quantum selection rules are also indicated.

B. Scattering theory and Phase-shift analysis

The best way to characterize the interaction between two particles is to make them collide and detect the product of the reaction (see illustration in Fig 4 (left)). In scattering theory, the cross section is best studied by introducing the eigenstates of the two-body problem, denoted by Ψ_{scat} . Since detectors are positioned at almost infinite distance, the probability to detect a particle at

a given position is only sensitive to the asymptotic behavior of the wave-function, i.e. (see for instance [9])

$$\Psi_{\text{scat}}(\mathbf{r}) \xrightarrow{r \rightarrow \infty} e^{ikz} + f(\theta, \varphi) \frac{e^{ikr}}{r}$$

where we recognize the superposition of the incident wave-function of a particle with an energy $E = \frac{\hbar^2 k^2}{2\mu}$ and a scattering wave. Then, the cross section detected at given angles θ and φ (where these angles correspond to spherical coordinates, z being the beam axis), is related to the scattering amplitude through the simple relation $\sigma_k(\theta, \varphi) = |f(\theta, \varphi)|^2$. Taking advantage of the nuclear inter-

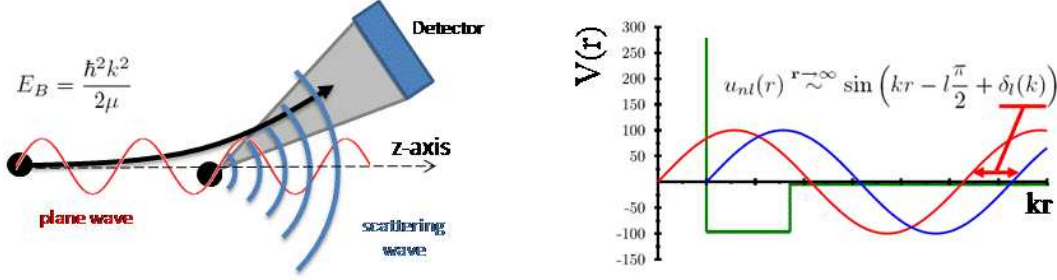


FIG. 4: (Color online) Left: Schematic illustration of the diffusion of one nucleon onto another nucleon with incident energy $E = \frac{\hbar^2 k^2}{2\mu}$. Here, μ is the reduced mass of the collision. Right: Illustration of the effect of the interacting potential. Here a schematic potential is used, the red curve is the free nucleon case while the blue curve corresponds to the eigenstate with two-body interaction. The phase-shift $\delta_l(k)$ is directly related to the difference between the asymptotic behavior of the two curves.

action spherical symmetry, the scattering wave-packet can be further decomposed on eigenstates with good angular momentum as

$$\Psi_{\text{scat}}(\mathbf{r}) = \sum_{nlm} c_{nlm} \frac{u_{nl}(r)}{r} Y_{lm}(\theta, \varphi).$$

We are then left with the study of the asymptotic behavior of the $u_{nl}(r)$ that is usually written as

$$u_{nl}(r) \xrightarrow{r \rightarrow \infty} \sin\left(kr - l\frac{\pi}{2} + \delta_l(k)\right)$$

where $\delta_l(k)$ denotes the phase-shift at a given l and energy (note that here for the sake of simplicity, spinless particles have been considered). The physical meaning of the phase-shift is illustrated in figure 4 (Right). As a final result, the cross-section integrated over the angles can be simply written as

$$\sigma(k) = \frac{4\pi}{k^2} \sum_l (2l + 1) \sin^2[\delta_l(k)].$$

In practice, phase-shifts are extracted from experimental analysis of nucleon-nucleon cross sections at different energies. Then parameters of the nucleon-nucleon interaction are adjusted to best reproduce their behavior. An example of result obtained with a conventional nuclear force, namely AV18 [10], is shown in figure 5 for various channels. A detailed discussion on different refinement to adjust forces can be found in [7].

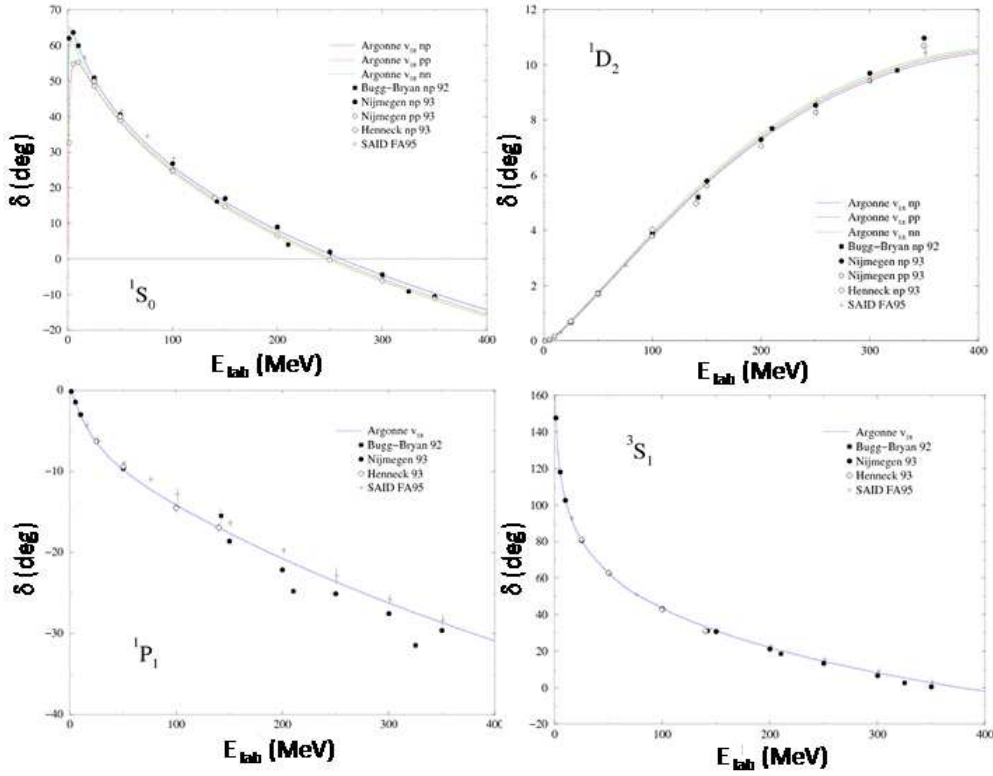


FIG. 5: (Color online) Illustration of a comparison between experimental and theoretical phase-shift as a function of laboratory colliding energy. The theoretical curves have been obtained with the AV18 interaction (Adapted from [10]).

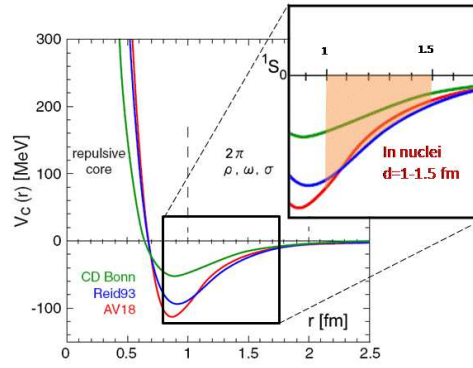


FIG. 6: (Color online) Comparison between three different conventional forces in the 1S_0 (spin singlet and s-wave) channel: CD Bonn [11], Reid93 [12], and AV18 [10]. The insert shows the difference for scales comparable to the relative distance between nucleons inside nuclei.

C. Critical discussion

Conventional nuclear forces have been adjusted for many years to reproduce scattering experiments and, most often, also deuteron properties. Several forces have been proposed that are dedicated to the same purpose. However, a direct comparison of these forces between each others

shows significant differences. This is illustrated in figure 6 where three different parameterizations are compared. These differences stem from different strategies used to design the forces themselves. One of the ambiguity of conventional forces is the absence of a constructive framework that would lead to a systematic improvement of the nuclear interaction and remove some of the ambiguities. Such a constructive framework is provided by the Effective Field Theory combined with Chiral perturbation theory (see discussion below).

A second difficult aspect, that is not specific to conventional nuclear forces, is related to in-medium effects. When two nucleons are in interaction inside a nucleus, the bare interaction is modified due to the presence of other surrounding nucleons. In figure 7, a schematic illustration of what are in-medium effects is given. When two nucleons are surrounded by other nucleons, the associated wave function looks very much like an independent particle case. This stems from a combined effects of the Pauli principle that blocks accessible configurations for the two nucleons and the properties of the force itself. This simple analysis has been used for instance to reconcile on one hand the strength of the interaction and the independent particle picture validity in nuclei (for further discussion see [13, 14]). It can be conclude from figure 7 that in-medium effects largely modify the interaction that is felt by nucleons compared to the original bare interaction and a proper dressing of the interaction is mandatory. Due to the repulsive core, specific methods based on the Bethe-Goldstone approximation have been developed [15]. These methods are rather involved and render the use of bare interactions in low energy nuclear model difficult. As we will see below, the recent introduction of soft interactions provides a suitable solution to these difficulties.

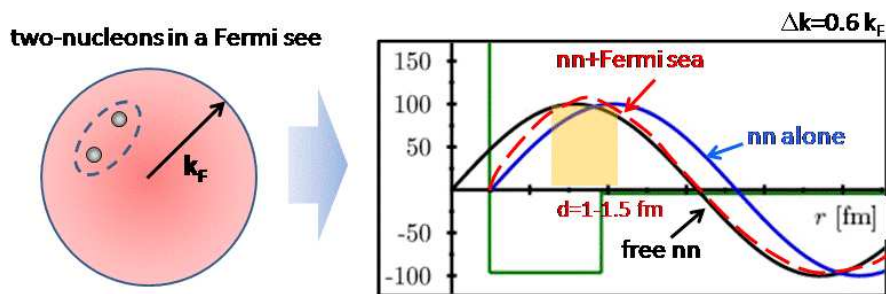


FIG. 7: (Color online) Left: Two nucleons plunged in a Fermi sea are considered. Only the interaction between the two nucleons is assumed to be non-zero, the surrounding nucleons affect the two nucleon subsystem through the Pauli principle by blocking accessible configurations. Left: a schematic nucleon-nucleon interaction is considered. The different curves correspond respectively to the two nucleon wave-function obtained for the free nucleon case (black solid line), i.e. without interaction and without the Fermi sea, the isolated two nucleon case (blue solid line), i.e. the two nucleon interacting through the bare interaction without the Fermi sea (this case is called "nn alone" in the figure), the two interacting nucleons plunged into the Fermi sea (green dashed line). As can be seen from the figure, contrary to the "nn alone" case, when the nucleons are plunged in a nuclear medium, the corresponding wave-function tends to the free nucleon case for $r > d$, where d is known as the healing distance.

III. FROM QUARKS TO NUCLEONS: NEW ASPECTS

Modern theories of nuclear forces aim at developing nucleon-nucleon interaction directly from the underlying QCD Lagrangian, denoted hereafter by \mathcal{L}_{QCD} . Here, basic but important notions on quarks and their organizations into mesons and hadrons are first recalled. Then, highlights on the different strategies used to derive NN interaction from QCD are given.

A. Reminder on basic aspects on quarks, mesons and baryons and Lattice QCD

Mesons and baryons are made from the aggregation of two or three quarks (or anti-quarks) respectively interacting through gluons. In figure 8, the constituents of different particles under interest are recalled. Low energy nuclear physics mainly focus on particles formed from quarks u (\bar{u}) and d (\bar{d}) forming for instance π , neutrons and protons. In Hyperons, that will be discussed below, one of the up or down quarks is replaced by a s quark.

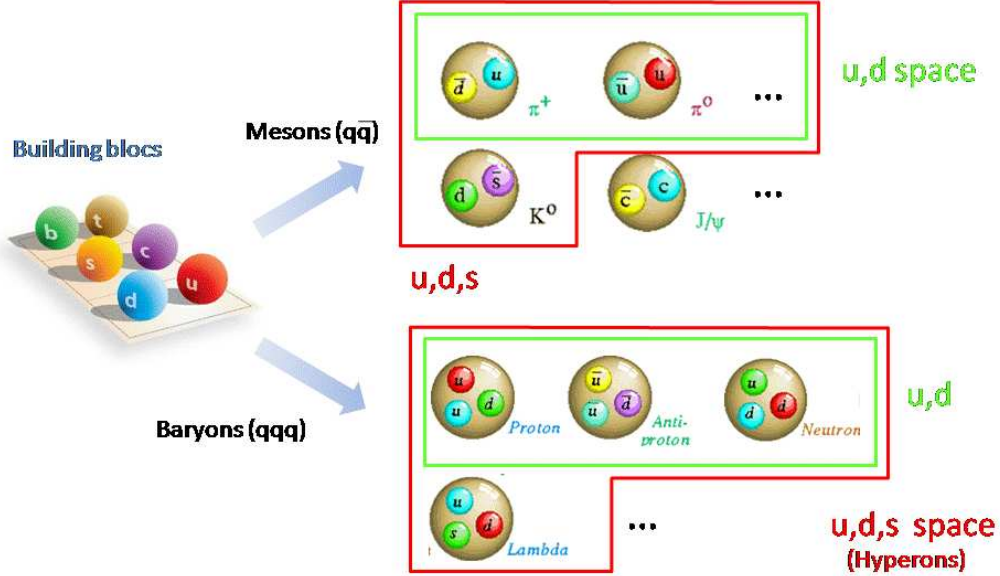


FIG. 8: (Color online) Illustration of the different quarks forming mesons and baryons.

The QCD Lagrangian writes as:

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{QCD}}^0 - \bar{q}Mq, \quad (2)$$

where the first part corresponds to the Lagrangian with massless quarks while the second part stands for the mass contribution. As far as the strong interaction is concerned, the different quarks (u, d, s) have identical properties, except for their masses. The quark masses are free parameters in QCD, i.e. the theory can be formulated for any value of the quark masses. One of the difficult aspect of QCD physics is that it is highly non-perturbative in the low energy regime. For many years, Lattice QCD approach, where quarks and gluons fields are considered on a discretized mesh [16, 17], has been thought as the only framework suitable for this problem. Lattice QCD calculations require large computational power. As a consequence, first lattice QCD calculation of the NN interaction have only recently been performed [1] (see figure 9). Lattice QCD application are numerical rather involved and approximations are still necessary to be able to obtain NN interaction like the one displayed in figure 9.

B. Quark masses, Chiral symmetry and Effective Field Theory

The introduction of Effective Field Theory has open an alternative to Lattice QCD to derive nucleon-nucleon interaction from the QCD Lagrangian. To understand recent developments on nuclear forces it is crucial to keep in mind the order of magnitude of quark masses:

$$\begin{array}{l} m_u \sim 2.4^{\pm 0.9} \text{MeV} \\ m_d \sim 4.75^{\pm 1.25} \text{MeV} \\ m_s \sim 100^{\pm 30} \text{MeV} \end{array} \left| \begin{array}{l} \text{Energy} \\ \text{GAP} \sim 1 \text{ GeV} \end{array} \right. \begin{array}{l} m_c \sim 1270_{-110}^{+70} \text{MeV} \\ m_b \sim 4200_{-70}^{+170} \text{MeV} \\ m_t \sim 171200^{\pm 2100} \text{MeV} \end{array} . \quad (3)$$

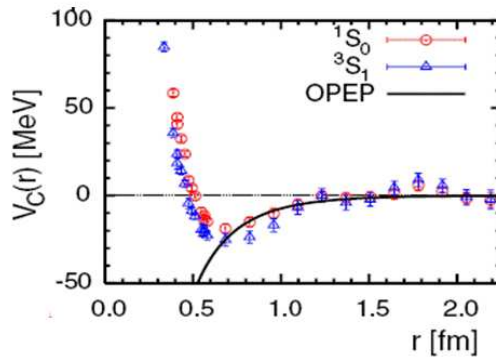


FIG. 9: (Color online) The lattice QCD result of the central (effective central) part of the NN potential in the 1S_0 and 3S_1 channel. The solid lines correspond to the one-pion exchange potential (OPEP) [1].

Several important comments can be made from these values: (i) Masses of (u, d, s) (called light quarks) are rather small even compared to the nucleon mass (ii) There exist a gap in energy around 1 GeV between these quarks and the (c, b, t) quarks (called Heavy quarks).

1. Chiral symmetry breaking and its consequences

Let us first, for simplicity, neglect the effect of (c, b, t) quarks and focus on the subspace (u, d, s) which is a priori relevant for low energy nuclear physics. Assuming that quarks are massless, the second part in Lagrangian (2) cancels out. One important properties of $\mathcal{L}_{\text{QCD}}^0$ is that it is invariant under Chiral symmetry. Because quarks have masses, the Chiral symmetry is explicitly broken, i.e. chiral transformation induces a mixing of Left-Handed (LH) and Right-Handed (RH) quarks. Chiral symmetry breaking plays a special role in the properties of nuclear forces, see illustration in figure (10). Chiral Perturbation Theory takes advantage of the smallness of (u, d, s) masses by treating the second term in equation (2) directly as a perturbation.

2. Effective Field Theory and Soft interactions

In the discussion above, (c, b, t) quarks have been completely disregarded. The main argument to do so is that the energy scale under interest is much smaller than the typical energy of these quarks. Effective Field Theory provides a framework to properly disregard some degrees of freedom when two separated scales (here in energy) coexist in a physical problem.

The idea behind EFT is to keep unchanged the physic at the energy scale under interest while providing more and more accurate approximations for the high energy scale [18, 19]. The strategy used is depicted in figure 11 for a toy model introduced by Meissner in [20]. As a result of the EFT, the interaction with a repulsive hard-core is replaced by a smooth interaction while the physics within a given range of energy, typically $\vec{q} \ll \Lambda$, is left unchanged.

The EFT has been used starting from the QCD Lagrangian to provide a systematic and constructive framework for the strong interaction, i.e.

$$\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}_{\text{EFT}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \dots \quad (4)$$

In that case, the typical energy range of interest is $|\vec{q}| \sim M_\pi = 140$ MeV, while the cut-off parameter is of the order of the gap in energy that separates the (u, d, s) quarks from the others, i.e. 1 GeV. An example of phase-shift analysis obtained using EFT up to $N^3\text{LO}$ (next-to-next-to-next to leading order) is shown in figure 12. From this figure, it can be concluded that the calculated phase-shift converges to the experimental one as more and more orders in the expansion are included.

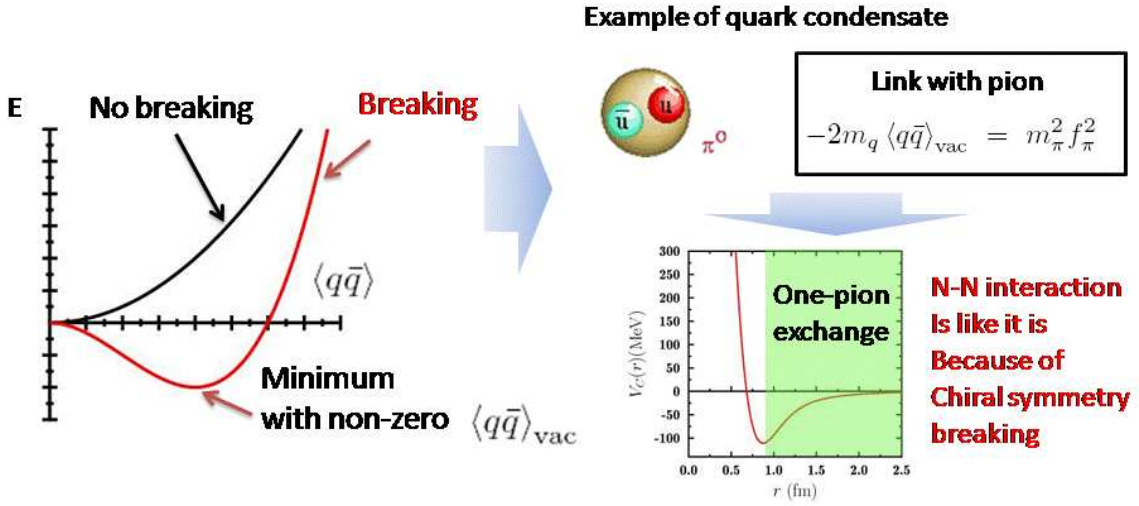


FIG. 10: (Color online) Left: Schematic illustration of the chiral symmetry breaking effect. As always, when a symmetry is broken one could defined an order parameter, here the quark-antiquark condensate $\langle q\bar{q} \rangle$. Without symmetry breaking, the energy landscape (here shown in one dimension for simplicity) has a minimum for zero value of the order parameter. When the symmetry is broken a minimum appear for non-zero value of $\langle q\bar{q} \rangle$. Right: the value of $\langle q\bar{q} \rangle$ at the minimum is directly related to the pion mass which is itself the main actor contributing to the long-range part of the strong interaction.

Therefore, on opposite to conventional forces, EFT provides a systematic framework to obtain more and more accurate approximations for the strong nuclear interaction starting directly from QCD. In addition, it also gives consistent three-body, four-body interactions automatically (see figure 12 (right)).

An important aspects brought by EFT is the possibility to obtain interaction optimized for a specific energy scale. New interactions based on chiral EFT and discussed above still contain information on high energy that is not necessary for low energy nuclear physics. A second class of interactions, specifically dedicated to nuclei have been introduced, the so called "soft interaction" or "low momentum interaction. This time, starting from either conventional interactions or interaction deduced from EFT, new interaction have been obtained using renormalization group (RG) techniques [21] while paying attention to keep the phase-shift at low energy unchanged when reducing further the cut-off compared to Λ_{QCD} .

The result that has been obtained is quite amazing. When the RG is systematically applied to different forces on the market, all interactions that were so different apparently lead to the same "universal" low momentum interaction. The conclusion from this study is that all forces were containing the same low energy information while the differences certainly stem from the high energy components which seems not to be constrained enough. These new soft interactions, besides removing the hard-core difficulty, also provide a suitable answer to the "non-uniqueness" puzzle of conventional forces.

IV. FROM NUCLEONS TO NUCLEI, HYPERNUCLEI AND STARS

Conventional nuclear forces were suffering from three major drawbacks (i) lack of systematic constructive framework, (ii) non-uniqueness, (iii) difficulty to use due to the hard-core. As discussed above, recent advances in the theory of nuclear forces overcome these difficulties. New forces, especially soft ones, are expected to be much easier to use. This has been nicely illustrated in infinite nuclear matter. Up to know, the presence of a hard-core has made any attempt to use the strict independent particle (Hartree-Fock or Hartree-Fock Bogolyubov) approximation to the

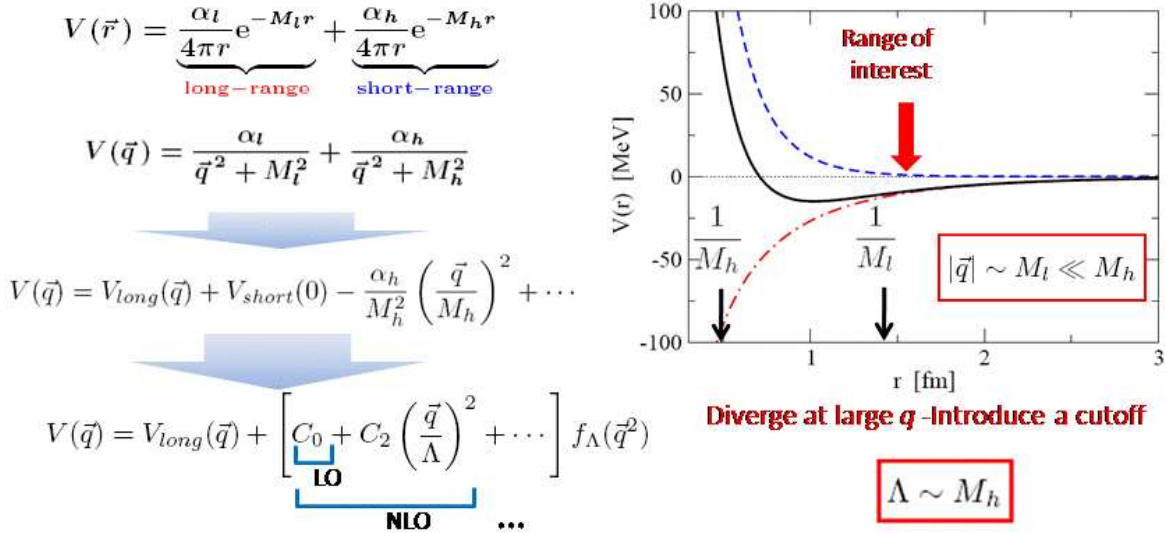


FIG. 11: (Color online) Illustration of the different steps used in Effective Field Theory on a schematic interaction (taken from ref. [20]). Here, the interaction (shown in the right side with solid line) writes as a sum of a long-range (dot-dashed line) and a short-range (dashed line) interaction respectively associated with the energy scale M_l and M_h . Within EFT framework, the short range contribution is expanded in powers of (\vec{q}/M_h) , where \vec{q} denotes the momentum transfer during the nucleon-nucleon interaction. Such an expansion is expected to be valid at small \vec{q} compared to the typical energy scale associated to the short-range interaction. Direct Taylor expansion of V_{short} presents a ultra-violet divergence as \vec{q} increases. To avoid this divergence, a smooth function f_Λ , associated to a cut-off parameter Λ is introduced, where $\Lambda \sim M_h$. Often, the expansion is directly made as a function of (\vec{q}/Λ) . The accuracy of the expansion depends of the expansion order, namely Leading Order (LO), Next to Leading Order (NLO) ...

exact nuclear many-body problem useless. The recently proposed soft-core interactions, however, seem to make the nuclear many-body problem perturbative [3, 22]. The result of Hartree-Fock calculation for nuclear matter equation of state using the V_{lowk} interaction (left) and HF + second perturbation theory + particle-particle ladder (right) are shown in figure 14 (taken from [22]). Two important remarks should be made at this point: (i) while no reasonable results could be obtained using conventional forces with Hartree-Fock, a stable solution is obtained here already at this level of approximation (ii) Perturbation theory converges rather fast to the expected saturation point. Note that similar results can eventually be obtained with conventional forces but using much more involved many-body techniques.

A. Ab initio calculation

Figure 14 illustrates that modern interactions, besides providing a direct connection with the underlying QCD Lagrangian, might also greatly simplify low energy nuclear physics calculations. In recent years, these forces have become a tool of choice for ab initio methods in light nuclei [23]. With the increasing computational power, several theories have been recently developed to provide ab initio calculations of light nuclei properties, like for instance the No-Core Shell Model (NCSM) [24], the Green-Function Monte-Carlo (GFMC) [25, 26], the Coupled-Cluster (CC) [27, 28]... A complete description of these theories is out of the scope of the present introduction. Nevertheless, figure 15 illustrates schematically some of these theories. During the past decades important efforts have been made to make ab initio methods reliable. A benchmark of several methods on the $A = 4$ [29] has shown that all methods are well under control and provide the same results if the same interaction is used as an input. This is illustrated in figure 16 where comparisons between NCSM

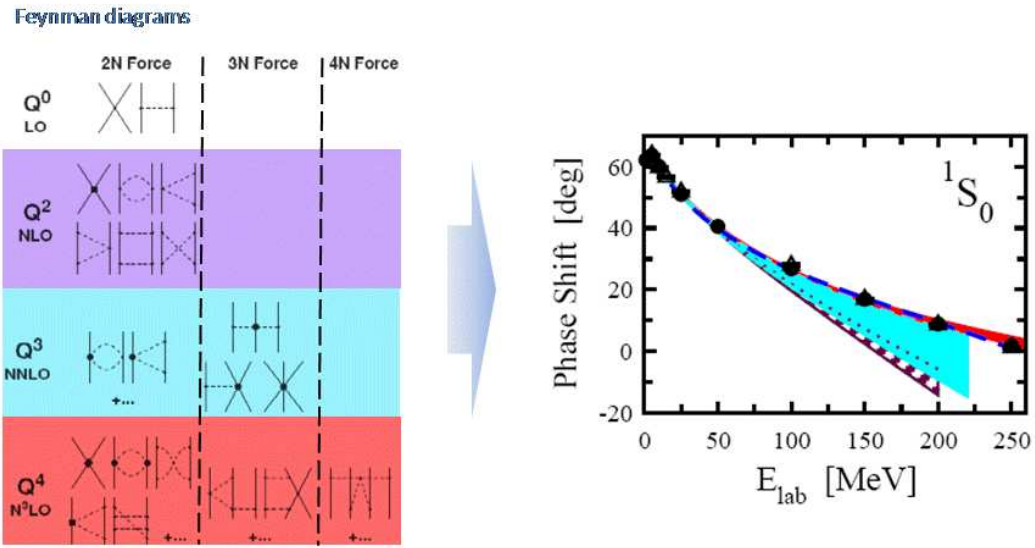


FIG. 12: (Color online) Left: Feynman Diagram representation of the different orders of EFT expansion. Right: Experimental phase-shift as a function of energy. The result of the calculation obtained at each order of the expansion are shown (from ref. [2]). Different colors correspond to different order shown with the same color convention on the left.

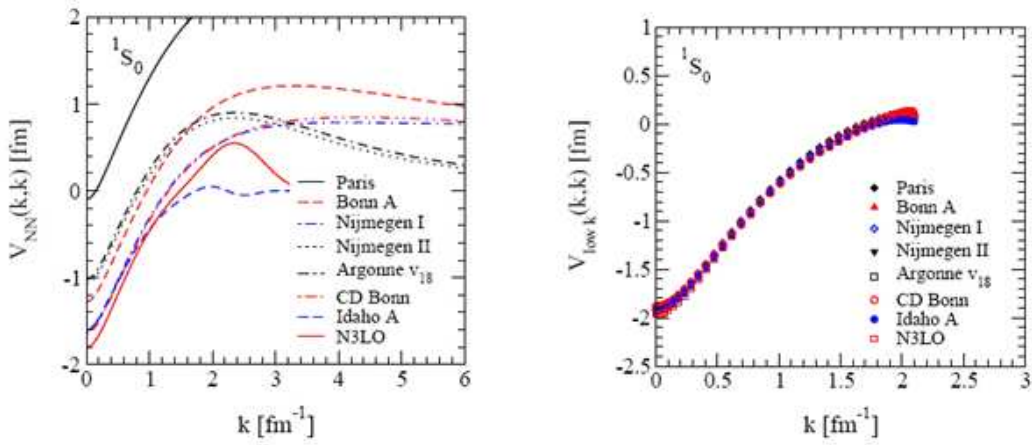


FIG. 13: (Color online) Left: Different conventional interaction as well as interaction obtained from EFT represented in momentum space. Right: New interactions deduced from the renormalization group procedure where the phase-shift has been constrained to remain unchanged [3]. In the latter case, all interactions that were originally completely different match with each other.

and GFMC results are shown. Ab initio methods have reached a high degree of accuracy and can now serve as a benchmark for nuclear forces.

B. Light nuclei and three-body forces

The actual trend of theoretical light nuclei studies is now to use a fully consistent bottom-up approach starting from QCD, deducing from it a bare nucleon-nucleon interaction dedicated to low energy nuclear physics and performing exact nuclear structure calculations. Recent discussions on

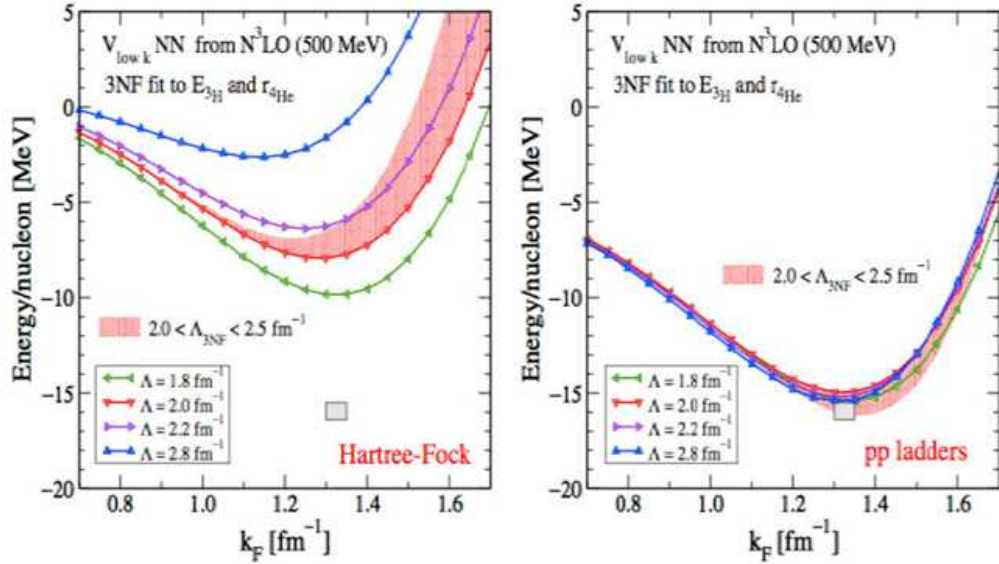



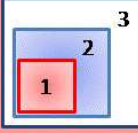


FIG. 14: (Color online) Energy per nucleon in symmetric nuclear matter as a function of the Fermi momentum calculated at the Hartree-Fock level (left), and including second order perturbation theory + particle-particle ladder contributions (right). The soft interaction used here is derived from the $N^3\text{LO}$ nucleon-nucleon interaction. In both cases, different curves correspond to different cut-off parameters (for more detail see [22]) Note that, a three-body force deduced from light nuclei properties is also used here (see discussion below).

the three-body force reflects the great interest of such a bottom-up approach. Applying ab initio methods to light nuclei with two-body forces only cannot reproduce experimental observation. This is shown in Figure 17 presenting the experimental or theoretical binding energy of the alpha particle versus the same quantity for the triton [31]. As seen in this figure, without 3-body interaction, both alpha and triton binding energy are underestimated while if it is included, the experimental data can be reproduced. It is worth mentioning that conventional forces discussed in section II, were already not able to reproduce light particle binding energy. However, due to the flexibility of such forces it is difficult to have a definite answer about the three nucleon interaction. In particular, one may have thought that a slight modification of the two nucleon force can eventually improve the agreement with experiments. The introduction of modern forces based on EFT leaves less room for adjustment showing that a 3-nucleon force is definitely missing. An illustration of most recent ab initio calculation starting from chiral perturbation theory and EFT up to $N^3\text{LO}$ is shown in figure 18. The agreement of the full 3-body calculation with the experiment show the amazing degree of accuracy of modern forces. From these different curves, one may also estimate the importance of different forces components contribution. In table I, the percentage of contribution of the two and three-body forces on the binding energy of triton and alpha particle are estimated. Using the difference between the calculated alpha particle binding energy and the experimental one, an upper estimate of a possible four-body force is also given.

C. Hypernuclei

A similar bottom-up strategy is also being followed to provide ab initio estimates for Hypernuclei starting from QCD ingredients. As discussed earlier, hyperons correspond to nucleon where one of the u or d quarks is replaced by a s quarks. Therefore, all the discussion made above should be

Name	Short description	Scale as	Up to
Few-body (Faddeev...)	$H\Psi = E\Psi$ 	M^A	$A = 2-4$
Green-Function Monte-Carlo (GFMC)	$\Psi(\tau) = e^{-(H-E_0)\tau}\Psi_T,$ $= [e^{-(H-E_0)\Delta\tau}]^n \Psi_T$ + auxiliary field 	$M!$ $(M-A)!A!$	$A < 12$
No-core Shell Model	$H\Psi = E\Psi$ 	4^A	$A < 16$
Coupled-Cluster (CC)	$ \Psi\rangle = e^S \Psi_0\rangle$ $S = S_1 + S_2 + \dots$ 	$(M-A)^4 A^2$	$A < 100$ Specific nuclei only

M : configuration space size

FIG. 15: (Color online) Schematic illustration of the Faddeev-Yakubowski, the Green-Function Monte-Carlo (GFMC), the No-Core Shell Model (NCSM) and the Coupled-Cluster (CC) theories (from top to bottom). In all cases, the basic equation that is used is shown as well as a logo summarizing the method, the computational cost in terms of system size and configuration space size as well as the actual (or estimated for the CC case) range of application are given in the last two rows respectively.

	triton	alpha
2-body force	92 %	89 %
3-body force	7 %	10 %
4-body force	-	< 1 %

TABLE I: Estimate of the percentage of contribution of 2-, 3- and 4-body forces in the binding energy of triton and alpha particles. Percentage here are obtained by using the formula $(E_{\text{Cal}} - E_{\text{exp}})/E_{\text{exp}}$, where E_{Cal} correspond to the calculated binding energy at different levels of approximation.

extended to the (u, d, s) quark space. Fortunately, the s quark mass (see (3)) is also much below the QCD gap (1 GeV) and one can take advantage of the EFT machinery to provide hyperon-nucleon or hyperon-hyperon interaction [33].

An illustration of calculated hyperon-nucleon cross section obtained from chiral EFT is presented in figure 19 and compared with experimental cross sections. These cross-sections have large errorbars due to the limited number of data set (around 30-40) existing for hyperons compared to the nucleon-nucleon case (between 2000 and 3000). Results obtained in ref. [33] show that at NLO, the theoretical prediction are already in rather good agreement with experimental data. In figure 19, an example of ab initio calculation for the hypertriton is also shown (lower right panel) giving a binding energy within the experimental errorbars.

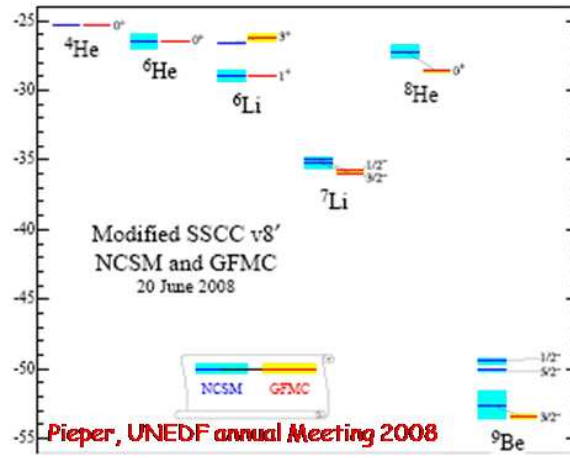


FIG. 16: (Color online) Comparison of results obtained with the NCSM and GFMC [30] for nuclei with $A \leq 9$.

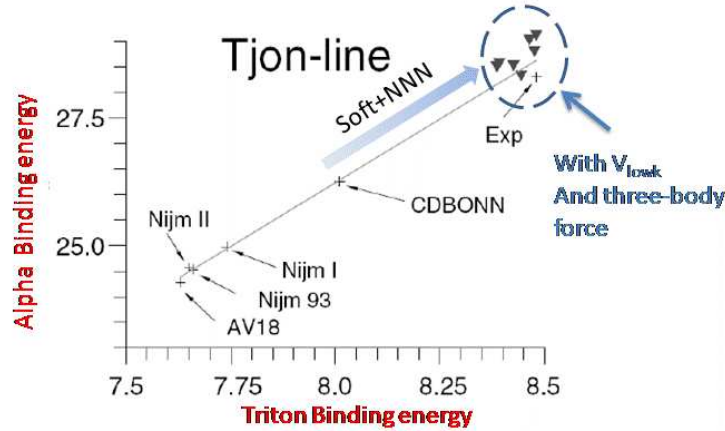


FIG. 17: (Color online) Experimental and theoretical binding energy of the alpha particle versus the same quantity for the triton [31]. Different theoretical points correspond to calculations with different nucleon-nucleon interaction. Solid triangles correspond to calculations using soft interaction and 3-body nuclear force.

D. New Trends in the description of medium and heavy nuclei

Due to their numerical complexity, ab initio methods are limited to rather light nuclei. One of the great challenges of today's nuclear physics is to provide a microscopic theory able to describe nuclei over the whole nuclear chart. Considering the first nuclear structure problem, two theoretical frameworks have become standard tools for low energy nuclear physicists, namely the Shell Model [5] and the Energy Density Functional theory [6, 34]. Figure 20 summarizes the range of applicability of most popular nuclear theories. Up to now, nuclear models contain a large number of phenomenological inputs (monopole interaction, residual interaction, components of the functionals ...) largely adjusted to reproduce experiments. As a result, the link with the underlying bare nucleon-nucleon interaction is often lost. In the meantime, recent studies have pointed out the lack of predicting power for not yet experimentally explored areas of the nuclear chart. Two strategies

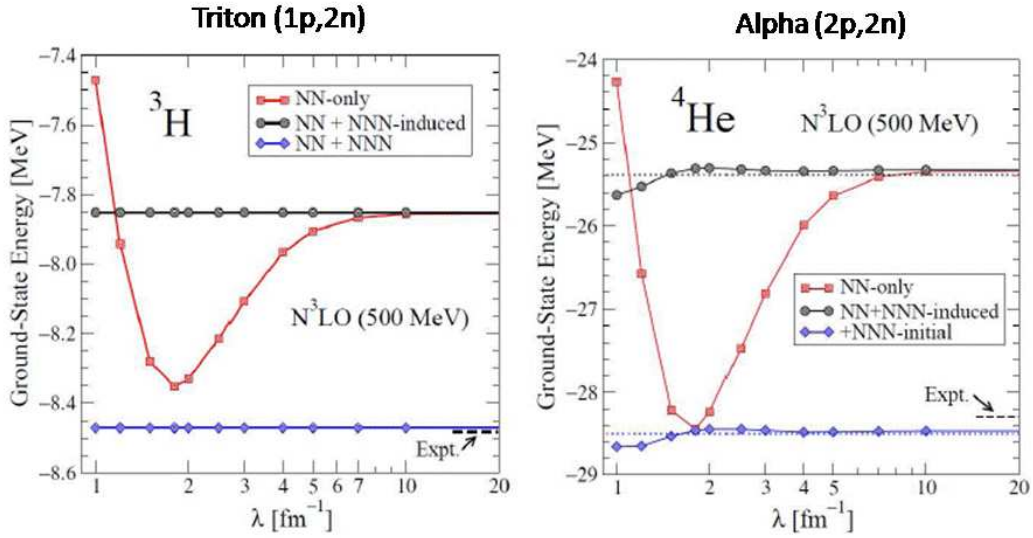


FIG. 18: (Color online) Example of recent application using modern forces described in section III combined with Similarity Renormalization Group (SRG) as a function of the SRG parameter λ (for more details see [32]). Left: triton binding energy, Right: alpha binding energy. When only two body interaction are included, an induced three-body interaction is already present coming from two-by-two interaction. The filled circles and filled squares correspond respectively to calculation using two-body interaction only with and without the induced 3-body force. Filled diamonds correspond to calculation when both the induced and bare three-body interaction are accounted for. Note that normally, for the alpha particle case, an induced 4-body interaction coming from the two and three-body force should a priori be included.

exist to overcome this difficulty: (i) either, new experiments specifically dedicated to unexplored region are performed in order to better constrain phenomenological inputs of nuclear models (ii) or new approaches, following the bottom-up spirit, where the connection with the bare interaction and QCD is kept, are developed.

In this section, recent examples demonstrating the new perspective offered by modern nuclear forces and using the second strategy are illustrated.

1. Nuclear Shell Model

a. Basic discussion on Shell Model: Here, the Shell Model approach is presented in a very schematic way and readers interested in all the subtle aspects of this approach can refer to the excellent recent review articles [5, 38, 39]. Standard Shell model starts from a single particle basis and construct from it Many-Body waves function written as

$$\Psi = \Phi_{[0p0h]} + \Phi_{[1p1h]} + \Phi_{[2p2h]} + \dots \quad (5)$$

where $\Phi_{[0p0h]}$ denotes the independent particle state, while $\Phi_{[1p1h]}$, $\Phi_{[2p2h]}$... denotes the mixing of different 1 particle-1hole, 2 particle-2 hole ...excitations. Many-Body states are obtained by direct diagonalization of an effective two-body Hamiltonian, that is schematically written as:

$$H = \text{SPME}(\varepsilon_i) + \text{TBME}(G) \quad (6)$$

where SPME and TBME stands respectively for Single-Particle and Two-Body Matrix Elements, denoted respectively by ε_i and G_{ijkl} . Due to the numerical complexity, standard Shell Model generally separates the single-particle space into an inert core and valence states that participate to correlations (see illustration 21). Accounting for the interaction symmetries, within a finite space

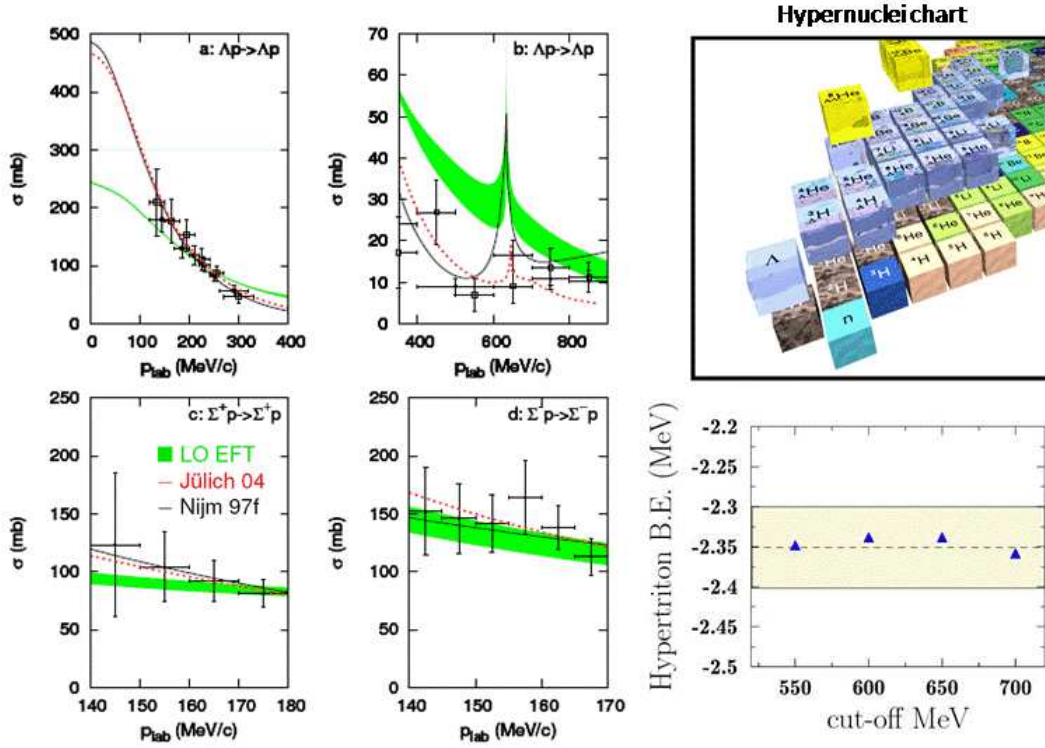


FIG. 19: (Color online) Left: Comparison of measured hyperon-nucleon cross-section with calculated ones estimated using EFT up to NLO (taken from [33]). Right-top: the hypernuclei chart. Right-bottom: Example of result of ab initio calculation for hypertriton. The dashed area corresponds to the experimental value with errorbars, while filled triangle are calculated values as a function of the cut-off parameter.

of single-particle, we are left with a finite number of input parameters $\{\varepsilon_i, G_{ijkl}\}$. In conventional Shell Model single particle energies are generally directly adjusted to reproduce experiments. G_{ijkl} matrix elements can a priori be deduced from the G-matrix theory starting from a bare interaction that is dressed to properly account for medium effects. However, direct use of G matrix components do not lead to satisfactory results. For this reason, G components are in practice slightly readjusted to better reproduce a set of spectroscopic properties in a number of nuclei relevant for the selected single-particle space. For instance, in the sd -shell, this leads to 66 parameters that have been adjusted on more than 600 levels to get a deviation lower than 200 keV. In figure 22 (left), the correlation between the G matrix components directly deduced from a conventional bare interaction and their values after re-adjustment is presented [37, 38]. Corresponding effective single-particle energies (ESPE) are also shown in this figure for the oxygen isotopic chain. Two important remarks could be drawn from Fig. 22: (i) there is a strong correlation between the initial value of the residual interaction and the final one showing that the re-adjustment is small. (ii) Even a small modification of the matrix elements leads to a rather large effect on ESPE and is for instance crucial to understand the appearance of new gaps.

The Shell Model is certainly the most precise tool available to the nuclear structure community. Insight into experimental observation can generally hardly be made without this approach. The understanding of new gaps related to the discovery of new island of stability strongly depends on the SM interpretation. Conjointly, one of the goal of studying isotopic and/or isotonic nuclei chains is to be able to uncover properties of the nuclear force in the nuclear medium. For instance, change of ESPE has led to important discussion on the spin-orbit coupling or more recently on the tensor part of the interaction [4, 40]. However, at the sight of the strong dependence of the

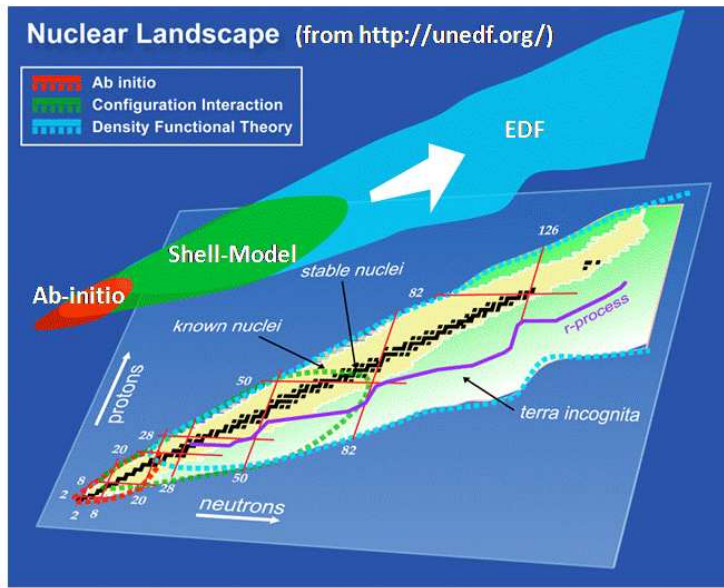


FIG. 20: (Color online) Summary of the different microscopic approaches dedicated to the nuclear many-body problem (adapted from the UNEDF (Universal Nuclear EDF) project website [35, 36]). While ab-initio methods are restricted to light nuclei, nuclear shell model is expected to describe nuclear structure in medium nuclei ($A < 100$) in the near future. Meanwhile, the EDF theory is the only theory able to treat nuclear system over the whole nuclear chart.

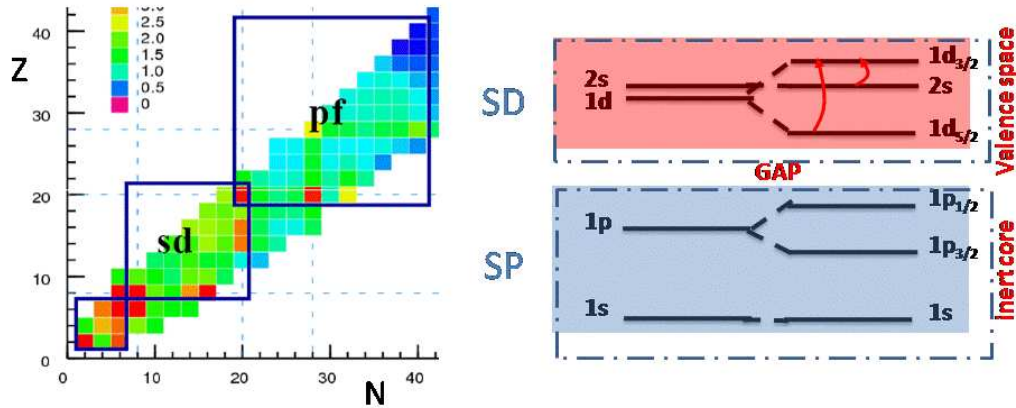


FIG. 21: (Color online) Left: Illustration of the different single-particle space generally used in the shell model (Adapted from [37]). Right: example of inert core and valence space used to describe *sd*-shell nuclei.

ESPE on the residual interaction (see figure 22) as well as the phenomenology introduced during the re-adjustment of the G matrix, one may worry about the possibility to draw precise conclusion on specific components of the interaction.

b. New trends in the nuclear shell model: The fact that a re-adjustment is needed to understand experiments reflects that part of the physics relevant for nuclear structure is missing in the bare two-body interaction input given to the Shell Model. According to recent discussions, the absence of three-body interaction in conventional Shell Model appears as a good candidate.

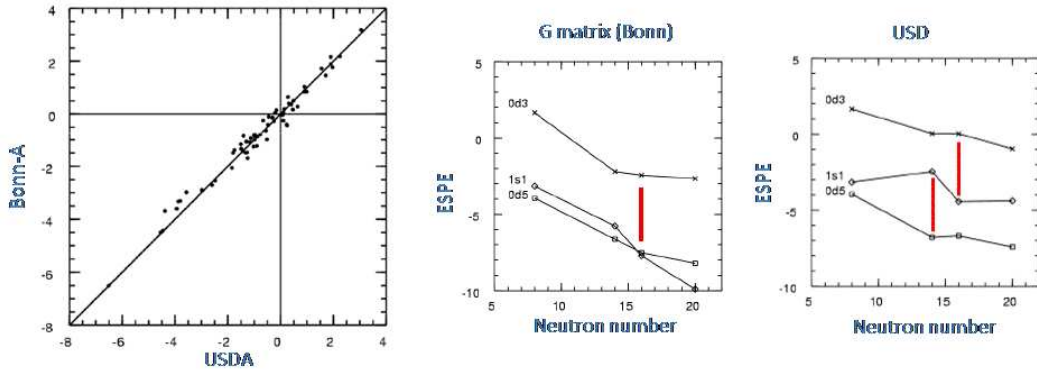


FIG. 22: (Color online) Left: Correlation between the residual interaction components obtained through the G-matrix technique starting from the Bonn interaction and the corresponding components after re-adjusting to better fit the experiments (Adapted from [37, 38]). Right: Effective Single Particle Energy (ESPE) deduced in both cases for the oxygen isotopic chain.

Taking advantage of the EFT constructive framework to provide three-body interaction (figure 12), the Shell Model theory has been recently applied using soft interaction and part of the three-body interaction (the component corresponding to a Δ excitation appearing at the N^2LO order). Corresponding ESPE are given in figure 23 and compared to ESPE obtained in phenomenological Shell-Model and/or with ESPE obtained when the three-body component is omitted (adapted from [41]). Although the application presented in figure 23 is still neglecting many components of the

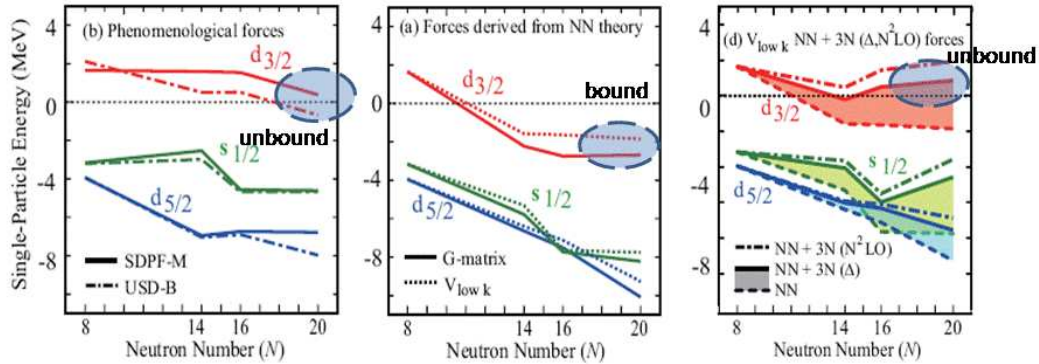


FIG. 23: (Color online) ESPE for the oxygen isotopic chain obtained in phenomenological shell model (Left), using directly the G matrix components deduced from either conventional or V_{lowk} two-body interaction (Middle) and the same G matrix components with the Δ N^2LO part of the three-body interaction (Right) (adapted from [41]).

three-body interaction that could be deduced from EFT, it paves the way for future application of the nuclear Shell-Model. It also brings many interesting aspects:

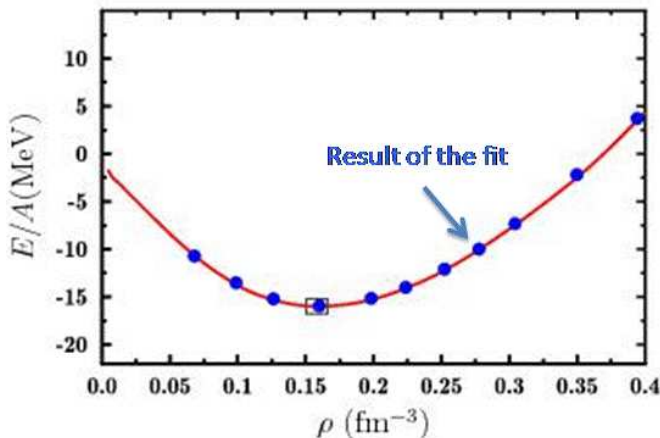
- The expertise acquired in the Phenomenological Shell-Model is crucial. Indeed, ESPE energies presented in left side of figure 23 corresponds to the best ESPE value optimized to reproduce actual nuclear data. It therefore will serve as a reference for future calculation directly based on modern forces without re-adjustment.
- Not surprisingly, new soft interaction leads sensibly to the same ESPE as conventional forces. Indeed, the same information on low energy nuclear physics is contained in both interactions.

The only expected difference is to simplify a priori the "dressing" of the force due to the absence of a hard-core.

- Although the effect of the three-body interaction is expected to be much smaller than the two-body contribution, it significantly affects the ESPE in nuclei.
- Last, the influence of the three-body interaction and the fact that (at least for the unbound state) it becomes closer to the phenomenological prescription tends to precise the re-adjustment role. Indeed, it is reasonable to conclude that the slight re-adjustment was a way to include three-body and higher-order effects that were not accounted for in the input bare two-body interaction at the first place.

The last remark also illustrates one of the drawback of phenomenological Shell-Model. Indeed by fitting directly experiments, adjusted residual interaction, even if they are very close to those obtained after the G-matrix application, mixes several effects. It is then very difficult to disentangle these effects and easy to misinterpret shell evolution. This is the challenge of future Shell Model application based on new forces and used without re-adjustment to be able to reproduce experiments and get better physical insight in nuclear structure study.

2. Energy Density Functional theory



Exercise : fit the curve with

$$E = \left\langle \frac{p^2}{2m} \right\rangle + U[\rho]$$

In nuclear matter:

$$\left\langle \frac{p^2}{2m} \right\rangle = \frac{3}{5} \left(\frac{3\pi^2}{2} \right)^{2/3} \rho^{5/3}$$

Fit with

$$U[\rho] = \sum_n c_n \rho^n$$

FIG. 24: (Color online) Consider a reference curve that gives the energy of symmetric nuclear matter as a function of its density (here, the red curve represent the ab-initio calculation performed in ref. [42]). The goal is to fit the curve by energy written as a functional of the density of an independent particle state. The first step is to write the functional as a sum of the kinetic and potential energy term. For independent particle system, the kinetic term directly express as a functional of ρ . As an illustration, it is assumed here than the potential part $U[\rho]$ simply writes as a polynomial of the density. It is straightforward to show that a fit (blue circles) with a fifth order polynomial gives a perfect reproduction of the curve.

a. Basic discussion on Energy Density Functional: The nuclear Energy Density Functional (EDF) is very close in spirit to the Density Functional Theory (DFT) introduced in electronic systems [43–46]. The great advantage of DFT (and EDF) is to establish a mapping between the original, most often intractable, many-body problem of interacting particles and a functional theory that can be solved using an independent particle method. In its simple form, where an auxiliary determinant state is used to construct the one-body density ρ , this mapping can be presented as:

$$E = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \iff \mathcal{E}_{\text{EDF}}[\rho] \iff \rho_{ij} = \langle \Phi | a_j^\dagger a_i | \Phi \rangle \iff |\Phi\rangle = \prod a_i^\dagger |-\rangle. \quad (7)$$

Historically, the introduction of functional theory was not motivated by firm existence theorem like the DFT in condensed matter but introduced more empirically having in mind the quite simple evolution of the ground state energies and universal behavior of the nuclei densities along the nuclear chart [47, 48]. Although most nuclear physics EDFs start from an effective interaction (Skyrme like [47] or Gogny like [49]), a more direct and simple illustration of EDF construction is given in figure 24 to illustrate the EDF spirit. When effective interaction are used, the parameters of the interaction are directly adjusted to reproduce properties of finite and infinite nuclear systems. This strategy is called phenomenological or empirical EDF hereafter.

The introduction of EDF in nuclear system in the early 70's was a major breakthrough. Today, the EDF is not restricted to ground state properties but is expected to provide a unified microscopic framework able to address the diversity of phenomena taking place in nuclei from nuclear structure to nuclear reactions or nuclear astrophysics: nuclear spectroscopy, small and large amplitude dynamics, equilibrium and non-equilibrium thermodynamics (see illustration in figure 25)...

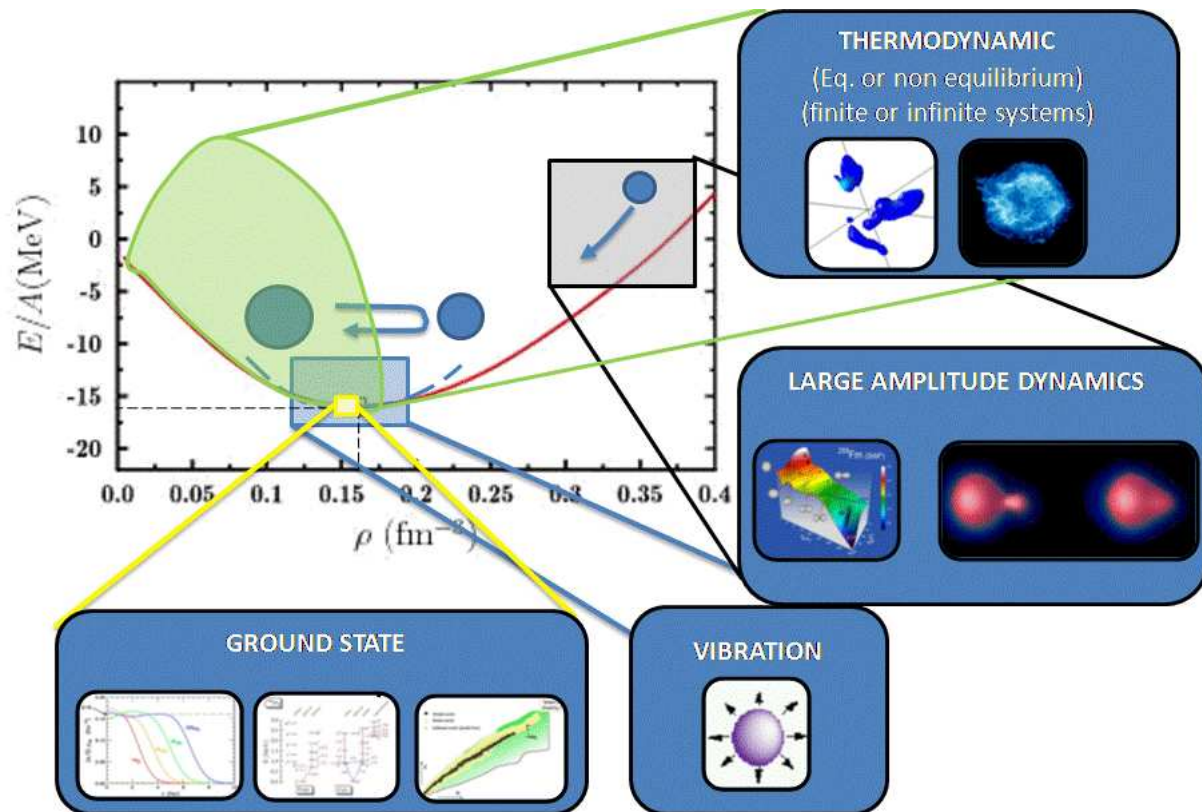


FIG. 25: **Range of application of the nuclear EDF:** (Color online) The red curve represents the equation of state in symmetric nuclear matter as a function of the system density. In the EDF theory, the energy is directly parametrized as a functional of the density. Nuclear structure study focus on or around the minimum. EDF is used extensively to get information on the ground state (yellow box) as well as excited state (see discussion in the text). It could also be used when the system is slightly shifted from the minimum leading to the onset of small amplitude vibrations (blue box). Time-Dependent version of the EDF also provides a description of nuclear dynamics like fusion or fission when the system is far away from the minimum (large amplitude collective motion [LACM]). Finally, EDF can also be extended to treat systems at finite temperature or entropy and provide information on the full phase-diagram and associated phase-transition.

Most recent EDF are able to reproduce ground state energy with a deviation around 500 – 600 keV over the whole nuclear chart which is already very good in view of the limited parameters number (less than 20) used to design the functional. Still, phenomenological EDF face the difficulty of large dispersion where no data exist (see examples in figure 26) showing the lack

of constraints in unexplored regions. Figure 26 clearly shows that the same EDF theory with different parameters values have been properly adjusted to provide a fair description of existing observations. However, they significantly differ from each others far from stability.

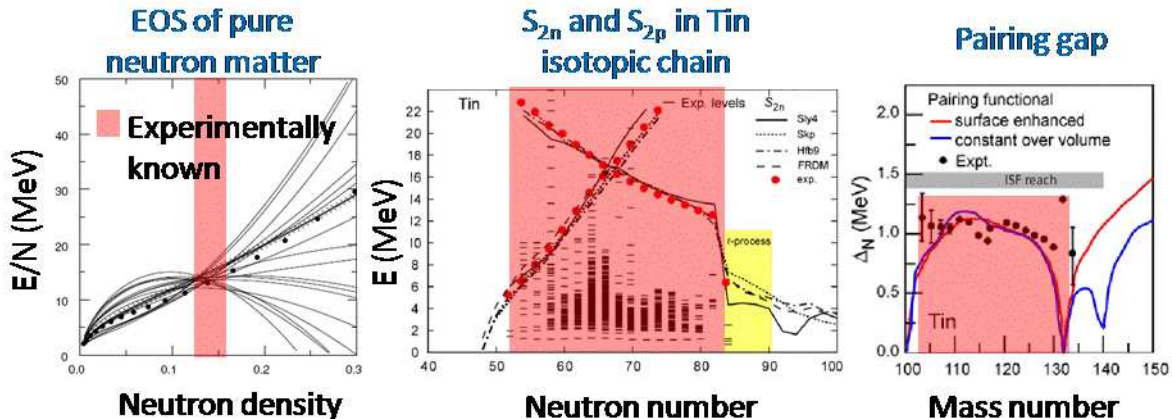


FIG. 26: (Color online) Left: Energy of the neutron matter as a function of density (taken from [50]), Middle: Two neutron and two proton separation energies along the Tin isotopic chain [51], Right: Pairing gap in the Tin isotopic chain [51]. In all cases, filled circles correspond to reference values obtained either by ab initio methods (left) or directly provided by experiments (middle and right). Well constrained regions are indicated by pink shaded area. In all cases, different curves correspond to different phenomenological approaches most often using different sets of Skyrme interaction parameters values.

b. New trends in EDF: For the future of EDF theory and to increase its predictive power, it is highly desirable to reduce this uncertainty. Two options are now being developed to reach this goal: either add new observations in unknown region by studying nuclear systems under extreme conditions (low/high density, large isospin...) or improve the theory to make it less phenomenological.

The first option goes with the use of new radioactive beam facilities that will open new perspectives for extracting specific contributions to nuclear properties. One example is the quest of symmetry energy extraction at various density. Large effort is actually devoted to this issue for instance using Heavy-Ion reactions [52–54] and/or Nuclear Astrophysics inputs [55–58].

Here, I will devote the rest of this section to the second option where the ultimate goal is to provide new EDF. Illustrations given in figure 26 point out the limitation of conventional EDF based on direct fit of experimental data. In addition, similarly to the conventional Shell-Model case, once a fitting procedure is used, it is very difficult, not to say impossible to make connection between the EDF parameters values and the bare nuclear force. For instance, in the original formulation of EDF based on Skyrme force, a three-body contact interaction is introduced. This interaction has nothing to do with the bare 3N interaction and is just a practical way to obtain a third order density dependence of the energy functional. Finally, recent pathologies observed in EDF when combined with configuration mixing (see for instance [59–61]) illustrate the lack of precise theoretical foundation for such a theory. As mentioned previously, on opposite to DFT, nuclear EDF has been introduced in a rather empirical way. This approach is being now revisited to put it on a firm theoretical ground. For instance, EDF is based on the existence of a functional of the intrinsic density for which, up to recently, an existence theorem was missing [62–64]. EDF practitioners use and abuse of symmetry breaking (particle number, spatial and rotational invariance...) to be able to grasp static and dynamical correlations in nuclei with relatively simple functional [6]. The possibility to use the symmetry breaking and the associated symmetry restoration within a functional framework is far from being trivial and is a key issue that has to be clarified in the near future [65].

Assuming that above aspects are clarified, the second challenge is to completely remove phenomenological aspects in EDF and directly connect terms entering in the functional to the parameters of the bare interaction derived from chiral EFT. Doing so gives the bottom-up picture displayed in figure 27 (left) and promotes the EDF framework to the rank of an ab initio theory. A discussion on the strategies to design ab-initio EDF taking advantage of modern nuclear forces specificities can be found in ref. [66]. Conjointly to this long term project, similarly to the Shell-Model case, attempts to combine conventional EDF with modern forces are being now developed. An illustration of the pairing gap obtained when the V_{lowk} interaction (and Coulomb for protons) is used in the pairing channel of the functional while keeping the mean-field channel unchanged (Skyrme force) [67]. Although the possibility to combine effective interaction and bare interaction

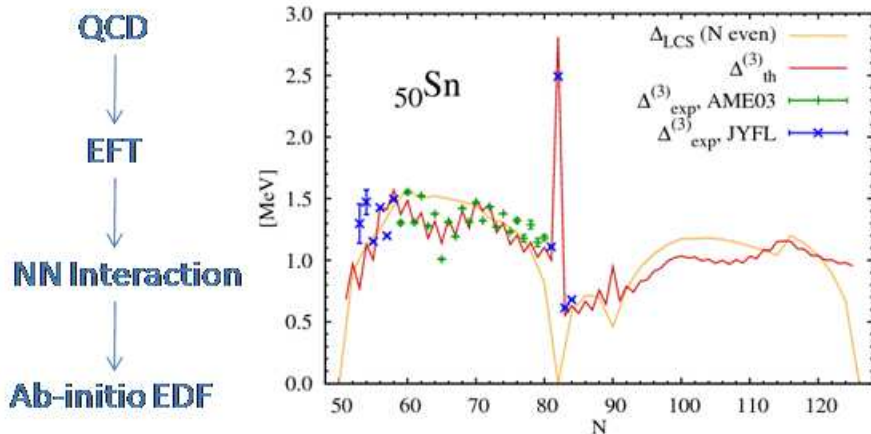


FIG. 27: (Color online) Left: Schematic representation of the full bottom-up approach connecting QCD to ab initio EDF. Right: Pairing gap in Tin isotopes obtained by directly plugging the bare V_{lowk} interaction (and Coulomb interaction for protons) into the pairing channel while keeping the mean-field channel unchanged. The final result denoted by $\Delta_{\text{th}}^{(3)}$ compares perfectly with the experimental gap $\Delta_{\text{exp}}^{(3)}$ [67].

into a single calculation is still to be clarified, the calculated gap with bare interaction is in very good agreement with experimentally known nuclei. In particular, the prediction is much better than the empirical predictions based on phenomenological contact pairing interactions. Future experiments with radioactive nuclei and the possibility to add more points on the neutron rich side will be crucial to further validate such calculations.

V. SUMMARY

In this introduction to the 2009 International Joliot-Curie School (EJC2009), recent advances related to nuclear interaction have been summarized starting from quarks and ending with finite or infinite nuclear systems. At different energy scale, selected new concepts and ideas have been discussed in a rather simple, sometimes oversimplified, way. Detailed discussion on the many facets of nuclear interaction can be found in dedicated lecture of this school. A road map of the different EJC2009 lectures as well as physical features each lecture is covering is given in figure 28. I hope that this introduction will be helpful to provide a coherent picture for the school.

Recent achievement in nuclear forces theory open new perspectives for the next decade of low energy nuclear physics, bringing together people from very different communities. Although many developments remain to be done, the possibility to directly use QCD to describe nuclear system is a major challenge that is within reach.

Acknowledgments

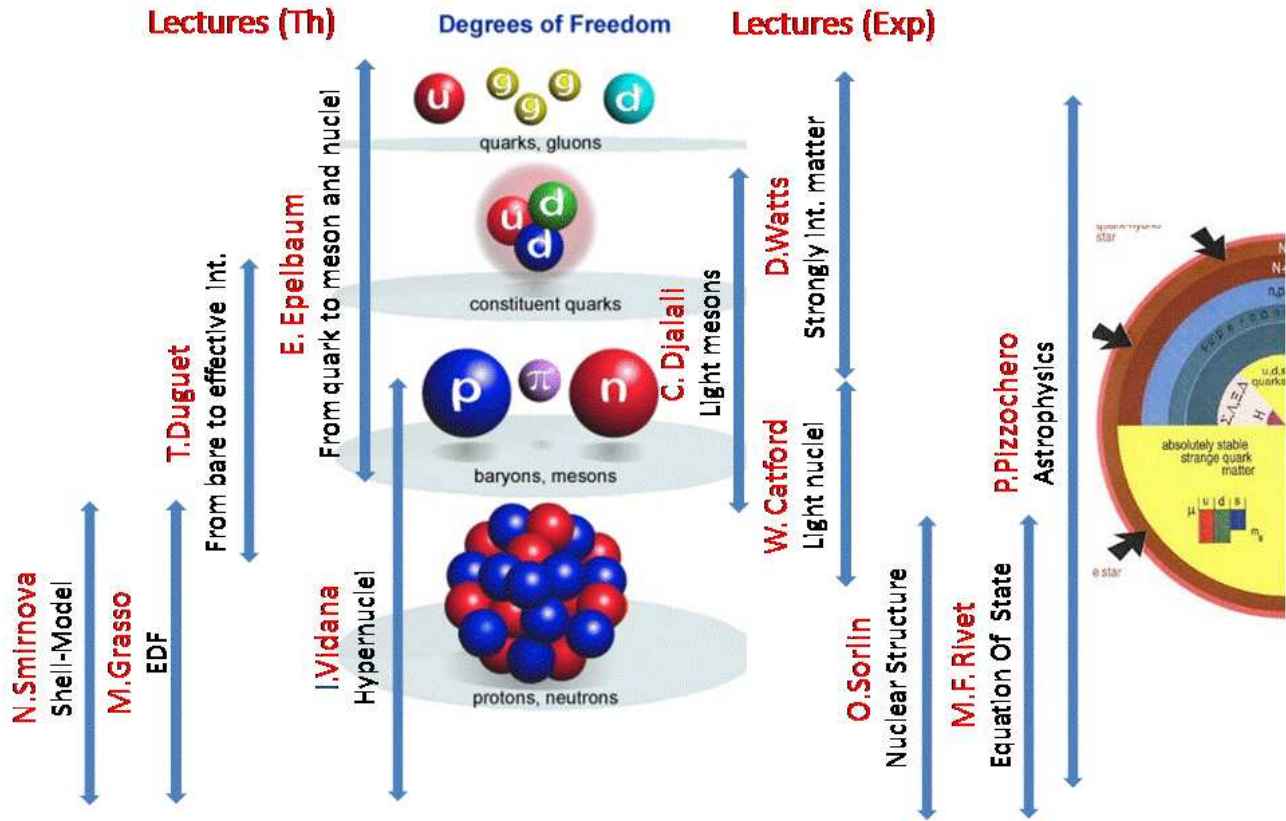


FIG. 28: (Color online) Schematic illustration of the different lectures given in the 2009 International Joliot-Curie School. Lectures displayed in the left and right side respectively correspond to more theoretical or more experimental.

I thank the organizing committee and different lecturers of the EJC2009 school for helpful discussions and for providing some of their transparencies to prepare this introduction. The author thank G. Hupin and G. Burgunder for proofreading the manuscript.

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